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Universality of the Weak Interactions, Cabibbo theory and where they led us(*)

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Summary. — Universality of the weak interactions is reviewed, with special emphasis on the origin of the Cabibbo theory of strange-particle β-decays and its role in the discovery of the unified Electroweak Theory. Achievements and present challenges of the Standard Theory of particles interactions are briefly illustrated. As a homage to Nicola Cabibbo, his leading role in the Roma school of theoretical physics and in the Italian science in general is reviewed. A selection of papers by Cabibbo and other authors, reprinted from *Il Nuovo Cimento* and historically related to the arguments considered here, is presented. The picture is completed with the classical paper by Cabibbo and Gatto on electron-positron collisions and Cabibbo's paper on the weak-interaction angle, reprinted from *Physical Review* and *Physical Review Letters*, respectively.

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1. – Universal Weak Interactions

In a 1961 book, Richard Feynman [1] vividly described his and Murray Gell-Mann's satisfaction at explaining the close equality of the muon and neutron beta decay Fermi constants. They [2] and, independently, Gershtein and Zeldovich [3] had discovered the universality of the weak interactions, closely similar to the universality of the electric charge and a tantalising hint of a common origin of the two interactions. But Feynman recorded also his disconcert following the discovery that the Fermi constants of the strange particles, *e.g.* the β-decay constant of the Λ baryon, turned out to be smaller by a factor of 4-5. It was up to Nicola Cabibbo [4] to reconcile strange-particle decays with the universality of weak interactions, paving the way to modern electroweak unification.

2. – Nicola Cabibbo: The beginning

Cabibbo's scientific life in steps:

– graduates in 1958, tutor Bruno Touschek;

– becomes the first thoretical physicist in Frascati, hired by G. Salvini;

– meets there Raoul Gatto (5 years elder) who was coming back from Berkeley and begins an extremely fruitful collaboration;

– witnesses exciting times in Frascati: the first e^+e^- collider, AdA (Anello di Accumulazione = storage ring), to be followed, later, by a larger machine, Adone (= larger AdA), reaching up to 3 GeV in the center of mass (= laboratory) frame; new particles (the η meson) studied at the electro-synchrotron, related to the newly discovered $SU(3)$ symmetry, etc.;

– publishes together with Gatto an important article on e^+e^- physics [5] (the Bible);

– in 1961, again with Gatto, investigates the weak interactions of hadrons in the framework of the newly discovered $SU(3)$ symmetry.

3. – The $V - A$ and Current $\times$ Current theory of the Weak Interactions

The Fermi weak interaction Lagrangian was simply the product of four-fermion fields ψ_i connected by Dirac matrices, which Fermi, to keep the analogy with electromagnetism, restricted to be γ_μ matrices. For the neutron β-decay:

$$\mathcal{L}_n = G\left[\bar{\psi}_p\gamma_\mu\psi_n\right] \times \left[\bar{\psi}_e\gamma^\mu\psi_\nu\right] \ + \ \text{h.c.} \tag{1}$$

Subsequent studies of nuclear decays and the discovery of parity violation, led to complicate the gamma matrix structure, introducing all possible kinds of relativistically invariant products of two bilinear fermion fields. At the end of the fifties, simplicity finally emerged, with the recognition that all β-decays could be described by a $V - A$ theory. Sudarshan and Marshak [6], and Feynman and Gell-Mann [7] proposed the general rule:

– *every ψ replaced by $a\psi$, with: $a = \frac{1-\gamma_5}{2}$.*

With this position, we are brought essentially back to Fermi. The Lagrangian in (1) reads now:

$$\mathcal{L}_n = \frac{G}{\sqrt{2}} \left[\bar{\psi}_p \gamma_\mu (1-\gamma_5)\psi_n\right] \times \left[\bar{\psi}_e \gamma^\mu (1-\gamma_5)\psi_\nu\right] \tag{2}$$

(the factor $1/\sqrt{2}$ is inserted so as to keep the constant G at the same value determined by Fermi from superallowed nuclear transitions).

The $V-A$ structure in eq. (2) is *almost* experimentally correct. The coefficient of γ_5 in the nuclear bilinear is in fact $g_A/g_V \simeq 1.25$ rather than unity, to be interpreted as a strong-interaction renormalisation.

