

The Moon's ashen light and libration in Leonardo and Galileo

Pasquale Tucci (*)

Università di Milano, Milano, Italy

Riassunto. Nell'articolo analizzo i contributi di Leonardo da Vinci (1452-1519) e di Galileo Galilei (1564-1642) all'interpretazione della luce cinerea e della librazione lunare. Entrambi fecero riferimento al più generale contesto delle relazioni tra Sole, Luna, Terra, pianeti ed evidenziarono alcune relazioni con il contesto teorico e osservativo di riferimento: quello geocentrico/Tolemaico per Leonardo e quello eliocentrico/Copernicano per Galileo. A Leonardo si attribuisce la prima interpretazione della luce cinerea. Ma essa differisce da quella Galileiana, oltre che da quella moderna. La librazione lunare è probabilmente riscontrabile in uno dei tre disegni leonardeschi della Luna, ma non fu notato dal Vinciano. Nelle immagini ad acquerello della Luna e in quelle pubblicate nel *Sydereus Nuncius* (1610) Galileo registrò, involontariamente, la librazione. Ma solo nel *Dialogo* (1632) troviamo una prima parziale descrizione e interpretazione del fenomeno. Mostrerò come le osservazioni della Luna ad occhio nudo di Leonardo non sono comparabili con quelle di Galileo eseguite con il telescopio. Il suo uso, comunque, non è sufficiente a spiegare la grande diversità tra le frammentarie argomentazioni di Leonardo e quelle molto articolate di Galileo. Solo l'emergenza di nuovi stili di pensiero e di pratiche visuali ed osservative aiutano a meglio comprendere il grande fenomeno che molti storici hanno chiamato "Rivoluzione scientifica", al fine di sottolinearne la novità nel campo del sapere filosofico e scientifico.

Abstract. In the paper I analyse the contributions of Leonardo da Vinci (1452-1519) and Galileo Galilei (1564-1642) to the interpretation of the lunar ashen light and libration. Both authors referred to the more general context of the relationships between the Sun, the Moon, the Earth, the planets and they highlighted some relationships with the theoretical and observational context of reference: the geocentric/Ptolemaic one for Leonardo and the heliocentric/Copernican one for Galileo. Leonardo is credited with being the first who interpreted the phenomenon of the ashen light for astronomical purposes. But Leonardo's interpretation differs from the Galilean one, as well as from the modern one. The lunar libration is probably found in one of Leonardo's three drawings of the Moon, but it was not noticed by the Vincian. In the watercolour images of the Moon (1609/1610) and in those published in the *Sydereus Nuncius* (1610) Galileo involuntarily recorded the libration. But only in the *Dialogo* (1632) do we find a first partial description and interpretation of the phenomenon. I will show how Leonardo's observations of the Moon with the naked eye are not comparable with those Galileo made with

(*) Retired.

the telescope. Its use, however, is not sufficient to explain the great diversity between Leonardo's fragmentary arguments and Galileo's highly articulated ones. Only the emergence of new styles of thinking and of visual and observational practices help to better understand the great phenomenon that many historians called "The Scientific Revolution", in order to underline its novelty in the field of philosophical and scientific knowledge.

1. Introduction

Many scholars in different branches of knowledge, natural philosophers, artists, laymen, included images of the Moon in their works in the century between Leonardo (1452-1519) and Galileo (1564-1642). It is evident, even to an observer not very attentive to details, that the images of the Moon that Galileo inserted in his *Sydereus Nuncius* (1610) [1, 2] had a quality and showed lunar surface appearances that only the use of a new observation instrument —the telescope— could highlight.

Reeves titled "The artist and the astronomer" the introductory chapter of her book [3] as to emphasise how, alongside the evident elements of discontinuity between artistic and astronomical practice, there were, in Galileo, elements of continuity that his biography highlights. Maffei, not unlike Reeves, following Leonardo's considerations in the *Trattato della pittura*, states: "... astronomy is not simply a consequence of optics, but of painting too." (see [4], p. 253).

The ashen light of the Moon was dealt with both by Leonardo and Galileo in the context of their reasoning about the similarity of the Earth and the Moon. The Vincian is credited with being the first who have consciously drawn and interpreted the phenomenon for astronomical purposes. The Pisan had observed with naked eyes and interpret the phenomenon at least since the 1606 and probably carried out experiments with Paolo Sarpi.

The "ashen light" or "secondary light" or "lumen cinereum" or "candore lunare" ("lunar candour") or "luce bronzina" ("bronzy light") or "Coppery light" is a faint luminosity that emanates from the dark part of the Moon near the conjunction. Currently it is attributed essentially to the reflection of Sun's rays on the Earth, especially on the clouds. The phenomenon is independent of the geocentric or heliocentric system used to interpret it. But, while Leonardo did not draw clear consequences against the Aristotelian dualism between the sublunar world, room of corruption, and the celestial world, theatre of perfection, in Galilean hands the phenomenon became an attack against the inadequacy of the Aristotelian physics.

The Moon would always show the same face towards the Earth if its orbit were perfectly circular and its shape were perfectly spherical, since its period of revolution around the Earth is equal to its period of rotation. The Moon, on the other hand, like any other celestial body orbiting around another, moves according to Kepler's laws. Its trajectory, therefore, is an ellipse travelled with variable speed. Its rotation axis is inclined by $\pm 6.8^\circ$ to the plane of orbit. These circumstances cause the Moon

to have small movements in longitude and latitude called librations: they allow us to observe more than half of its surface. Moreover, the position of an observer on the Earth's surface contributes to the lunar libration: it is also called parallactic libration. What we call parallactic libration (or diurnal libration) has been known since Ptolemy's model as lunar parallax, even though the Alexandrian astronomer gives a too large value of it. Several astronomers tried to correct the Ptolemaic value of the lunar parallax before Copernicus. Refer to the literature on the subject for an in-depth discussion of the topic [5–8]. The lunar parallax is independent of the geocentric or heliocentric system. Even in Copernicus, in fact, the lunar theory is geocentric. The analysis of the lunar libration becomes complex if we take into account the Sun-Earth-Moon system, that is, if we pass from a two-body system to a three-body one.

Galileo probably drew the libration in the ink-and-wash images of the Moon and in those ones of the Moon published in the *Sydereus Nuncius* (1610) [1, 2]. Galileo thought that the Moon did not rotate and that its revolution around the Earth occurred along a perfectly circular orbit. Therefore, as G. Righini [9, 10] argued, Galileo despite having “unwittingly” (see [9], p. 74) recorded the lunar libration in the drawings of the Moon published in *Sydereus Nuncius* (1610), gave neither a description nor an explanation. The libration led Galileo, first in the *Dialogo* (1632) [11] and later, in the Letter to Micanzio (1637) and in the Letter to Antonini (1638), when he was blind and “imprisoned” in Arcetri, to reaffirm his thought that the Moon was completely similar to the Earth, in opposition to the Aristotelian dichotomy between the sublunar and the celestial world. And in accordance with the Copernican thesis that the Moon had the same features as the Earth: “. . . as Aristotle says in a work on animals, the Moon has the closest kinship (*cognitionem* in Latin) with the Earth.” (see [12], p. 32). Copernicus' reference to Aristotle, however, would require an in-depth discussion that would take me too far from the limits of this paper. Suffice it to say here that Copernicus explained the falling of bodies by the Aristotelian theory of “natural place”, which was based on the Aristotelian distinction of “natural” and “artificial” movements. According to Ziesel, Copernicus accepted “. . . the theory of Aristotle and classical astronomy that celestial bodies move in circles and that this movement is something “natural”, whereas rectilinear motion belongs only to terrestrial bodies and is “artificial”. . . (see [13], pp. 115-116).

Moreover, to Galileo, the superiority of the Copernican system over the Ptolemaic one had an unequivocal observational basis since the end of 1610 when he observed the phases of Venus through a telescope ⁽¹⁾ [14, 15].

Subsequent developments in physical theories have challenged the concept of superiority of one system over another. In fact, in Newtonian mechanics, all frames of

⁽¹⁾ However it should be noted that the phases of Venus could be correctly deduced from the Tyconic system which hypothesized that the planets revolved around the Sun and that the Sun and the Moon revolved around the Earth placed in the centre of the Universe. Galileo was convinced that it was a pure mathematical compromise.

reference, as long as they are inertial, are equivalent. So it does not matter if we describe the universe in an inertial frame of reference centred on the Earth or on the Sun or on the centre of mass of the planetary system ⁽²⁾. What we can say at best, as Infeld suggested (see [16], p. 75) is that the inertial frame of reference centred in the centre of mass of the planetary system is more inertial than the one centred on the Earth; namely that Newton's laws are better verified in the reference frame centred on the centre of mass of the planetary system rather than in the Ptolemaic one.

But after General Relativity (GR) the inertial reference frame was not necessary: there were no differences among the Ptolemaic system, the Copernican one or the Tychonic one or that one centred in the centre of mass of the planetary system. GR was able to explain some natural phenomena that Newtonian mechanics was unable to explain: deflection of the rays of light from their path near a massive body, the motion of Mercury perihelion, the shifting of the light spectrum toward the red, until the gravitational waves recently detected.

2. Leonardo's drawings of the Moon

In 1987 Reaves and Pedretti published a paper in which they drew the attention of the historians of astronomy on Leonardo's three drawings of the Moon [17]. They were included in the Codex Atlanticus (CA): two of them were in F.310*r* and the other in F.674*v*, made visible only after restoration of the CA [18] (fig. 1). The first two drawings were made between 1505 and 1508 when Leonardo was either in Florence or in Milan. The third one was drawn about 1513 or 1514 when Leonardo was in Milan or in Rome [17].

The three drawings were the first to be carried out with astronomical intentions. Although Leonardo's drawings were not accompanied by any consideration or comment, nonetheless his manuscripts contain a large amount of speculations on the Moon, characterized by:

- a) geometric studies of the relations between Earth, Sun and Moon;
- b) hypotheses on the constitution of the lunar soil deduced from observations compared with similar observations of the Earth's surface.

Leonardo did not leave any treatise or any organic discussion on any topic. Vasari, a few decades after Leonardo's death, reported that "... Lionardo began many things for understanding of art, & none ever finished" ⁽³⁾ (see [19], p. 5). It is not by chance that Kemp speaks about Leonardo's "... congenital inability to arrive at a definitively fixed and classified order for the subject he wishes to treat" (see [20], p. 26). Maffei's

⁽²⁾ The centre of mass of the planetary system lies inside the Sun but not in the centre of the Sun.

⁽³⁾ "... Lionardo per l'intelligenza de l'arte cominciò molte cose, & nessuna mai ne finì" (see [19], p. 5).

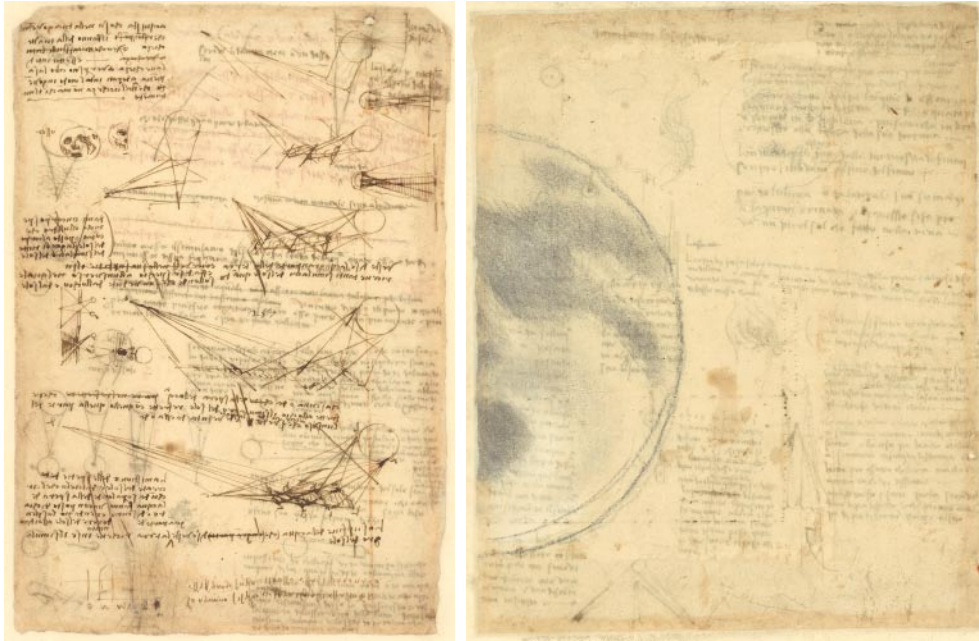


Fig. 1. – Leonardo's drawings of the Moon. Codex Atlanticus F.310r and F.674v. Reproduced with permission © Veneranda Biblioteca Ambrosiana/Mondadori Portfolio.

too writes about “. . . an extraordinarily interwoven body of perspicacious deductions and colossal misrepresentations . . .” (see [4], p. 251). Nevertheless Leonardo's role in the Renaissance lunar topic is quite peculiar as regards the approach to natural phenomena even though he was neither a natural philosopher nor an astronomer (see [21], p. 8). According to Kemp, Leonardo developed “. . . extraordinary visions of nature in extreme action over a vast number of years” (see [20], p. 41). Also Whitaker underlines how Leonardo viewed the spots (*maculae*) on the Moon with a scientific rather than imaginative eye (see [22], pp. 8-9).

Leonardo's drawings of the Moon seemed inscribed in the context of “a man in the Moon”, an ancient tradition dating back to Plutarch [23,24]. Also many artists had preceded Leonardo, among them Giotto (fig. 2) and van Eyck (fig. 3) as stressed by several scholars [21, 25, 26]. However the first two Leonardo's images, although inspired by the tradition of “a man in the Moon”, contain observational elements that can be compared with features afterwards discovered on the Moon. In fig. 4 and fig. 5 it is possible to spot some “*maria*” (seas) in Leonardo's drawings.

2.1. The Moon's ashen light in Leonardo

Considerations around the Moon are found in various Leonardian Codices. In the Leicester Codex [27] the phenomenon of ashen light is accompanied by a beautiful image repeatedly reported by several scholars. It also contains many reasonings about



Fig. 2. – Giotto di Bondone (ca. 1265-1337). “Il giudizio universale” (Last Judgement), one of the frescoes (ca. 1305), 10.0 × 8.4 m of the Cappella degli Scrovegni, Padova. On the right an enlarged detail of the Moon. Courtesy of Comune di Padova - Assessorato alla Cultura, Settore Cultura Turismo Musei Biblioteche.

the phenomenon: I will mainly mention them in my discussion, referring to the rich bibliography found in Maffei [28] to get a complete picture of the role the Moon had in the development of Leonardo’s thought and of the influence it subsequently exercised.

Paolo Frisi was the first to draw the attention of scholars to Leonardo’s interpretation of the ashen light (see [29], pp. 31-32). Leonardo realised that the spots on the



Fig. 3. – Jan van Eyck (ca. 1390-1441), The Crucifixion, ca. 1440-41. The MET, New York. 56.5 cm × 19.7 cm. On the right an enlarged detail of the Moon, red circled in the painting.

Moon, later on named *maculae*, were stable and did not resemble the vapours that would exist in the sublunary world ⁽⁴⁾.

According to the Vincian, on the Moon there were “*maria*” (seas) which reflected Sun rays in all directions due to the wind which blew on their surface, causing a wave motion. To a fictional “adversary” who denied the existence of water on the Moon

⁽⁴⁾ Leonardo, Notebook F, Institut de France Paris 84a; Ms2177-folio83/84-manuscript F. <https://www.photo.rmn.fr/CS.aspxVP3=SearchResult&VBID=2C05PCTBS3CLV&SMLS=1&RW=1161&RH=937&PN=3#/SearchResult&VBID=2C05PCTBS3CLV&SMLS=1&RW=1161&RH=937&PN=4>

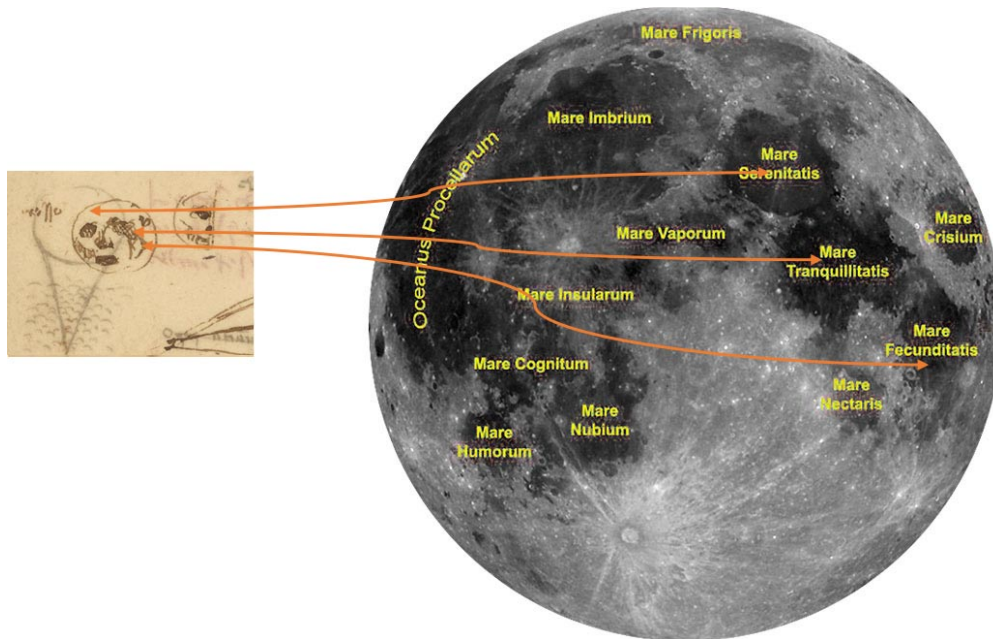


Fig. 4. – Some “*maria*” (seas) located in Leonardo’s drawings CA F.310r.



Fig. 5. – Some “*maria*” (seas) located in Leonardo’s drawings CA F.674v.

because in this case it would fall on the Earth, the centre of the Universe towards which all heavy things tend, Leonardo would reply that the Moon attracted both water and earth towards its centre as it happened on the Earth (see [30], p. 82). Moreover the Moon did not shine with its own light. It received the light of the Sun and the one mirrored by the terrestrial oceans. This last effect changed according to the position of the Sun, the Earth and the Moon. The brightness that appeared between the horns of the growing new Moon derived from the light reflected from the terrestrial seas. So Leonardo guessed that the Moon did not shine with its own light and this was certainly contrary to the Aristotelian dichotomy between sublunar and celestial world. At the same time he thought that the Earth was at the centre of the Universe. Furthermore Leonardo thought that the terrestrial seas reflected sunlight better than the soil of the Earth. Leonardo's reasoning was based on two points: 1) the ashen light was due to the reflection of the Sun's rays on the Earth towards the Moon and 2) the reflection was more effective for the seas than for the earth.

Depending on whether the emphasis is on point 1) or on point 2) Leonardo was, from time to time, a brilliant precursor of scientific reasoning, or an artist who made some good considerations on natural phenomena. But it is underestimated, in both cases, that Galileo's explanation, as we shall see, was very different from that of Leonardo and was based on the assumption that rough surfaces diffused light in all directions while shiny surfaces would have reflected light in a single direction. Galileo highlighted the difference, in the *Dialogo* [11], with an experiment.

In fig. 6 a page of the Leicester Codex is shown ⁽⁵⁾. The last drawing shows the ashen light of the Moon, clearly visible in a modern photo, fig. 7.

This is my summary of Leonardo's sentences ⁽⁶⁾:

Sec. IV The shining part of the Moon is formed by water that mirrors the light it receives from the Sun. The water of the Moon has a wave motion that reflects the light that arrives from the Sun and makes it less luminous than the Sun.

It remains now to show if the Moon is a heavy or light object: because if it were heavy . . . it would fall towards the centre of the Universe and would be united to the Earth. At least the water should fall towards the centre of the Universe and, in this case, the Moon would be uncovered and appear black. But since it appears shining to us, it is clear that the Moon is covered with water, air and fire and thus remains in space.

Sec. VIII The Moon does not shine with its own light. It receives the light of the Sun and the one mirrored by the terrestrial oceans. This last effect changes according to the position of the Sun, the Earth and the Moon.

Sec. XIV Some argue that the Moon shines with its own light but this is a false opinion.

The brightness that appears between the horns of the new moon derives from the light reflected from the terrestrial seas.

⁽⁵⁾ The red writings are have been added by me.

⁽⁶⁾ The transcriptions of Leonardo's considerations are taken from [31].

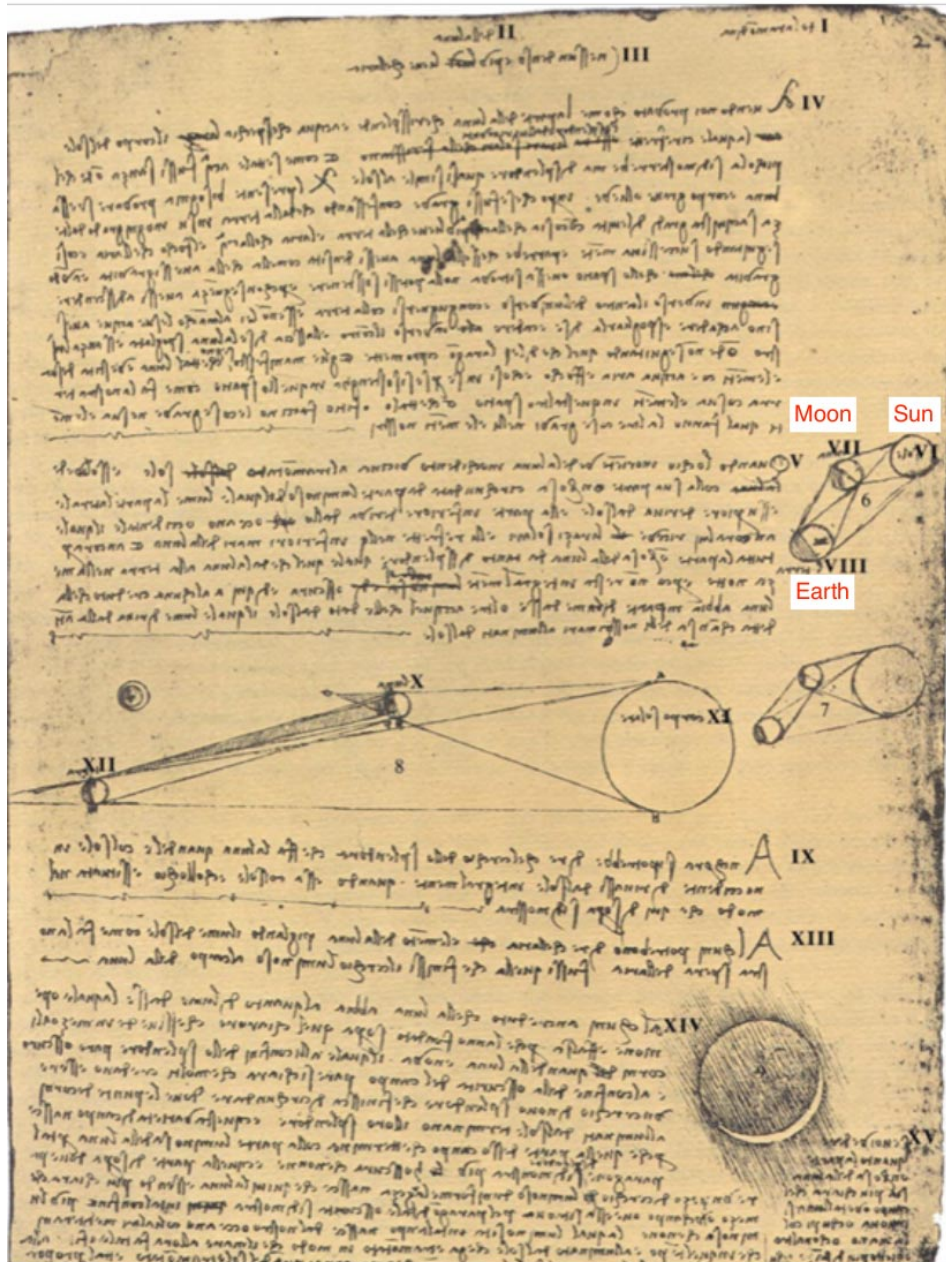


Fig. 6. – The page F.2r of Leonardo's Leicester Codex [27]. The words in red have been added by me.



Fig. 7. – A modern photo where the phenomenon of the ashen light is clearly visible. © Tunc Tezel (TWAN), <https://apod.nasa.gov/apod/ap190504.html>.

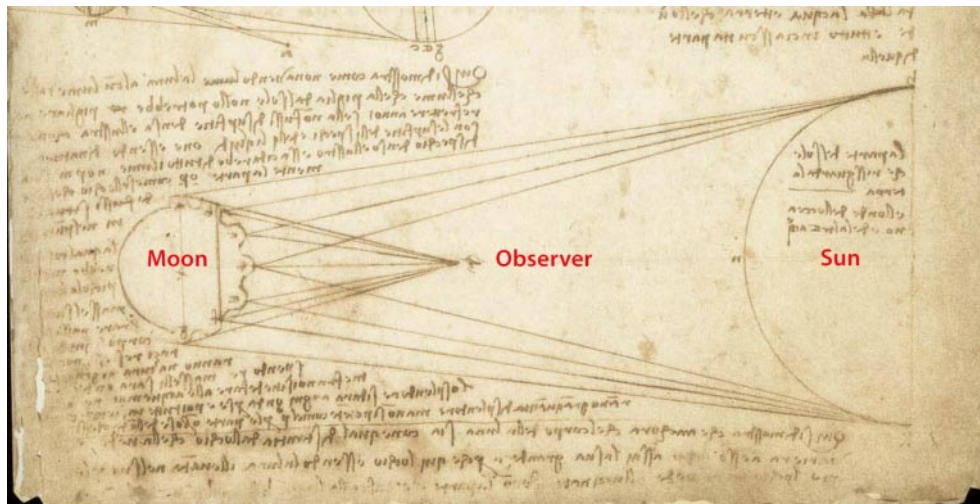


Fig. 8. – Leicester Codex F.1r [27]: Leonardo's model of the lunar seas as a tooth wheel. The words written in red have been written by me.