Under the $(1-\gamma_5)$ rule given above, only vector and axial vector currents survive in the Fermi interaction. Equation (2) further suggests the Current $\times$ Current hypothesis:

– *the Lagrangian of the full weak interactions, describing muon, meson, etc. β-decays, has the form*

$$\mathcal{L}_W = \frac{G}{\sqrt{2}} J^\mu \times J^+_\mu, \tag{3}$$

with J_μ the sum of $n-p$, $e-\nu_e$, etc. contributions. Omitting gamma matrices:

$$J = (\bar{\nu}_e e) + (\bar{\nu}_\mu \mu) + (\bar{p}n) + X. \tag{4}$$

X represents the contribution of the current to strange-particle decays and we have to consider now what properties the term X might have (I follow here almost *verbatim* the considerations made by Feynman in [1]).

A first observation is that if we insert the form (4) into (3), the terms corresponding to electronic and muonic decays of strange particles will appear with the same coefficient. This corresponds to the so-called *electron-muon universality*, which indeed is very well satisfied in strange-particle β-decays.

Second, semi-leptonic decays of strange particles seem to be suppressed with respect to nuclear β-decays, which implies the term X to appear with a small coefficient, of the order of 0.1.

However, if that were the case, a similar suppression should hold for the term $X \times (\bar{n}p)$, which, judging from K_S decay, does not seem to be the case.

Here ends Feynman's analysis of 1961. In modern terms, the suppression of the semi-leptonic strange-particle decays got mixed with the $\Delta I = 1/2$ *enhancement* of non-leptonic decays, resulting in what seemed to be, at the time, a really inextricable mess.

4. – Gell-Mann and Levy's ansatz

An observation made in 1960 by M. Gell-Mann and M. Levy [8] is often quoted as a precursor or source of inspiration for Cabibbo. This is justified to some extent, but the role of Gell-Mann and Levy's observation need not be overestimated. Gell-Mann and Levy's paper is quoted by Cabibbo and was well known to all those working in the field.

In the GML paper, the weak current is written in the Sakata model, with elementary P, N and Λ. All hadrons are supposed to be made by these three fundamental fields.

GML observe that one could relate the reduction of the Λ coupling with respect to the muon coupling by assuming the following form of the weak vector current:

$$V_\lambda = \frac{1}{\sqrt{1-\epsilon^2}}\left[\bar{P}\gamma_\lambda\left(N+\epsilon\Lambda\right)\right]. \tag{5}$$

But... nobody knew how to proceed from the GML formula to a real calculation of meson and baryon decays, for two reasons:

i) The Sakata model was already known to be substantially wrong, due to the absence of positive-strangeness baryons. Thus, inclusion of the decays of the $S=-1$ and $S=-2$ hyperons was completely out of reach.

ii) The important point of the non-renormalisation was missed. In Gell-Mann and Levy's words [8]: *There is, of course, a renormalization factor for that decay, (i.e.,* Λ *decay) so we cannot be sure that the low rate really fits in with such a picture.*

5. – $SU(3)$ symmetry and weak interactions

Gatto and Cabibbo [9] and Coleman and Glashow [10] observed that the Noether currents associated to the newly discovered $SU(3)$ symmetry include a strangeness-changing current that could be associated with strangeness-changing decays, in addition to the isospin current responsible for strangeness–non-changing beta-decays (CVC). The identification, however, implied the rule $\Delta S = \Delta Q$ in the decays, in conflict with some alleged evidence of a $\Delta S = -\Delta Q$ component, indicated by the single event $\Sigma^+ \to \mu^+ + \nu + n$ reported in an emulsion experiment [11]. In addition, the problem remained how to formulate correctly the concept of CVC and muon-hadron universality in the presence of *three* Noether currents:

$$V_\lambda^{\text{lept}} = \bar{\nu}_\mu\gamma_\lambda\mu + \bar{\nu}_e\gamma_\lambda e \quad (\Delta Q = 1), \tag{6}$$

$$V_\lambda^{(1)} + iV_\lambda^{(2)} \quad (\Delta S = 0,\ \Delta Q = 1), \tag{7}$$

$$V_\lambda^{(5)} + iV_\lambda^{(6)} \quad (\Delta S = \Delta Q = 1). \tag{8}$$

6. – Enters Cabibbo

In his 1963 paper, Nicola made a few decisive steps.