In the Leicester Codex F.1r (fig. 8) [27] the Moon was not represented as Leonardo saw it but was modelled as a toothed wheel. The aim was to demonstrate how Sun rays, reflected from the lunar surface, reached the observer placed in A from several directions. If the surface of the lunar seas was flat, the light would be reflected only in one direction. The Moon was a *quasi dentata rota* paraphrasing Leonardo. The drawing was very unusual and it is as if a dragon's crest had grown out of the Moon (see [28], p. 308).

2.2. The Moon's libration in Leonardo

The Moon, as mentioned above, in a first approximation always turns the same face towards the Earth, but, due to the libration, a terrestrial observer sees more than half of the Moon, about 59%.

In order to detect the libration, an observer must record several images of the Moon taken in succession. We have no documentation concerning a Leonardo series of images of the Moon. Nor in his writings do we find the slightest hint that Leonardo had noticed the phenomenon [21,32,33]. Leonardo probably unwittingly recorded the phenomenon. In the 1993 Reaves [34] dated the drawing F.674*v* of the Atlanticus Codex (fig. 1). Having calculated the libration $+7^\circ$ in latitude and -2° in longitude (all angles to $\pm 2^\circ$), he found that the possible dates were 1513 November 14 and December 12. They were in good agreement with Leonardo's own notations in his notebooks [34] ⁽⁷⁾. Leonardo's drawing would be therefore accurate enough to deduce quantitative evaluations from it. However, there is a sentence in which Leonardo seemed to have noticed the phenomenon of the lunar libration:

If you keep the details of the spots of the Moon under observation you will often find great variation in them, and this I myself have proved by drawing them (see [31], vol. II, p. 167).

But in another passage he stated:

... the spots on the moon, as they are seen at full moon, never vary in the course of its motion over our hemisphere (see [31], vol. II, p. 166).

Whitaker argues that the contradiction "... may possibly be explained by the fact that the markings so noticeable at the full phase are almost lost when they are situated near the terminator (the sunrise or sunset line) at other phases" (see [22], p. 9; [32], p. 516 (footnote)).

3. The Moon's ashen light in Galileo

Galileo was not the first to observe the phenomenon of the ashen light that had been described since ancient times (see [3], pp. 28-29). Nor was he the first to give an astronomical interpretation of it. As we have seen, Leonardo firstly described the phenomenon by relating it to the arrangement of the Earth, the Sun and the Moon in the heavens and to the roughness of the Moon which was covered, according to the Vincian, with wind-rippled waters that spread in all directions the sunlight reflected from the Earth toward the Moon.

⁽⁷⁾ I would like to thank the anonymous referee for his relevant comments and suggestions, included the bibliographic reference for Reaves.

We do not know if and how Galileo became aware of Leonardo's point of view ⁽⁸⁾. But, as we will see in detail, not only did Galileo describe the phenomenon with a precision that only the telescope, although weak, allowed him to do, but he inscribed it in the context of his opposition to Aristotelian cosmology, which stated that the Moon and the Earth were made up of different materials.

It is known, however, that Galileo had been in close friendships, since the '90s of 1500, with Paolo Sarpi, who had come to very similar conclusions to those of Leonardo:

Therefore the Moon is opaque, but not shiny. . . . If she were a polished mirror, she would still shine during the day, and not only in the shadows, indeed with a double light, although in the same solar light, because so do the mirrors and the waters, making the solar light duplicated [my translation] (see [35], p. 10).

The two interpretations diverged on the different aptitude of the Earth and water to reflect the Sun's rays. And it is probable, as Reeves suggests, that Galileo and Sarpi worked together in 1606-1607 and carried out experiments to determine whether the land or the water reflected better the light (see [3], p. 34). As we will see, Galileo will dwell on this point in the *Sydereus Nuncius*.

The Galilæian interpretation of the ashen light does not need rotation and revolution of the Earth around the Sun. It rather highlighted a symmetry of behaviour of the Earth and the Moon as regards the reflection of sunlight. Aristotelian distinction between the sublunar world and the world beyond the Moon was questioned: implicitly by Leonardo and explicitly by Galileo. On the other hand, Galileo's interest in Copernicus's *De Revolutionibus* dated back to several years earlier [36]. So it is very likely that Galileo inserted his interpretation of ashen light not only in the context of the critique of the Aristotelian cosmology but also in the broader development of the Copernican heliocentric system.

Galileo had observed lunar ashen light by naked eyes many years before the use of the telescope, but he stressed in the *Sydereus Nuncius* that the phenomenon, observed through a telescope, highlighted the similarity between the Earth and the Moon:

In this place I wish to explain the cause of another lunar phenomenon worthy of notice. This phenomenon was observed by us not recently but rather many years ago, shown to some close friends and pupils, explained, and given a causal demonstration (see [2], p. 26 or p. 14R).

If we analyse the way in which the *Sydereus Nuncius* printing had occurred since the manuscript was in the hands of Tommaso Baglioni for the composition between the 15 January and 12 March 1610, we get evidence that the ashen light represented, to Galileo, an important topic for his commitment to Copernican system (see [38], p. 137). The initial manuscript underwent several changes (see [38, 39] and [40], Vol. II). One of these was the interpolation of the description and interpretation of

⁽⁸⁾ We do not know which particular Leonardo's note Galileo had access to, so we are unable to understand the precise extent of Galileo's debt to Leonardo [37].

ashen light. Earthshine, according to the Pisan scientist, was a powerful evidence in favour of Copernicus' claim that there was no difference between the Earth, the Moon and the planets. In the *Sydereus Nuncius* Galileo claimed:

I thought it not unsuitable to be repeated here, and especially so that the relationship (*cognatio* in latin) and similarity between the Moon and Earth may appear more clearly ⁽⁹⁾. (see [2], p. 26 or p. 14R).

It was precisely the ashen light that impressed so much Henry Wotton, the English ambassador at the Republic of Venice, when he transmitted to the Earl of Salisbury a dispatch on the new discoveries that were being made in Venice ⁽¹⁰⁾.

As we have seen, Galileo had carried out naked eyes observations of the Moon and had explained its ashen light several years before the use of the telescope. His allegiance to the development of heliocentric Copernican system was long standing ⁽¹¹⁾. Anyway Galileo's acceptance of the heliocentric Copernican system, never explicitly treated in any published work, was a private affair as if Galileo preferred to keep a low profile on this thorny topic. But now, in 1610, after observing with a new instrument phenomena such as the similarity between the Moon and the Earth and, above all, the four satellites of Jupiter, Galileo felt the need to make his thoughts public in a book that he knew would have changed the way of observing and interpreting the movements of celestial bodies.

In the *Sydereus Nuncius*, the reference to Copernicus was explicit when Galileo, discussing about the satellites of Jupiter, mentioned the Copernican system:

We have moreover an excellent and splendid argument for taking away the scruples of those who, while tolerating with equanimity the revolution of the planets around the Sun in the Copernican system, are so disturbed by the attendance of one Moon around the Earth while the two together complete the annual orb around the Sun that they conclude that this constitution of the universe must be overthrown as impossible. For here we have only one planet revolving around another while both run through a great circle around the Sun: but our vision offers us four stars wandering around Jupiter like the Moon around the Earth while all together with Jupiter tra-

⁽⁹⁾ Giudice [41], p. 55, notes that the word used by Galileo —*cognatio*, relationship or kinship in English— is the nominative singular of the one used by Copernicus —*cognitionem*— previously quoted in this paper (third page).

⁽¹⁰⁾ An excerpt from the letter is published by Reeves who writes that the ambassador was so impressed by the Galilean interpretation of the ashen light that in his *Elements of Architecture*, published 15 years later, he described Galilean interpretation (see [3], pp. 9-10).

⁽¹¹⁾ There are at least two episodes, prior to *Sydereus Nuncius*, in which Galileo expresses a favourable evaluation of the Copernican heliocentric system. The first is the letter written by Galileo to Kepler on 4 August 1597 (see [43], pp. 67-68) and the other refers to fragments of the University lectures given by Galileo in December 1604, to notes and considerations relating to the *Progymnasmata* of Tycho Brahe and some observations and calculations drawn up in different periods, datable approximately between 1604 and 1607 [my translation] [42], p. 240. However, Galileo' interest in the Copernican thesis had been clear since 1590 when he left Pisa to occupy the chair of mathematics at the Padua University [36].

TABLE I. – Printed images of the Moon in the *Sydericus Nuncius*. SN F.10r and SN F.10v lower are the same (see [1], pp. 11–15).

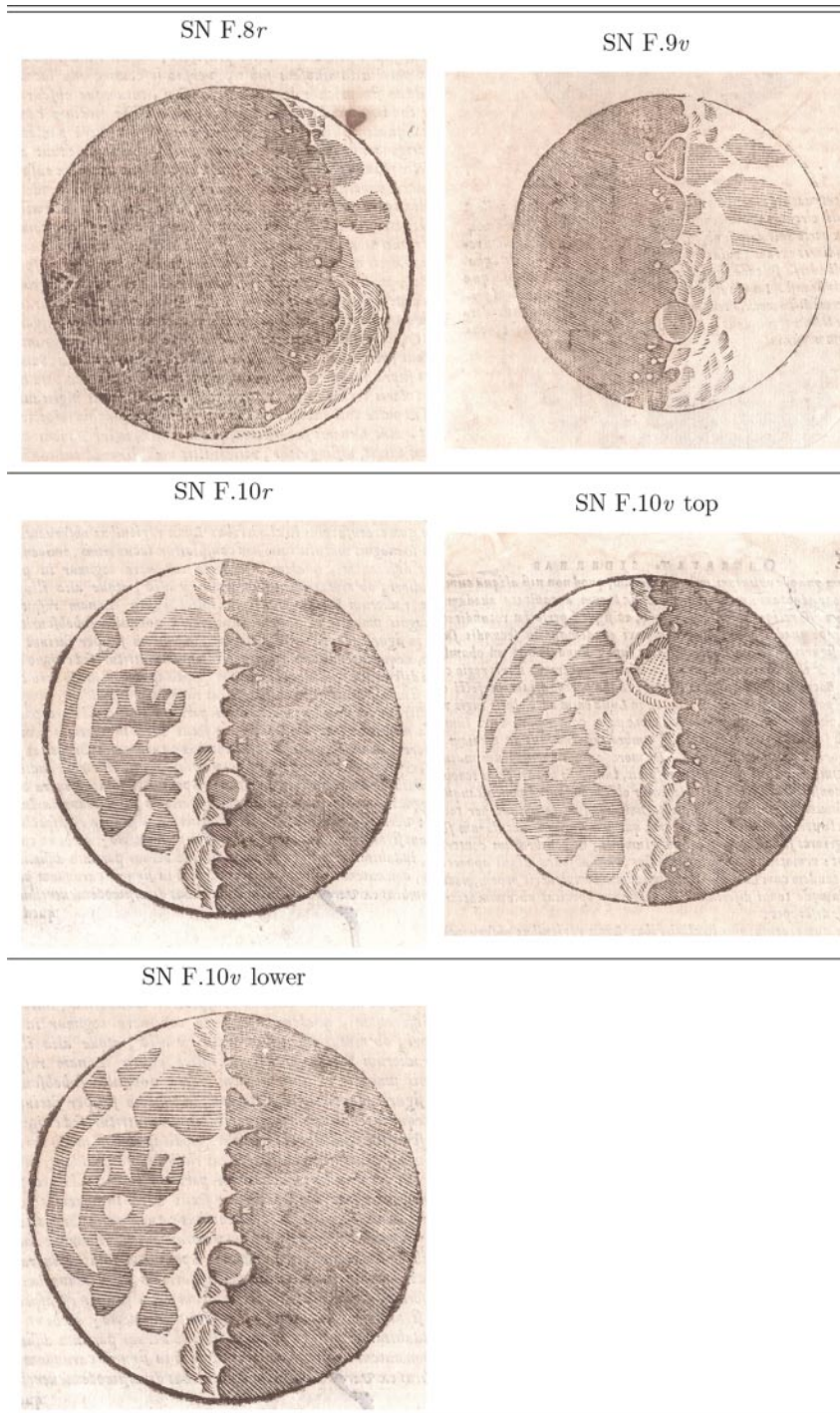




Fig. 9. – Galileo Galilei, *Sidereus Nuncius*, cc. 8r-29v, autograph sketch, 1610 in Galileo Galilei, *Opere di Astronomia*, manoscritto composito, secc. XVII-XIX; Firenze, BNC, Galileiano 48, c.28r, 29r, 29v. By permission of the Ministero per i Beni culturali e per il turismo/Biblioteca Nazionale di Firenze.