– He decided to ignore the evidence for a $\Delta S = -\Delta Q$ component. Nicola was a good friend of Paolo Franzini, then at Columbia University, and the fact that Paolo had a larger statistics without any such event was crucial.

– He ignored also the problem of the normalisation of non-leptonic processes and of the $\Delta I = 1/2$ enhancement.

– He formulated a notion of universality between the leptonic current and one, and only one, hadronic current, a combination of the $SU(3)$ currents with $\Delta S = 0$ and $\Delta S = 1$: the hadronic current has to be equally normalized to each component of the lepton current (electronic or muonic). Axial currents are inserted via the $V-A$ hypothesis.

In formulae, Cabibbo wrote:

$$V_\lambda^{(\text{hadron})} = a\left[V_\lambda^{(1)} + iV_\lambda^{(2)}\right] + b\left[V_\lambda^{(5)} + iV_\lambda^{(6)}\right], \tag{9}$$

with

$$a^2 + b^2 = 1, \tag{10}$$

to ensure equal normalization of the hadronic with respect to either the electron or the muon component of the leptonic vector current, eq. (6).

Adding these hypotheses to the $V - A$ formulation of the weak interactions, Cabibbo thus arrived to the final expression of the total leptonic and hadronic weak currents:

$$J_\lambda^{\text{lept}} = \bar{\nu}_\mu\gamma_\lambda(1-\gamma_5)\mu + \bar{\nu}_e\gamma_\lambda(1-\gamma_5)e, \tag{11}$$

$$J_\lambda^{(\text{hadron})} = \cos\theta\left[J_\lambda^{(1)} + iJ_\lambda^{(2)}\right] + \sin\theta\left[J_\lambda^{(5)} + iJ_\lambda^{(6)}\right], \tag{12}$$

$$J_\lambda^{(i)} = V_\lambda^{(i)} - A_\lambda^{(i)}. \tag{13}$$

In the above equations, $A_\lambda^{(i)}$ denotes an octet of axial-vector currents. While the normalization of the vector currents is fixed by the very notion of CVC, the axial currents are not conserved and their normalization constants are free parameters, not determined by the $SU(3)$ symmetry. The angle θ is a new constant of Nature, since known as the *Cabibbo angle*.

In the Cabibbo theory:

– Currents belong to $SU(3) \times SU(3)$;

– partial conservation of the vector and axial vector currents protects the normalization of strength;

– the Gatto-Ademollo theorem [12] holds: vector current matrix elements are not renormalized to first order in $SU(3)$ breaking.

The phenomenological success of the Cabibbo theory for semi-leptonic decays has made it clear that the $I = 1/2$ enhancement of non-leptonic decays must have a different origin than the normalization of the strange-particle current, X. This was understood later as a renormalization group effect, as first guessed by K. Wilson [13] and computed in QCD by M. K. Gaillard and B. W. Lee and by G. Altarelli and L. Maiani [14].

As of today, the agreement of the Cabibbo theory with experiments has been but reinforced by the most recent data from Frascati, FermiLab and CERN [15].

7. – The weak current of baryons and the unitarity limit

The form of $J_\lambda^{(\text{hadron})}$, well readable in terms of the $SU(3)$ symmetry, leads to a remarkably complicated form of the current in terms of individual baryon fields (to be