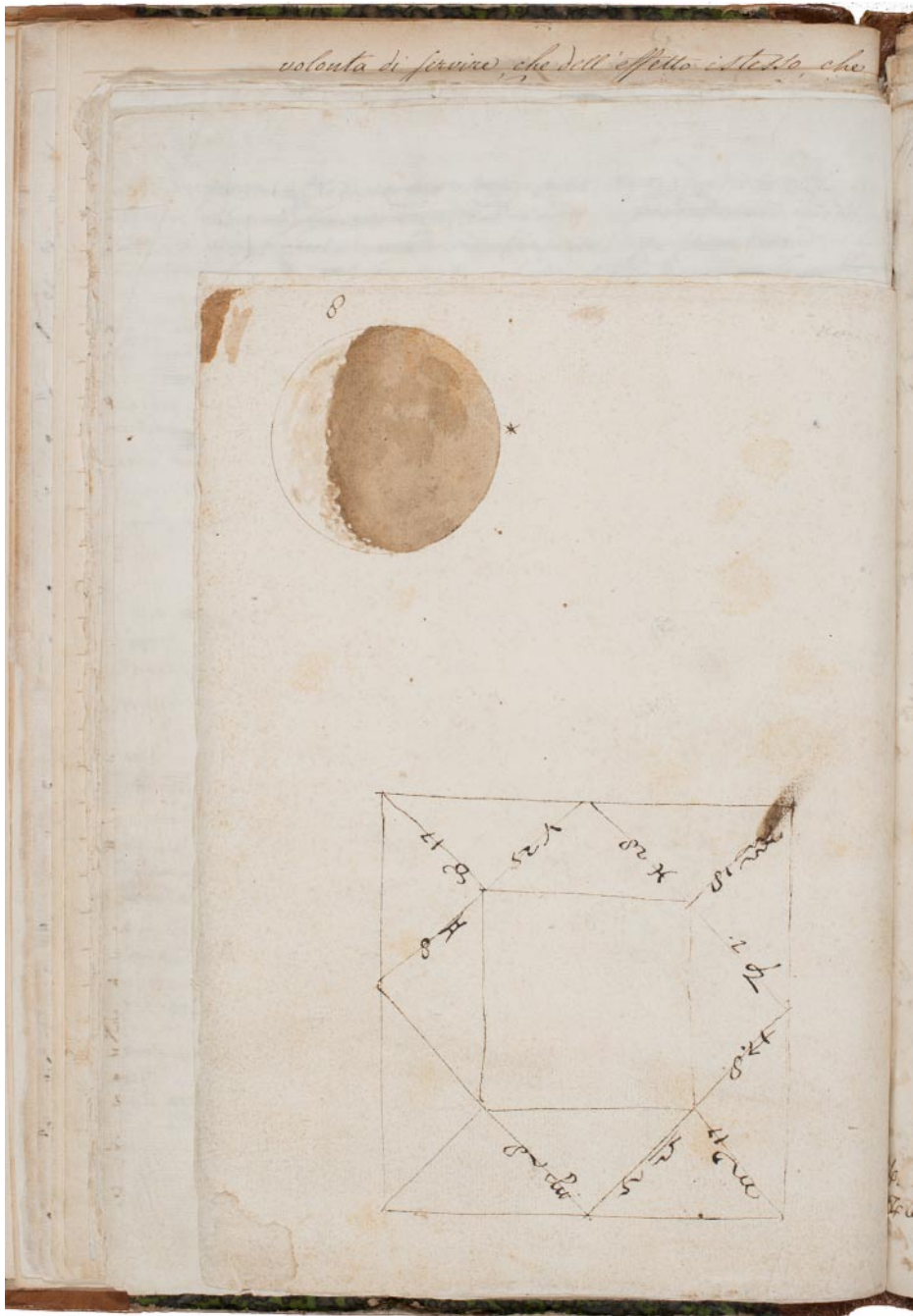


Fig. 10. – Galileo Galilei, *Sidereus Nuncius*, cc. 8r-29v, autograph sketch, 1610 in Galileo Galilei, *Opere di Astronomia*, manoscritto composito, secc. XVII-XIX; Firenze, BNC, Galileiano 48, c.28r, 29r, 29v. By permission of the Ministero per i Beni culturali e per il turismo/Biblioteca Nazionale di Firenze.

test image, then repeated in “1”. On why Galileo did not number “2” the “upper right” image, see Gingerich [47] ⁽¹⁴⁾.

Galileo's representations of the Moon have been analysed and cited by an endless number of scholars. What interests us, however, in the present context, is whether the ashen light is clearly visible in drawn or printed images. To this end attempts to date them are worth analysing and the correspondence between ink-and-water drawings and printed ones, because a dating procedure would give us information on the quality of the images and on their adequacy to show faint phenomena such as that of ashen light.

The five printed images were analysed in depth by G. Righini [9,10] with the aim of trying to date them. His approach was purely astronomical. The first image represents the waxing Moon on the fourth or fifth day after the new Moon. G. Righini established the dates of the new Moon between July and December 1609, having identified this dates interval on the basis of historical reconstructions of Galileo's activity. For dating the second and the third image G. Righini used a very original method. He claimed that in Galileo's images it was possible to detect the phenomenon of lunar libration (I shall talk about it later), unwittingly noticed by the Pisan scientist. From this G. Righini deduced a method for dating the images: his results are shown in table II.

Gingerich criticised these results as they refer to images of the Moon which were too inaccurate for a quantitative analysis. G. Righini had deduced from his analysis that Galileo was a remarkably faithful recorder of his visual experiences. Gingerich instead argued that the accuracy required for Righini's method—to the extent of concluding that Galileo unknowingly recorded Moon's libration—was very far from the one achieved by Galileo. In particular, Gingerich argued that the crater that was seen in the second (F.9v) and third images (F.10r) was completely unrealistic. Moreover, three possible dates corresponded to the first image: 29 January 1610, 29 March 1610, 2 October 1609 (see [48], pp. 86-87).

Bredenkamp ⁽¹⁵⁾ [50] suggested a different dating: the first image (F.8r) of the *Sydereus Nuncius* 31 December 1609, the second one (F.9v) 02 January 1610, the third one (F.10r) 15 January 1610, the fourth one (F.10vtop) 16.01.1610 (see [50], pp. 180-181). But, as Gingerich pointed out (see [51], p. 161), he entirely neglected

⁽¹⁴⁾ Nine drawings of the Moon are in a copy of Galileo's letter dated 7 January 1610 to an unknown correspondent, doubtfully identified by Favaro in Antonio de' Medici (see [44], pp. 273–278). In spite of the poor quality of the images due to printing systems, they have been analysed by Needham (see [40], pp. 93-101). Afterwards, the images drawn in the letter, now in the Historical Archives of the Pontificia Università Gregoriana (APUG 530, ff. 155r-156v), have been analysed by Bredenkamp who had the opportunity to consult the catalogue of the 2015 exhibition *Magistri astronomiae dal XVI al XIX secolo: Cristoforo Clavio, Galileo Galilei e Angelo Secchi*, where the letter was displayed (see [49], pp. 100–110).

⁽¹⁵⁾ Bredenkamp was a staunch and disinterested supporter of the authenticity of a copy of the *Sydereus Nuncius* which later on turned out to be a forgery. A story of this unbelievable fact can be found in a essay review of Van Helden [54].

TABLE II. – *Dating of the printed images of the Moon in the Sydereus Nuncius according to G. Righini. The diameter of the images 1, 3, 4, 5 is 80 mm; image 2 has a diameter of 76,5 mm [9,10].*

SN F.8r	02.10.1609
	waxing Moon the fourth or fifth day after new Moon
SN F.9v	03.12.1609
	first quarter
SN F.10r	18.12.1609
	last quarter
SN F.10v top	18.12.1609
	shortly before the last quarter
SN F.10v lower	18.12.1609
	last quarter "editorial mistake"

astronomical constraints, relying his dating only on Galileo's letter of 07 January 1610 [44].

Drake re-examined Righini's chronology and, on the basis of his knowledge of Galileo's biography, observed that Galileo could not have started his observations before 1 December 1609, shortly after sunset when the age of the Moon was 5 days. He moved Righini's date 02.10.1609 to 29.01.1610 (see [45], p. 154), he accepted other G. Righini's dating and considered G. Righini's work a contribution "... which remains without rival in scope and scholarship in the vast literature regarding this important phase of Galileo's activities." [52]












The problem of dating both ink-and-wash and printed images was reconsidered by Whitaker, an astronomer specialized in lunar studies, who argued that he had been surprised to find that the ink-and-wash drawings and printed images were "... more accurate in their *qualitative* portrayal of actual phases (their geometrical accuracy is clearly much poorer) than I had hitherto thought" (see [53], p. 158). Whitaker compared Galileo's images of the Moon with the photographic ones taken at the same conditions of Galileo's observations. See table III. Whitaker confirmed Righini's dates for the second and fourth printed image of the *Sydereus Nuncius*; for the third image he confirmed Righini's date if for "shortly before the last quarter" we mean one day before. For the first image Whitaker proposes a different date: 30.11.1609. Moreover he finds links between Galileo's printed and drawn images of the Moon.

In the images of the *Sydereus Nucnius* there was an error, probably due to the printer, in the up-down orientation of image 2 (F.9v) which must be reversed. Nevertheless, if we compare the second image (F.9v) with a photo of the Moon (at low resolution as suggested by Gingerich) at the same phase, we can "... muzzle the gainsayers who assert that Galileo images bear no resemblance to reality" (see [22], p. 21).

Gingerich and Van Helden (see [39], p. 258) took Whitaker's chronology as definitive, except for the date of the second image of the *Sydereus Nuncius* (2 December 1609 instead of 3 December 1609). Moreover Whitaker, according to the two authors, roughly confirmed G. Righini's dates. More recently A. Righini revised G. Righini's dates, using the JPL *Horizon* project website and the *Stonyhurst* disc, and corrected the date provided by G. Righini of the first image of the *Sydereus Nuncius* from 2 October 1609 to December 1 1609, with a mean absolute error of 2.5% in units of lunar diameter. Such a little error means, according to the author, that Galileo's images were accurate enough to perform measurements on them and, contrary to what Gingerich claimed, they were not just attempts to give the idea of mountains and plains [33,55]. It is worth underlining that while for most scholars the fifth image of the *Sydereus Nuncius* is the same as the third and this was a mistake, A. Righini argues that, perhaps, Galileo deliberately duplicated the third image in order to speed up the publication of the book (see [33], p. 30).

So, due to the studies on dating of both the images drawn by Galileo and those printed in the *Sydereus Nuncius*, we can be enough confident that Galileo was quite accurate to record not only the ashen light but also the lunar libration, this last one although unknowingly.

TABLE III. – Dating and correspondence between Galileo’s ink-and-wash drawings and printed images of the Moon in *Sydereus Nuncius* according to Whitaker [22, 53].

Galileo’s ink-and-wash drawings numbering	Date	<i>Sydereus Nuncius</i>
Sheet 28r [upper left] 	30.11.1609	
Sheet 28r [upper right] 	01.12.1609	
Sheet 28r drawing 1 	30.11.1609	SN F.8r 
Sheet 28r drawing 3 	02.12.1609	
Sheet 28r drawing 4 	18.12.1609	SN F.10v top 
Sheet 28r drawing 5 	17.12.1609	SN F.10r 
Sheet 29v drawing 8 	19.01.1610	
	03.12.1609	SN F.9v 

3.2. The ashen light of the Moon in Galileo's "Sydereus Nuncius"

In addition to the famous and fascinating images and drawings of the Moon, Galileo had left us dozens of pages in which lunar phenomena are analysed in detail. He dedicated to the Moon about a third of the 60 pages of the *Sydereus Nuncius* [1,2], from the end of page 10 to the middle of page 30, including the famous five images. About 4 pages (from the top of page 26 to page 29 in the middle) were assigned to the ashen light. But while the reference to printed images was clear when Galileo exposed the novelties that the telescope had allowed him to detect—in particular complete similarity of the Moon to the Earth with its mountains, valleys, clefts, sinuosities—he did not relate the ashen light to any of the images.

Swerdlow argues that non-uniformities of shading in the dark part of the first image (F.8r) may be intended to show the effect of the ashen light (see [56], p. 249). This is certainly possible but one needs to keep in mind that at least two other craftsmen (not necessarily different) contributed to the printing: the engraver and the printer. And again we have to consider that in the several reprints of the *Sydereus Nuncius* the images have undergone various transformations and therefore we should know which edition of the *Sydereus Nuncius* has been consulted.

Knowing, therefore, that the ashen light is a tenuous phenomenon, it seems appropriate to analyse also the drawings that Galileo made during the observations or immediately afterwards. Not all the images of the Moon in the 80 known copies of the 550 of the first edition of the *Sydereus Nuncius* are perfectly identical, if only for their different state of preservation. Not to mention the images of the Moon in the various subsequent editions. Galileo's ink-and-wash drawings of the Moon are, on the other hand, unique. Since in some of them the phenomenon of the ashen light can be identified, the historiographical problem arises of understanding why there was not any printed image that referred to the ashen light, even though there was a very detailed description of the phenomenon.

In the *Sydereus Nuncius* Galileo noted that, when we observe the Moon before and after the conjunction, and when it is not far from the Sun, we not only see its shining horns but also a faint circle which separates it from the darker surrounding ether. Moreover, looking more closely at the phenomenon, we observe also the entire face of the Moon, though it was not illuminated by the Sun. This last phenomenon can be observed with difficulty, as Galileo noted, due the bright horns. But it can be observed if the horns are concealed by a roof or a chimney or another obstacle between our sight and the Moon. The observer then will discover that this region of the Moon, although deprived of sunlight, also shines with a considerable light. The phenomenon was observed by Galileo with the naked eye many years before, and, as we have already seen, was firstly interpreted by Leonardo.

Galilean explanation assumed that the Earth was able to reflect the Sun's rays, a point of view strongly criticised by "... those who claim that the Earth is to be excluded from the dance of the stars, especially because she is devoid of motion and light" (see [2], p. 30). Galileo claimed that in his *Systema Mundi* he would have demonstrated a very strong reflection of solar light from the Earth.

As the phenomenon of the ashen Moon is more visible in the phases of the waxing Moon, the watercolour drawings which I have analysed are the “upper right”, the “1”, the “3” and the “5”. This latter can be excluded because in the waxing gibbous phase the phenomenon of the ashen light is very weak. See fig. 9 and fig. 11.

I should also exclude the drawing “3” because Galileo drew the dark background of the sky only in correspondence with the illuminated part of the Moon, unlike the other two, where the dark background of the sky is drawn behind the whole Moon. Both in the “upper right” drawings and in the “1” drawings we note how the part of the Moon not illuminated by the Sun is a little bit lighter than the dark background of the sky. This greater candour could indicate that Galileo actually perceived and drew the ashen light. Describing the image “1” Bredekamp states that “. . . strangely, the deepest dark appears outside, on the upper and lower edges of the picture . . .”. Indeed if, as I suppose, Galileo wanted to highlight the phenomenon of the ashen Moon, I do not find strange that he coloured the sky with a much darker color in order to enhance the faint luminosity of the Moon illuminated by the Sun’s rays reflected from the Earth towards the Moon. It’s the same artifice used by Galileo in the image “upper right” where, as Bredekamp rightly writes, “. . . after the glaringly illuminated crescent and the abrupt contrast of the dividing line to the lunar night, the refined treatment of lighting continues with a diminishing of the darkness to reproduce the reflected light of the Earth” (see [57], pp. 173-174).

So, assuming that Galileo actually drew the ashen light, why is there no record of this phenomenon in the printed images of the *Sydereus Nuncius*? And why did not Galileo add another image that would have better illustrated the phenomenon described at length in the text?

In this regard, Molaro argues that it does not seem plausible, as someone could suppose, that the more common Venetian woodcut would have been more suitable than etchings to highlight details and nuances present in the ink-and-wash drawings. The problem should be therefore, according to the author, not technical. Those who accepted that the Earth could reflect the Sun’s rays were considered supporters of Copernicanism. For this reason Galileo would have hesitated to illustrate with an image the theory of the origin of ashen light (see [58], pp. 83-84). To this hypothesis it can be objected that Galileo wanted to publish the text as soon as possible to claim discovery priority, comforted, among other things, by Cosimo II de’ Medici who had accepted that the book could be published under Medici patronage.

Moreover, in addition to the point already mentioned, there are two other parts of the *Sydereus Nuncius* in which the reference to the Copernican system is clear although not explicit. The first one is in the dedication to Cosimo II de’ Medici:

. . . four stars . . . make their journeys and orbits with a marvellous speed around the star of Jupiter . . . with mutually different motions, like children of the same family, while meanwhile all together, in mutual harmony, complete their great revolutions every twelve years about the centre of the world, that is, about the Sun itself (see [2], p. 4, F.3r).

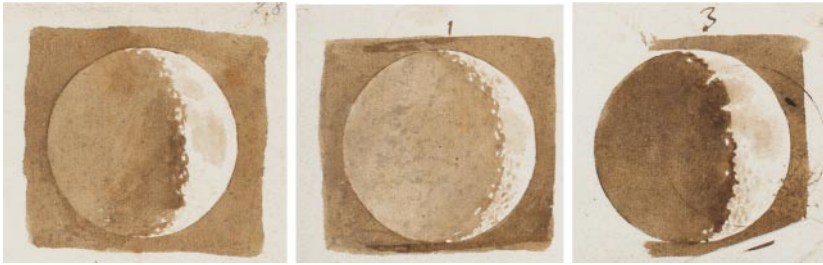


Fig. 11. – The three drawings where the ashen light should be detected.

A second reference is in the part of the *Sydereus Nuncius* which deals with the ashen Moon:

We will say more in our *System of the World*, where . . . we will demonstrate that she [the Earth] is movable and surpasses the Moon in brightness, and that she is not the dump heap of the filth and dregs of the universe . . . (see [2], p. 30, F.16r).

The reference to the Earth as a “wandering star” is all the more significant as it is not necessary for the interpretation of the phenomenon of the ashen light.

Evidently Galileo wanted to underline his full endorsement of the heliocentric Copernican system ⁽¹⁶⁾ (see [59], note 84, p. 76). It was then evident for a reader educated enough to read a book written in Latin, that two of the key points of the text—similarity between the Moon and the Earth and the Jupiter’s satellites—had been interpreted by Galileo in light of Copernican system.

In conclusion, Galileo, especially after the Medicis’ patronage, did not seem reluctant at all to admit his acceptance of the Copernican system. He did not insert an image of the Moon that highlighted the ashen light because he was in a hurry and he was not very satisfied with the quality of the images in the *Sydereus Nuncius*. When the Pisan scientist sent to Cosimo II de’ Medici a still fresh copy of his booklet, he accompanied it with a letter, dated 19 March 1610, to Vinta, the First Secretary of State of the Grand Duchy. In it Galileo expressed his disappointment at the poor quality of the images:

It is necessary that Your Most Illustrious Lord offers my apologies to the Highnesses if the work doesn’t come out printed with that magnificence and decorum that such a larger-than-life object would have required, because the time constraints did not allow it, nor did I want to put off the publication, so as not to take the risk that someone else has encountered the same and made me worry. . . . It will also be necessary in a very short time to reprint the work, completed with many observations, which I’m continuing, and with many and beautiful copper-cut images by a talented man [my translation] (see [61], pp. 299, 300).

⁽¹⁶⁾ A detailed analysis of Galileo’s adhesion to Copernicanism in the *Sydereus Nuncius* would take us too far from the purpose of this paper. For information on this topic see [38, 59, 60].

The poor quality of the images was, therefore, attributed by Galileo to haste even if he did not exclude, in the case of reprinting, to contact a more skilled printer. The lack of an image of the Moon under the effect of the ashen light could also be attributed to the same cause.

3.3. The ashen light of the Moon in Galileo's "Dialogo"

As Galileo had foretold in the *Sydereus Nuncius*, he detailed his thought on ashen light in the First Day of the *Dialogo* [11]. The similarity between the Earth and the Moon was the context, introduced by Salviati —the Galileian— who listed seven topics in favour of it ⁽¹⁷⁾. To our purposes, the sixth argument is interesting: As the Moon supplies us with the light received from the Sun so the Earth reflects the solar rays towards the Moon.

Salv. . . . To get back to our original discussion, I state that the sixth agreement between the Moon and the Earth is that just as the Moon supplies us with the light we lack from the Sun a great part of the time, and by reflection of its rays makes the nights fairly bright, so the Earth repays it by reflecting the solar rays when the Moon most needs them, giving a very strong illumination —as much greater than what the Moon gives us, it would seem to me, as the surface of the Earth is greater than that of the Moon (see [62], p. 67).

Simplicio —the Aristotelic— disagreed on some proofs Salviati brought to support similarities between the Earth and the Moon. Specifically he found unconvincing the interpretation of the ashen light of the Moon. Simplicio said that the Moon could not receive light from the Earth which was completely dark, opaque and unfit to reflect the sunlight as the Moon reflected it. And he added:

Simp. . . . I consider the light which is seen over the rest of the face of the Moon (outside the horns brightly illuminated by the Sun) to be the Moon's own proper and natural light, and it would be quite a feat to make me think otherwise (see [62], p. 70).

Salviati replied that Simplicio's assumption —the Moon was polished and smooth as a mirror and, as such, fitted to reflect the sunlight, while the Earth, due to its roughness, had no power to make a similar reflection— was wrong. An easy experiment showed the opposite. Salviati hanged a mirror on a rugged wall illuminated by Sun's rays and showed that the surface of the wall always looked equally illuminated, no matter from what place you observed it, and somewhat lighter than that of the mirror from every place except that small area where the reflection from the mirror struck.

If the surface of the Moon were a mirror observed from the Earth, it would intolerably dazzle us in one point but it would be much darker in other points. So, since the Moon reflected the Sun's rays in all directions, it looked more like the wall than

⁽¹⁷⁾ English quotations of the *Dialogo* are taken from [62].

the mirror. A rough wall scattered its light more diffusely unlike the mirror, which diffused it more vividly (see [62], pp. 73, 77 and [11], pp. 98, 102).

But, Sagredo —the subtle moderator of the debate— asked, and Simplicio agreed with him, as the Moon was spherical and not flat, it ought to reflect on the Earth light rays coming from the Sun to all directions, and not to a restricted area like those of a flat mirror (see [62], p. 73 and [11], p. 98). It was easy for Salviati to argue that only a very small part of the Moon would show itself to the eyes of a particular person on the Earth, although the entire lunar hemisphere would be exposed to Sun's rays. The rest would remain, to the observer's eyes, unilluminated and therefore invisible, since that little area which gave reflection would be lost by reason of its smallness and great distance (see [62], pp. 74-75 and [11], pp. 99-100).

Similar arguments were used by Salviati against Simplicio's objections dealing with the reflection of light on copper or silver plates. And Salviati, rather ironically, concluded:

Salv. . . . So if anyone, seeing from a distance a little gilded plate shining all over, should imagine that the same phenomenon would have to occur with a plate as large as the Moon, he would be as much deceived as if he were to think that the Moon is no larger than the bottom of a bathtub. (see [62], p. 79)

Simplicio was not satisfied and came back citing a "recent booklet" in which the author —Locher— argued that ashen light derived from the illumination of the Sun itself, which penetrated its whole body because the substance of the lunar globe was somewhat transparent. But this more vividly illuminated the surface of the hemisphere which was exposed to Sun's rays, and the interior, soaking up this light like a cloud or crystal, transmitted it and made the Moon visibly lighted ⁽¹⁸⁾. Moreover, Simplicio again said, during the solar eclipses, when the Moon was under the Sun's disc, it was seen to be translucent, especially around the extreme edges. Then, since this phenomenon could be derived neither from the Earth, nor from the stars nor from the Moon itself, it necessarily followed that it came from the Sun. This hypothesis, according to Locher, explained all the observed lunar phenomena: the appearance of ashen light more vividly along the extreme edges; the observation of a brighter circle along the extreme edge of the Moon, during a solar eclipse, in that part which was under the Sun's disc, and not in the part outside this disc; and so on ⁽¹⁹⁾.

Salviati easily proved that all Locher's arguments were false or contradictory and concluded that reflection from the Earth is very powerful on the Moon and he delved into the question of the solidity of the material which the Moon was made of.

Simplicio argued:

⁽¹⁸⁾ It is the so called "theory of imbibition", not longer used to explain the light of the Moon, but the ashen light.

⁽¹⁹⁾ The booklet which Simplicio referred to was *Disquisitiones mathematicæ* written by Locher, a pupil of Scheiner [63].

- that the Moon was made of solid material because it was polished and smooth;
- that the reflection from the Earth's seas ought to be stronger as they have a smooth surface.

While Salviati claimed:

- that it was solid because it was mountainous
- that the Earth's seas reflected less light than land which was rough and dark (see [62], p. 97 and [11], p. 122).

Dealing with the first question, if the Moon and the Earth were made of fluid material, they would level out and it would be impossible to see on the Moon mountains and their shadows (see [62], p. 97 and [11], p. 122).

As concerns the second question, Salviati claimed that, having considered the events that take place in mirrors, we could understand quite well that the reflection of light coming from the seas would be less than that coming from the land ⁽²⁰⁾ (see [11], p. 22). While seas, if unshaken by the wind, reflect solar rays in one direction only, land reflects light in all directions due its roughness. To make his reasoning even more understandable, Salviati poured water on the floor and asked his interlocutors to observe the reflection of solar rays from the dry part and the wet part of the ground. Like the experiment with the mirror, the dry ground diffused the light in all directions while the wet ground reflected the solar rays only in one direction. Salviati could conclude therefore that just as the surface of the Earth's oceans seen from the Moon would appear even (except for islands and rocks), so it would appear less bright than that of the land, which was uneven and mountainous (see [62], pp. 97-98 and [11], pp. 122-123).

But did similarities between the Moon and the Earth go so far as to consider the Moon covered with forests or inhabited by human beings? No, on this point Salviati was cautious because the richness of nature is unimaginable for us. So Sagredo commented:

It always seems to me extreme rashness on the part of some when they want to make human abilities the measure of what nature can do. ... On the contrary, there is not a single effect in nature, even the least that exists, such that the most ingenious theorists can arrive at a complete understanding of it. This vain presumption of understanding everything can have no other basis than never understanding anything. For anyone who had experienced just once the perfect understanding of one single thing, and had truly tasted how knowledge is accomplished, would recognize that of the infinity of other truths he understands nothing (see [62], p. 101 and [11], pp. 126-127).

⁽²⁰⁾ As we have seen previously, Leonardo also posed the same problem and solved it by stating that lunar seas did not constitute a flat surface but were rippled by the wind.