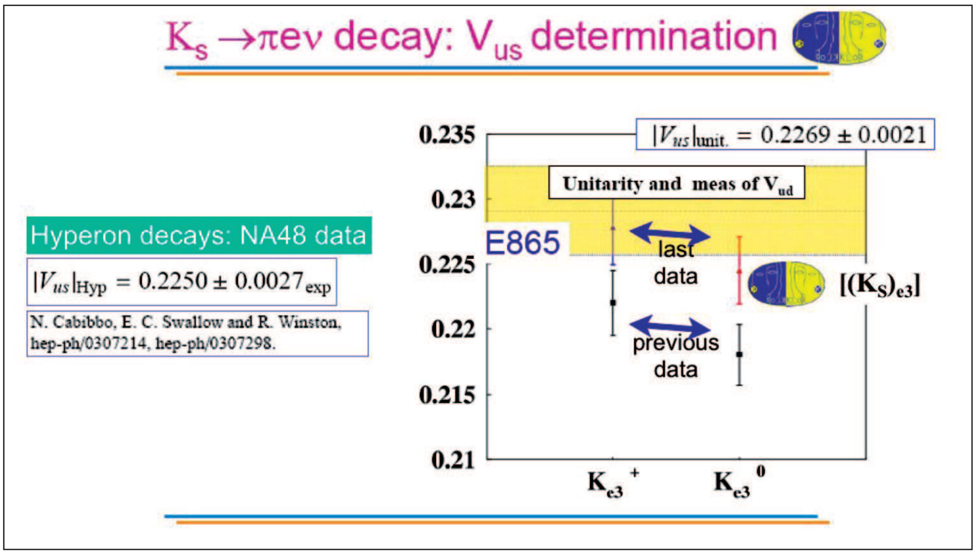

Fig. 1. – Test of Cabibbo unitarity. The grey band indicates the range of $V_{us} = \sin\theta$ predicted from the value of $V_{ud} = \cos\theta$ measured in superallowed nuclear transitions. Indicated are also the latest values of $\sin\theta$ obtained from $Kl3$ decays by the experiments E865 (Fermilab) and KLOE (Frascati); the value of V_{us} from strange-hyperon decays is also reported.

compared with the Gell-Mann and Levy's form):

$$
\begin{aligned}
(14)\qquad J_\mu^{(\mathrm{had})} &= \cos\theta\; \bar{p}\gamma_\mu \left[1-(F+D)\gamma_5\right] n + \sin\theta \left\{ -\sqrt{\frac{3}{2}}\bar{p}\gamma_\mu \left[1-(F+\frac{1}{3}D)\gamma_5\right]\Lambda \right\} \\
&+ \sin\theta \left\{ -\bar{n}\gamma_\mu \left[1-(F-D)\gamma_5\right]\Sigma^- - \bar{\Sigma}^+\gamma_\mu \left[1-(F+D)\gamma_5\right]\Xi^0 \right\} \\
&+ \sin\theta \left\{ \sqrt{\frac{3}{2}}\bar{\Lambda}\gamma_\mu \left[1-(F-\frac{1}{3}D)\gamma_5\right]\Xi^- \right\} \\
&+ \cdots .
\end{aligned}
$$

We have used particle's names to indicate the corresponding fields; F and D are phenomenological coefficients related to axial current renormalization, see the comment made after eq. (2). Experimental data require [16]: $F \simeq 0.46$; $D \simeq 0.80$; $\sin\theta \simeq 0.22$.

The first term in eq. (14) describes the $\Delta S = 0$, $n \to p$ transition and is normalized by the factor $\cos\theta$ which is, of course, less than unity. Thus the Cabibbo theory may explain the observed reduction of the nuclear Fermi constant with respect to the muon one, a fact noticed already by Feynman in [1] following the precise measurement by V. Telegdi and coworkers [17]. The effect was not so clear at that time, as it had to be disentangled by competing electromagnetic radiative corrections, which were not under control in the early sixties. The situation is much clearer today, with precise data coming from superallowed Fermi nuclear transition and radiative corrections under control.

As shown in fig. 1, the determination of the angle from the baryonic $\Delta S = 1$ and the latest data on $Kl3$ decays presented by the Fermilab, E865, and Frascati, KLOE, experiments, agree exceedingly well with what predicted from the superallowed nuclear transitions [18].

8. – Cabibbo theory with quarks

Gell-Mann-Levy's formula was given a new life in the context of the quark model, after the consolidation of the Cabibbo theory. If quarks and flavor-singlet gluons are the fundamental particles, as we know today, β-decays of baryons and mesons simply reflect the two transitions:

$$d \to u; \qquad s \to u. \tag{15}$$

Note that this is similar to Fermi's idea that β-decays of nuclei are simply the manifestation of the $n \to p$ transition.