3.4. The ashen light of the Moon in the “Lettera al Principe Leopoldo”

Prompted by the request of Prince Leopoldo de' Medici ⁽²¹⁾ to communicate his thoughts about Liceti's book ⁽²²⁾ on the ashen light of the Moon, the seventy-six year old Galileo, now completely blind, dictated to Vincenzo Viviani (1622-1703), his “dear friend” ⁽²³⁾ and devoted disciple, a memoir which would turn out to be the last one of the Pisan scientist.

Galileo's answered Prince Leopoldo on 13 March 1640 [64] and the Memoir was addressed on 14 May 1640. Paolo Frisi refers to the Memoir in his *Elogio del Galileo* (see [29], pp. 32-33). The first printed version of the memoir was published by Liceti in his book *De Lunæ subobscura luce* [65], after repeated requests of authorization to Galileo, rather reluctant to make public what he considered a private affair between himself and the Prince.

The Letter to the Prince is full of considerations on the relationships among perception, natural philosophy and mathematics. But I will not analyse this topics, which would take me away from the main subject of my paper. Galileo, in the letter to the Prince, came back to discuss about the Moon after that his first telescopic images of the lunar soil, published almost thirty years before [69], had placed him in the narrow circle of the great thinkers who had revolutionized the philosophical, cultural and scientific thought. With thorough arguments Galileo reiterated the thesis that the Moon's ashen light originated from the reflection of the Sun's rays on the surface of the Earth. Moreover the Pisan scientist demolished, with sharp quills ⁽²⁴⁾, Liceti's thesis, who maintained that the Moon gave back, in the dark, the light stored when hit by sunlight: a behaviour quite similar to that one of the “bolognese stone” ⁽²⁵⁾ [70].

In the memoir Galileo does not bring new proofs supporting his thesis. The goal is to reiterate interpretation of the ashen light that Liceti had criticized and to demolish Liceti's one. Galileo therefore continued to support his thesis with proofs he had

⁽²¹⁾ Leopoldo de' Medici (1617-1675) was the brother of Ferdinando II de' Medici (1590-1621), Grand Duke of Tuscany from 1621, who thought highly of Galileo and protected him, within the limits of his possibilities [66]. At the death of Leopold, his personal patrimony constituted, among other things, by the lens of Galileo, the telescopes, the jovialabe and the glassworks of the Accademia del Cimento, became part of the Medici collections and are currently exhibited in the “Museo Galileo” in Florence.

⁽²²⁾ In the chapter L of his book, Liceti criticized Galileo on the ashen light of the Moon [65]. Fortunio Liceti (1577-1657) was professor of philosophy first at Pisa University and then at Bologna University where he taught from 1637 to 1645. Afterwards he taught medicine in Padua University where he was colleague of Galileo for one year. Their relationship, testified by several letters, was based on esteem and friendship which, however, did not influence their deep divergences as regards to natural philosophy, mathematics and astronomy [67].

⁽²³⁾ Galileo spoke of Viviani as “dear friend” in several letters. See [68], p. 473.

⁽²⁴⁾ Leopoldo had invited Galileo to soften his criticisms to Liceti, “ad astenersi dalli aculei” [to refrain from quills] as the Pisan scientist had interpreted Prince's suggestion.

⁽²⁵⁾ The bolognese stone is barium sulphate which, once calcinated, becomes phosphorescent in the dark, after his exposition to light. The phenomenon was discovered at Monte Paderno, near Bologna, around the 1604 [71]. Galileo learned of it through Giovanni Antonio Magini, mathematician of the Bolognese Studio (see [68], p. 469).

already given in the *Sydereus Nuncius* and in the *Dialogo*. On this occasion he added methodological considerations on the relationships between natural philosophy and mathematics. By proposing stringent logical arguments Galileo showed the fallacy of Liceti's arguments against him.

Galileo then moved on to Liceti's hypothesis that the ashen light depended on the reflection of Solar rays on the ether surrounding the Moon. According to Galileo, there was an indisputable proof against this hypothesis: during an eclipse, the ether surrounding the Moon remained outside the shadow cone and continued to be illuminated. The ether could have reflected the Sun's light on the dark part of the Moon, but observations did not highlight this phenomenon, except to a very low extent such as to be absolutely inferior to that one of the ashen light. Galileo, continuing in his inexorably rational analysis, taught Liceti a true lesson, with considerations of natural philosophy taking into account mathematical, quantitative reasoning. Liceti, in fact, argued that ashen light would be weaker than terrestrial illumination during the full Moon.

"Senses —Galileo wrote— can go wrong and be in need of correction, to be obtained through the use of right rational speech" (see [68], p. 511).

Liceti's mistake was to form

"... judgment ... from the first and sensual appearance, reasoning as a physicist and natural [philosopher] See therefore Your Highness how mathematical reasoning serves to dodge those rocks, in which sometimes the pure physicist takes the risks of colliding and breaking" (see [68], pp. 515, 521).

At the end of the memoir, Galileo takes into account Liceti's statement that the Moon can be compared to the Bologna's stone. According to this hypothesis, the Moon would preserve for some time the tenuous light reflected by the ether that surrounds her and that this light would vanish after some time that the Moon was in the shadow. Galileo had no doubts in saying that this hypothesis was a "poetical graceful joke" (see [68], p. 540).

4. The Moon's libration ("titubazione") in Galileo

There are three types of libration: 1) optical libration in longitude, 2) optical libration in latitude, 3) optical diurnal libration, called also parallaxic libration.

Optical libration in longitude was discovered by Hevelius (see [72], p. 96). It derives from the variation, according to Kepler's second law, of the angular velocity of the Moon along its orbit, being it a little eccentric. A terrestrial observer sees an angular shift of surface objects either to the western or the eastern edge of the lunar disk. The maximum angular shift relative to the centre of the Moon is of $\pm 7.9^\circ$ (see [32], p. 495).

Optical libration in latitude was discovered by Hevelius too (see [72], p. 96). It depends on the Moon's axis which is not normal to the plane of the orbit. A terrestrial observer sees the North and the South poles of the Moon alternately inclined towards the Earth ($\pm 6.8^\circ$ maximum [72]).

Optical diurnal libration, also called parallactic libration, was discovered by Galileo (as we will shortly see). It depends on the position of the observer on the Earth and from the position of the Moon on the horizon of the observer. Its maximum value is equal to $57'$ [72].

While the librations listed above are referred to as optical, there is another type of libration, called physical, inherent in the movement of the Moon.

The librations were first observed by Galileo (see [29], p. 33) the one in longitude in 1632 and the one in latitude in 1637, but Hevelius was the first to portray the effects of these in images (see [22], p. 57).

In 1693, J.D. Cassini (see [73], p. 35) published some empirical laws, now called Cassini's laws, which described approximately the rotation of the Moon, whose deviation from laws was nothing but the physical libration [74]. They are as follows (see [75], p. 50):

- 1) The Moon rotates eastward, about a fixed axis, with constant angular velocity and in a period of one sidereal month.
- 2) The inclination I of the Moon's axis of rotation to the ecliptic remains constant.
- 3) The poles of the Moon's axis of rotation, of the ecliptic and of the lunar orbit lie in the same plane, and on one great circle, in that order ⁽²⁶⁾.

Optical librations were analysed by Newton in the light of his gravitation theory ⁽²⁷⁾ (see [76], Prop. 17, Theo. 15, p. 820). Later in the same volume Newton, discussing the problem of the figure of the Moon, mentioned also the possibility of its physical librations caused by the attraction of the Earth but "... neither Jean Dominique Cassini himself, nor his son Jacques ever published the observations ...". This is why for Newton, following Hevelius, the axis of rotation of the Moon was still perpendicular to its orbital plane (see [75], p. 58). Newton's solution was however considered incomplete. In 1764 the Académie des Sciences in Paris offered Lagrange a special prize for his solution of the problem after Cassini's laws, then unknown to Newton. Further difficulties arose when the gravitational influence of the Sun was also taken into consideration. The whole problem was still considered unsolved in 1818 if Laplace felt the need to propose to the Académie des Sciences in Paris to set up a new prize to be awarded to whoever succeeded in constructing lunar tables based solely on the law of universal gravitation [77].

⁽²⁶⁾ Eckhardt (see [60], p. 7) claims that the third law was added only in the twentieth century, giving implausible reasons for it. Actually, already for Tisserand (see [78], T.II, p. 444) Cassini's laws were three.

⁽²⁷⁾ Newton's libration theory appeared in the Appendix to Nicholas Mercator's *Institutionum astronomicarum libri duo* (see [72], p. 97).

4.1. The libration in Galileo's *Dialogo*

As previously mentioned, G. Righini dated the second and third image (F.9v and F.10r) of the *Sydericus Nunciatus* having found that Galileo, unwittingly, recorded the phenomenon of the libration in latitude. So, according to G. Righini, already from his first telescopic observations of the Moon, Galileo had highlighted the phenomenon of “optical libration” —namely a combination of the libration in longitude and the libration in latitude— but he had not realised their meaning and origin. Indeed in the *Sydericus Nunciatus* there was no reference to it. In the *Dialogo* (1632), instead, and afterwards in the letters to Fulgenzio Micanzio (1637) and Alfonso Antonini (1638), Galileo set out in detail his ideas on the subject.

It was Salviati who introduced the subject of a new event observed on the Moon by his “Academic friend” (Galileo): 1) we do see somewhat more than half the Moon and 2) the Moon’s motion bears an exact relation to the centre of the Earth (see [62], pp. 65-66). For an observer on the surface of the Earth, the line joining his eye with the center of the Moon did not cross in the same point on the Moon as the line joining the center of the Earth with the center of the Moon. So Galileo highlighted the difference between geocentric observation —the observer at the centre of the Earth— and topocentric observation —the observer anywhere on the Earth’s surface—. When the observer is at the zenith, geocentric and topocentric observation coincided.

In other words the observer saw some parts of the edge of the Moon’s hemisphere and at the same time some parts were hidden for him. If the observer had been at the centre of the Earth he would have always seen the same face of the Moon. It is evident that Galileo was speaking of the “parallactic libration” (also called “diurnal libration”) and that he did not grasp the different consequences of libration in longitude and latitude.

Galileo also recorded in his observations, the phenomenon of the optical libration in longitude:

Now the telescope has made it certain that this conclusion is in fact verified. For there are two special markings on the moon, one of which is seen to the north-west when the moon is on the meridian, and the other almost diametrically opposite. The former is visible even without a telescope, but not the latter. The one toward the north-west is a small oval spot separated from three larger ones. The opposite one is smaller, and likewise stands apart from larger marks in a sufficiently clear field. In both of these the variation mentioned already is quite clearly observed; they are seen opposite to one another, now close to the edge of the lunar disc and now farther away. The difference is such that the distance between the north-westerly spot and the edge of the disc is at one time more than twice what it is at another. As to the other spot, being much closer to the edge of the disc, the change is more than threefold from one time to the other. From which it is obvious that the moon, as if drawn by a magnetic force, faces the earth constantly with one surface and never deviates in this regard (see [62], pp. 66-67).

Galileo, according to Righini (see [10], p. 37) attributed it to the movement of the observer due to the Earth’s rotation. According to Włodarczyk, instead, [32] Galileo

was speaking about the combination of the libration in longitude and that one in latitude.

As we have seen, G. Righini's approach to the phenomenon of libration in Galileo was strongly criticized by Gingerich, partly for philosophical considerations and partly because it presupposed an accuracy that Galileo's images of the Moon did not have.

Whitaker recognized that 1974 Righini's work was an "ingenius paper" by an author, I would add, who first had had the courage and skill to tackle a topic that seemed beyond the scope of a quantitative evaluation. However the core of Whitaker's criticism regarded Galileo, who would not have recorded at all, even if unwittingly, the effects of the libration in latitude. G. Righini based his calculations on the position of the large crater in image 2 (F. 9v) and 3 (F.10r) of the *Sydereus Nuncius*. His results were in agreement with the data of the libration but, if I have understood correctly Whitaker's reasoning, agreement was due to chance: if Righini had been forced in the absence of the crater, to rely on the borders of the mountain common to the Maria Serenitatis and Inbrium, the value of motion in latitude would have occurred in the opposite direction.

In technical language, Whitaker reposed the old problem, already raised by Hevelius, that Galileo did not have a good telescope or did not care enough for his observations or he was ignorant of the art of painting ⁽²⁸⁾, in contrast instead with those who, like Righini, maintained that Galileo's printed images of the Moon represented a transposition on paper of what Galileo actually had observed and not an invention or a remake as claimed by Gingerich.

As we have seen G. Righini and Włodarczyk did not agree on the interpretation of Galileo's claims. And other dissenting statements could be quoted. Their dissent is not due to justified methodological problems relating to the modalities of historical research. In fact, it is Galileo who was imprecise when speaking about libration. The Pisan believed that the Moon did not rotate on its axis and that its trajectory was perfectly circular. With these assumptions, the phenomenon of libration could not be correctly set, much less interpreted.

As for the ashen light it is possible to hypothesize that both the printed images and those drawn by Galileo are sufficiently accurate to carry out quantitative investigations such as those of G. Righini or Whitaker. However, the fact remains that Galileo was not able to correctly interpret the phenomenon of libration.

4.2. The libration in Galileo's letters to Micanzio and to Antonini

After the publication of the *Dialogo* (1632), and the tragic consequences that followed, Galileo returned to the subject of the libration in a letter to Micanzio [79],

⁽²⁸⁾ "Hinc vero plane colligitur, Galilæum, aut satis idoneo Perspicillo caruisse, aut iisdem observationibus suis non satis vacare potuisse, aut quod potissimum, artem pictoriam & delineatoriam ignorasse; quæ alias huic operi admodum inservit, uti non minus visus acutus, Patientia & Labor" (see [80], p. 205).

a Venetian friend of his. In the letter he spoke of two wonders he had discovered: 1) three variations of the Moon's face and 2) they have three different periods.:

I have discovered a very wonderful observation in the face of the Moon, in which, although it has been looked at infinite times, I do not find that any change has been observed, but that the same face always represents itself in the same view; which I find not to be true, on the contrary that she is changing our appearance with all three possible variations, making towards us those mutations that does who, exposing his face to our eyes, and as they say "in maestà" (front face), is changing in all possible ways, that is, turning it a little now to the right and now to the left, or actually raising and lowering it, or finally tilting it now to the right and now to the left shoulder.

All these mutations are seen to take place in the face of the moon, and the great and ancient spots, which can be seen in it, make what I am saying manifest and meaningful to us. We add a second surprise, and it is that these three different mutations have three different periods: it is important that one changes from day to day, and thus comes to have its diurnal period; the second is changing from month to month, and has its menstrual period; the third has its annual period, according to which its variation ends [my translation] (see [79], pp. 214-215).

So Galileo spoke of three mutations with different periods: daily, monthly and yearly. The first two are librations: the first one is nothing more than the diurnal libration (or parallactic libration), while the second is a combination of the libration in longitude and that in latitude. The third one has nothing to do with librations.

In a letter to Antonini he will no longer speak of annual mutations, testifying to be clearly aware of the difference between "mutations" of the Moon appearances, due to the relative motions of Earth-Moon-Sun system and the "librations" which allow the terrestrial observer to see more that 50% of the lunar face, but whose origin is not specified by Galileo except in the case of the change of the observer position on the terrestrial surface (parallactic libration) [81].

In the letter to Antonini, written on February 20th, 1638, "from the Arcetri prison", Galileo stated that the first reason that led him to support a similarity between the Earth and the Moon was that the main philosophers had supported it:

I therefore say to VS Ill.ma that the first reason that led me to estimate that the correspondence and, as they say, kinship ⁽²⁹⁾ between the Moon and the Earth was great, was that this conclusion was commonly held and pronounced by the main philosophers. In the second place, this concept was confirmed in me by the diversity of the spots that can be seen on the face of the moon, very similar to those which on earth would appear through the continents and seas, when they were gazed at from a great distance; but then this opinion is greatly enhanced by more minute details that can be seen in it, very similar to ours on earth: I say from the wide flat

⁽²⁹⁾ In Italian "cognazione" is the same word, in Latin, used by Copernicus (in latin "cognationem" accusative of "cognatio", relationship or kinship in english) See p. 4 of this paper. It is worth stressing that Galileo employed the same Copernican word used twenty-eight years before in the *Sydereus Nuncius*.

countryside, and from the long stretches of mountains and groups of reefs, which are equally and very similar both on them. ...

The Moon, therefore, with a diurnal period raises and lowers its face in setting and being born; and with the menstrual period she turns it to the left and to the right.

...

I wanted with more accurate observations to go and find other peculiarities ... but fate prevented me from doing this, since for about six months I had a congestion in my eyes that deprived me of the use of the telescope ... [and] that it ended in a total blindness [my translation] (see [81], pp. 292, 294, 296).

In the letter Galileo reported the theses that he had already maintained in the letter to Micanzio. However, there was a difference: this time he did not talk about the annual libration.

5. Conclusions

In a previous paper I have underlined how the different quality of the images of the Moon by Leonardo and by Galileo were not assigned only to the use of the telescope [82]. In the century between the two representations, some great events had taken place that had changed our way of interpreting natural phenomena, giving rise to what historians have defined “The Scientific Revolution” with the purpose of stressing the discontinuity that had occurred in the history of philosophical and scientific thought. A phenomenon that did not concern only the analysis, representation, transformation and modelling of natural phenomena, but involved all aspects of our being social individuals.

In the *Sydereus Nuncius* Galileo destroyed the Aristotelian dichotomy between celestial and terrestrial phenomena. He was not the first to deduce that the Moon was completely similar to Earth. Leonardo as well maintained the same thesis, but he did not derive from it a critique either of the Ptolemaic system or to Aristotelian dichotomy between sublunar and celestial world. Galileo, thanks to his Copernican views, his mechanical and mathematical background, his drawing abilities, announced the proof of the fallacy of the Ptolemaic system. We know, after General Relativity, that a scientific choice between a frame of reference centred on the Sun or centred on the Earth no longer makes sense: but, four centuries ago, the claim that the Copernican system was not only more effective in describing astronomical phenomena but was also true led Galileo to condemnation and abjuration and, in the 1633, his *Dialogo* was put on the *Index librorum prohibitorum*.

The seven original drawings of the telescopic observations of the Moon show clearly that Galileo was an expert in watercolour technique and, in particular, in *chiaroscuro* technique. With the deft brushstrokes of a practiced watercolourist, Galileo laid at least half a dozen different grades of washes, imparting to his images an attractive soft and luminescent quality (see [83], p. 229).

For Galileo, *diseño* was a tool to exercise his sight and hands and not just a means of making beautiful figures. The skills of the astronomer had to be combined with

those of the painter. Galileo aimed to prove that the lunar surface was very similar to the Earth's one. Galileo's drawings of the Moon were not a pure converting into an image what his eyes had seen with the help of the telescope. He interpreted the observational data in the light of a theoretical context [84]. In the *Sydereus Nuncius* Galileo gave quite a long description of the phenomenon of the ashen light of the Moon.

We do not know if Galileo knew Leonardo's hypothesis; in any case, as I tried to show, his proof was very different. Galileo not only proved that the Moon was another Earth, a strand of thought introduced by Copernicus: from this point of view Leonardo had been the first who has highlighted similarities between the Moon and the Earth. But Galileo, unlike Leonardo, thought that rough surfaces reflected light better than flat and shiny surfaces and proved this in the *Dialogo*, through an experiment probably actually carried out with Paolo Sarpi many years earlier. Whereas Leonardo had been forced to introduce a mechanical model of the Moon as a "toothed wheel" in order to account for the phenomenon of the ashen light.

As far as the lunar libration was concerned, Leonardo certainly did not notice it. On the other hand, it is controversial whether Galileo actually realised that he had observed the new phenomenon of the lunar libration before 1632. G. Righini argued that in the *Sydereus Nuncius* the images of the Moon showed that Galileo, although without realizing it, recorded the phenomenon and, on the basis of it, proposed their dating. Galileo, however, noticed the phenomenon not before 1632 and, in any case, gave an incomplete interpretation of it. His bias that the orbits of the planets and the Moon must be circular and travelled with uniform speed prevented him from analysing the phenomenon in the light of Kepler's laws.

In summary, if we analyse in detail Leonardo's fragmentary arguments about the Moon and do the same with those proposed by Galileo in the *Sydereus Nuncius*, in the *Dialogo* and in the Letters, we notice a difference that only the emergence of new styles of thinking and visual and observational practices can explain.

* * *

I thank the anonymous referee who provided me with useful annotations and Gianfranco Truffa for his careful reading of the text and for his valuable suggestions.

Bibliography

- [1] GALILEO GALILEI, *Sydereus Nuncius* (Ex Typographia HH. de Ducijs, Bononiæ) 1655.
- [2] ALBERT VAN HELDEN (Editor & translator), *Galileo Galilei Sidereal Messenger* (The University Chicago Press, Chicago) 1989.
- [3] EILEEN REEVES, *Painting the Heavens. Art and Science in the age of Galileo* (Princeton University Press, Princeton) 1997.
- [4] RODOLFO MAFFEIS, "Water as Microscope of Nature", in PAOLO GALLUZZI (Editor), *Leonardo da Vinci's Codex Leicester* (Giunti, Firenze) 2018, pp. 251–265.
- [5] VICTOR ROBERTS, "The Solar and Lunar Theory of Ibn ash-Shātir: A Pre-Copernican Copernican Model", *ISIS*, **48** (1957) 428.

- [6] EMMANUEL A. PASCHOS e PANAGIOTIS SOTIROUDIS, *The schemata of the stars. Byzantine astronomy from A.D. 1300* (World Scientific Publishing, Singapore) 1998.
- [7] F. JAMIL RAGEP, "Copernicus and his Islamic predecessors. Some historical remarks", *History of Science*, **45** (2007) 65.
- [8] GEORGE SALIBA, *A History of Arabic Astronomy. Planetary Theories during the Golden Age of Islam* (New York University Press, New York/London) 1994.
- [9] GUGLIELMO RIGHINI, "New light on Galileo's Lunar observations", in MARIA LUISA RIGHINI e WILLIAM R. SHEA (Editors), *Reason, Experiment, and Mysticism in the Scientific Revolution* (Science History Publications, New York) 1975, pp. 59–88.
- [10] GUGLIELMO RIGHINI, "Contributo alla interpretazione scientifica dell'opera astronomica di Galileo", *Supplemento agli Annali dell'Istituto e Museo di Storia della Scienza*, **2** (1978) 5.
- [11] GALILEO GALILEI, "Dialogo sopra i due massimi sistemi del mondo" (1632) in ANTONIO FAVARO (Editor), *Le Opere di Galileo Galilei* (Barbèra, Firenze) 1890-1909, 20 Vols, vol. VII, 1897, pp. 21–489.
- [12] EDWARD ROSEN (Editor & translator), *Nicolaus Copernicus. De Revolutionibus Orbium Coelestium, Libri VI. Nuremberg, 1543*, <http://people.reed.edu/wieting/mathematics537/DeRevolutionibus.pdf>.
- [13] EDGAR ZILSEL, "Copernicus and Mechanics", *J. History Ideas*, **1** (1940) 113-118.
- [14] GALILEO GALILEI, "Lettera di Galileo a Benedetto Castelli in Brescia. Firenze, 30 dicembre 1610", in ANTONIO FAVARO (Editor), *Le Opere di Galileo Galilei* (Barbèra, Firenze) 1890-1909, 20 Vols, vol. X, 1900, pp. 499–502.
- [15] GALILEO GALILEI, "Lettera di Galileo a [Cristoforo Clavio in Roma]. Firenze, 30 dicembre 1610", in ANTONIO FAVARO (Editor), *Le Opere di Galileo Galilei* (Barbèra, Firenze) 1890-1909, 20 Vols, vol. X, 1900, pp. 502–505.
- [16] LEOPOLD INFELD, "From Copernicus to Einstein", in BIEŃKOWSKA BARBARA (Editor), *The scientific world of Copernicus. On the occasion of the 500th Anniversary of his birth 1473-1973* (D. Reidel Publishing Company, Dordrecht-Holland/Boston-U.S.A.) 1973, pp. 62–83.
- [17] GIBSON REAVES e CARLO PEDRETTI, "Leonardo da Vinci's drawings of the surface features of the Moon", *J. History Astron.*, **18** (1987) 55.
- [18] FRANCESCO BARBIERI, "Il restauro del Codice Atlantico di Leonardo da Vinci", *Accademie e Biblioteche d'Italia*, **L** (1982) 98.
- [19] GIORGIO VASARI, *Delle vite de' più eccellenti Pittori Scultori et Architettori*, Primo volume della Terza Parte (Giunti, Firenze) 1568.
- [20] MARTIN KEMP, "This will be a collection without order. Compilation and chaos in the Codex Leicester", in PAOLO GALLUZZI (Editor), *Leonardo da Vinci's Codex Leicester* (Firenze, Giunti) 2018, pp. 23–41.
- [21] FLAVIA MARCACCI e SARA TAGLIALAGAMBA, "2019, la Luna, Leonardo", in FLAVIA MARCACCI e SARA TAGLIALAGAMBA (Editors), *Atti del Convegno Leonardo e la Luna. Alle origini della scienza moderna. Cinquantenario di Leonardo e cinquantennio dell'allunaggio. Pontificia Università Lateranense, Roma, 20 novembre 2019*, in *Giornale di Astronomia*, **46** (2020) 3, pp. 2–12.
- [22] EWEN A. WHITAKER, *Mapping and naming the Moon* (Cambridge University Press, Cambridge) 1999.
- [23] NATACHA FABBRI, "The Moon as another Earth: what Galileo owes to Plutarch", *Galilæana, J. Galilæan Studies*, **IX** (2012) 103.
- [24] NATACHA FABBRI, "Looking at an Earth-Like Moon and Living on a Moon-Like Earth in Renaissance and Early Modern Thought", in CECILIA MURATORI e GIANNI PAGANINI (Editors), *Early Modern Philosophers and the Renaissance Legacy* (Springer International Publishing, Cham) 2016, pp. 135–151.
- [25] SCOTT L. MONTGOMERY, *The Moon and the Western imagination* (The University of Arizona Press, Tucson) 1999.
- [26] JAY M. PASACHOFF e ROBERTA J.M. OLSON, "Depictions of the Moon in Western Visual Culture", in PETER READ (Editor), *Oxford Research Encyclopedia of Planetary Science* (Oxford University Press, Oxford) 2016, pp. 135–151, <https://doi.org/10.1093/acrefore/9780190647926.013.55>.
- [27] LEONARDO DA VINCI, (*Hammer Codex*). *Drawings and Writings* (WebUp London) 2016.

- [28] RODOLFO MAFFEIS, “«Quasi dentatæ rotæ»: Leonardo disegna la Luna”, in FABIO FROSINI e ALESSANDRO NOVA (Editors), *Leonardo da Vinci on Nature. Knowledge and Representation* (Marsilio, Venezia) 2013, pp. 303–337.
- [29] PAOLO FRISI, *Elogio del Galileo* (Nella Stamperia dell’Enciclopedia, Livorno) 1774.
- [30] DOMENICO LAURENZA, *The Codex Leicester. Leonardo da Vinci* (Giunti, Firenze) 2018.
- [31] JEAN PAUL RICHTER (Editor), *The Literary works of Leonardo da Vinci* (Sampson Low, Marston, Searle & Rivington, London) 1883, 2 Vols.
- [32] JAROSŁAW WŁODARCZYK, “Libration of the Moon, Hevelius’s theory, and its early reception in England”, *J. History Astronomy*, **42** (2011) 495.
- [33] ALBERTO RIGHINI, “The telescope in the making, the Galileo first telescopic observations”, in CESARE BARBIERI, SUKANYA CHAKRABARTI, MARCELLO CORRADINI e MONICA LAZZARIN (Editors), *Galileo’s Medicean Moons: their impact on 400 years of discovery* (Cambridge University Press, Cambridge) 2010, pp. 26–32.
- [34] GIBSON REAVES, “The use of the lunar librations to determine the date of Leonardo da Vinci’s drawing of the moon in the Codex Atlanticus 674 verso”, in “Proceedings of 182nd AAS Meeting, Berkely, California”, *Bull. Am. Astron. Soc.*, **17** (1985) 844.
- [35] ROMANO AMERIO, *Fra Paolo Sarpi. Scritti filosofici e teologici. Editi e inediti* (Gius. Laterza & Figli, Bari) 1951.
- [36] ANNA DE PACE, “Galileo lettore di Copernico” (Casa Editrice Leo S. Olschki, Firenze) 2020.
- [37] SVEN DUPRÉ, “Galileo’s telescope and celestial light”, *J. History Astronomy*, **34** (2003) 369.
- [38] DAVID WOOTTON, “New light on the composition and publication of the *Sidereus Nuncius*”, *Galilæana. Journal of Galilæan Studies*, **VI** (2010) 123.
- [39] OWEN GINGERICH e ALBERT VAN HELDEN, “From Occhiale to printed page: the making of Galileo’s *Sidereus Nuncius*”, *J. History Astronomy*, **34** (2003) 251.
- [40] PAUL NEEDHAM (Editor), *Galileo makes a book: the first edition of Sidereus nuncius, Venice 1610* (Akademie Verlag GmbH, Berlin) 2011, Vol. II of HORST BREDEKAMP (Editor), *Galileo’s O* (De Gruyter and Akademie Verlag GmbH, Berlin) 2011–2014, vols I–IV.
- [41] FRANCO GIUDICE, “Galileo’s cosmological view from the *Sidereus Nuncius* to *Letters on Sunspots*”, *Galilæana. J. Galilæan Studies*, **XI** (2014) 49–63.
- [42] MASSIMO BUCCIANINI, “Galileo e la Nova del 1604”, in MASSIMO BUCCIANINI e MAURIZIO TORRINI (Editors), *La diffusione del copernicanesimo in Italia 1543–1610* (Leo S. Olschki, Firenze) 2010, pp. 237–248.
- [43] GALILEO GALILEI, “Lettera di Galileo a Giovanni Kepler in Graz, n. 57, Padova, 4 Agosto 1597”, in ANTONIO FAVARO (Editor), *Le Opere di Galileo Galilei* (Barbèra, Firenze) 1890–1909, 20 Vols, vol. X, 1906, pp. 67–68.
- [44] GALILEO GALILEI, “Lettera di Galileo a [Antonio de’ Medici in Firenze?], n. 259, [Padova], 7 gennaio 1610”, in ANTONIO FAVARO (Editor), *Le Opere di Galileo Galilei* (Barbèra, Firenze) 1890–1909, 20 Vols, vol. X, 1900, pp. 273–278.
- [45] STILLMAN DRAKE, “Galileo’s first telescopic observations”, *J. History Astronomy*, **7** (1976) 153–168.
- [46] ELIZABETH CAVICCHI, “Painting the Moon”, *Sky and Telescope*, **LXXXII** (1991) 313.
- [47] OWEN GINGERICH, “The mystery of the missing 2”, *J. History Astronomy*, **9** (2012) 91.
- [48] OWEN GINGERICH, “Dissertatio cum Professor Righini et Sidereo Nuncio”, in MARIA LUISA RIGHINI e WILLIAM R. SHEA (Editors), *Reason, Experiment, and Mysticism in the Scientific Revolution* (Science History Publications, New York) 1975, pp. 77–88.
- [49] HORST BREDEKAMP, *Galileo’s thinking hand* (De Gruyter, Berlin/Boston) 2019.
- [50] HORST BREDEKAMP, *Galilei der Künstler: Der Mond, die Sonne, die Hand* (Akademie Verlag, Berlin) 2007.
- [51] OWEN GINGERICH, “The curious case of the M-L Sidereus Nuncius”, *Galilæana. J. Galilæan Studies*, **VI** (2009) 141.
- [52] STILLMAN DRAKE, “Éloge. Guglielmo Righini”, *ISIS*, **70** (1979) 552.
- [53] EWAN A. WHITAKER, “Galileo’s lunar observations and the dating of the composition of Sidereus Nuncius”, *J. History Astronomy*, **9** (1978) 155.
- [54] ALBERT VAN HELDEN, “Unmasking Galileo forgery”, *J. History Astronomy*, **45** (2014) 361, 507.
- [55] ALBERTO RIGHINI, “Sulle date delle prime osservazioni lunari di Galileo”, *Giornale di Astronomia*, **35** (2009) 7.

- [56] NOEL M. SWERDLOW, “Galileo’s discoveries with the telescope and their evidence for the Copernican theory”, in P. MACHAMER (Editor), *The Cambridge Companion to Galileo* (Cambridge University Press, Cambridge) 1998, pp. 244–270.
- [57] HORST BREDEKAMP, “Gazing hands and blind spots: Galileo as a draftsman”, *Science Context*, **13** (2000) 423.
- [58] PAOLO MOLARO, “On the Earthshine depicted in Galileo’s watercolors of the Moon”, *Galilæana. J. Galilæan Studies*, **X** (2013) 73.
- [59] ISABELLE PANTIN (Editor), *Galileo Galilei. Sidereus Nuncius. Le Messager Céleste* (Les Belles Lettres, Paris) 1992.
- [60] DONALD H. ECKHARDT, “Theory of the libration of the Moon”, *The Moon and the Planets*, **25** (1981) 3.
- [61] GALILEO GALILEI, “Lettera di Galileo a [Belisario Vinta in Pisa]. Padova, 19 marzo 1610”, in ANTONIO FAVARO (Editor), *Le Opere di Galileo Galilei* (Barbèra, Firenze) 1890-1909, 20 Vols, vol. X, 1900, pp. 297–302.
- [62] GALILEO GALILEI, “Dialogue Concerning the Two Chief World Systems - Ptolemaic & Copernican” (1967), Second edition (University of California Press, Berkeley and Los Angeles) 1967. English translation of the “Dialogue . . .” made by Stillmann Drake.
- [63] JOHANN GEORG LOCHER, “Disquisitiones mathematicæ de controversiis et novitatibus astronomicis”, *Le Opere di Galileo Galilei* (ex typographeo Ederiano apud Elisabetham Angermariam, Ingolstadii) 1614.
- [64] GALILEO GALILEI, “Lettera a [Leopoldo de’ Medici in Pisa], n.3982, Arcetri, 13 Marzo 1640”, in ANTONIO FAVARO (Editor), *Le Opere di Galileo Galilei* (Barbèra, Firenze) 1890-1909, 20 Vols, vol. XVIII 1898, pp. 165–166.
- [65] FORTUNIO LICETI, *De Lunæ subobscura luce* (Nicolai Schiratti, Utini) 1642.
- [66] PAOLO GALLUZZI, “L’Accademia del Cimento: «Gusti» del Principe, Filosofia e Ideologia dell’esperienza”, *Quaderni Storici*, **16** (1981) 788.
- [67] GIUSEPPE ONGARO, “LICETI, Fortunio”, in *Dizionario Biografico degli Italiani* (Istituto della Enciclopedia italiana, Roma) 1961-2020, Vol. 65, 2005, [https://www.treccani.it/enciclopedia/fortunio-liceti_\(Dizionario-Biografico\)](https://www.treccani.it/enciclopedia/fortunio-liceti_(Dizionario-Biografico)).
- [68] GALILEO GALILEI, “Lettera al Principe Leopoldo di Toscana in proposito del cap. L° del Lithosphorus di Fortunato Liceti”, in ANTONIO FAVARO (Editor), *Le Opere di Galileo Galilei* (Barbèra, Firenze) 1890-1909, 20 Vols, vol. VIII, 1898, pp. 467–556.
- [69] ANTONIO FAVARO (Editor), “Galileo Galilei Sidereus Nuncius 1610”, in *Le Opere di Galileo Galilei* (Barbèra, Firenze) 1890-1909, 20 Vols, vol. III parte prima, 1892, pp. 53–96.
- [70] FORTUNIO LICETI, *Lithosphorus, sive de lapide bononiensi* (Nicolai Schiratti, Utini) 1640.
- [71] SUSAN GÓMEZ, “The Bologna Stone and the nature of light. The Sciences Academy of Bologna”, *Nuncius. Annali di Storia della Scienza*, **6** (1991) 3.
- [72] ALAN GABBEY, “Innovation and Continuity in the History of Astronomy: The Case of the Rotating Moon”, in PETER BARKER e ROGER ARIEW (Editors), *Revolution and continuity. Essays in the History and Philosophy of early modern science* (The Catholic University of America Press, Washington D.C.) 1991, pp. 95–129.
- [73] JEAN DOMINIQUE CASSINI, “De l’origine de l’Astronomie et de son usage dans la geographie et dans la navigation”, in *Recueil d’Observations faites en plusieurs voyages par ordre de sa Majesté pour perfectionner l’astronomie et la geographie. Avec divers Traitez Astronomiques* (de l’Imprimerie royale, Paris) 1693, pp. 1–43.
- [74] KAROL KOZIEL, “Libration of the Moon”, in ZDENĚK KOPAL (Editor), *Physics and astronomy of the Moon* (Academic press, New York and London) 1962, pp. 27–29.
- [75] ZDENĚK KOPAL e ROBERT CARDER, *Mapping the Moon. Past and present* (Springer-Science+Business Media, B.V., Dordrecht) 1974.
- [76] ISAAC NEWTON, *The Principia. Mathematical Principles of Natural Philosophy* (University of California Press, Berkeley-Los Angeles-London) 1999. A new translation by I. Bernard Cohen and Anne Whitman assisted by Julia Budenz. Preceded by A GUIDE TO NEWTON’S *PRINCIPIA* BY I. Bernard Cohen.
- [77] GUIDO TAGLIAFERRI e PASQUALE TUCCI, “Carlini and Plana on the theory of the Moon and their dispute with Laplace”, *Annals of Science*, **56** (1999) 221.
- [78] FELIX TISSERAND, *Traité de Mécanique céleste* (Imprimerie Gauthiers-Villars et fils, Paris)

- 1891, 2 Vols.
- [79] GALILEO GALILEI, “Lettera di Galileo a [Fulgenzio Micanzio in Venezia], n. 3595, Arcetri, 7 Novembre 1637”, in ANTONIO FAVARO (Editor), *Le Opere di Galileo Galilei* (Barbèra, Firenze) 1890-1909, 20 Vols, vol. XVII, 1906, pp. 214–215.
- [80] JOHANNES HEVELIUS, *Selenographia sive Lunæ descriptio; . . .* (typis Hunefeldianis, Gedani) 1647.
- [81] GALILEO GALILEI, “Lettera di Galileo ad Alfonso Antonini in Udine, n. 3684, Arcetri, 20 Febbraio 1638”, in ANTONIO FAVARO (Editor), *Le Opere di Galileo Galilei* (Barbèra, Firenze) 1890-1909, 20 Vols, vol. XVII, 1906, pp. 291–297.
- [82] PASQUALE TUCCI, “Leonardo’s and Galileo’s drawings of the Moon”, in ADELE LA RANA e PAOLO ROSSI (Editors), *Proceedings of the 39th Annual conference: Pisa, 9-12 Settembre 2019* (Pisa University Press, Pisa) 2020, pp. 5–11.
- [83] SAMUEL Y. EDGERTON JR., “Galileo, Florentine «Disegno», and the «Strange Spottedness» of the Moon”, *Art Journal*, **44** (1984) 225.
- [84] MICHELE CAMEROTA, *Galileo Galilei e la cultura scientifica nell’età della controriforma* (Roma, Salerno Editrice) 2004.