In the quark picture, the Cabibbo weak current takes the form

$$\begin{aligned} J_\lambda &= \cos\theta \left[\bar{u}\gamma_\lambda(1-\gamma_5)\,(d + \tan\theta s)\right] = \\ &= \bar{u}\gamma_\lambda(1-\gamma_5) d_C, \end{aligned} \tag{16}$$

which coincides with Gell-Mann and Levy's with: $(P, N, \Lambda) \to (u, d, s)$. The Cabibbo angle, θ, is seen as the mixing angle expressing the weakly interacting down-quark, d_C, in terms of the mass-eigenstate fields: d, s.

9. – Equal normalization?

It was clarified by Cabibbo himself, in his 1964 Erice lectures, that the condition (10) implies that the weak charges are the generators of a *weak isospin* $SU(2)$ group. In $SU(3)$ space, θ determines the orientation of the weak $SU(2)$ group with respect to the *strong* $SU(2)$ group, which is determined by the medium-strong interactions which break $SU(3)$ to the familiar isotopic spin symmetry. In the absence of the medium-strong interactions, one could identify the weak isospin group with the isospin symmetry and strange particles would be stable under weak decays.

The interplay of the weak and medium-strong interactions to determine the value of θ proved to be far reaching. It has remained in the present unified theory in the form of a misalignment between the weak isospin subgroup of the flavor symmetry and the quark mass matrix, which arises from the spontaneous symmetry breaking of the weak isospin gauge symmetry.

10. – The angle as a dynamical effect of strong vs. weak interactions

Cabibbo entertained for sometime the idea that the value of the weak angle, θ, could be determined by theoretical considerations. The fact that the angle indicates the direction of the weak isospin group in $SU(3)$ space could be seen as a kind of *spontaneous magnetization* in $SU(3)$ space and its value should arise as a solution of a self-consistency equation for the symmetry-breaking parameter, presumably an $SU(3)$ symmetric equation. This led to the problem of finding the *natural solutions* of equations invariant under a given group, G. The problem was tackled theoretically by L. Michel and L. Radicati [19], who investigated the natural minima in $SU(3)$, always finding trivial minima corresponding to $\theta = 0$ or π. Cabibbo and myself [20] extended the analysis to the chiral symmetry group $SU(3) \times SU(3)$ with two possible symmetry-breaking structures, transforming as

$$(3, \bar{3}) \oplus (\bar{3}, 3) \quad \text{or} \quad (8, 1) \oplus (1, 8), \tag{17}$$

but again finding only trivial results.

In modern terms, computing the Cabibbo angle means to determine theoretically the structure of the quark mass matrix, which, with three quark flavour, would correspond to the first choice in the previous equation. Attempts in this direction have met with some success [21], which amounts to justifying the *empirically valid* relation:

$$\sin\theta \simeq \sqrt{\frac{m_u}{m_s}} \tag{18}$$

between θ and the up and strange-quark masses, but a really convincing theory has not emerged yet and θ is still to be considered an undetermined constant of Nature.

Historically, the attempt to compute the Cabibbo angle was one of the motivations that led to the discovery of the GIM mechanism. One should not give up the idea that sometimes we shall be able to compute the pattern of symmetry-breaking quark masses and therefore to compute the Cabibbo angle. The more so, since, after the discovery of neutrino oscillations, the problem reproposes itself for the neutrino mass matrix.

Michel and Radicati ideas have been later used to justify the natural symmetry-breaking patterns of Unified and Grand Unified theories.

11. – Closing up on Cabibbo theory

From its very publication, the Cabibbo theory has been seen as a crucial development. It indicated the correct way to embody lepton-hadron universality and it enjoyed a heartening phenomenological success, which in turns indicated that we could be on the right track towards a fundamental theory of the weak interactions.

The authoritative book by A. Pais [22], in its chronology, quotes the Cabibbo theory among the most important developments in postwar Particle Physics.

In the *History of CERN*, J. Iliopoulos [23] writes: *There are very few articles in the scientific literature in which one does not feel the need to change a single word and Cabibbo's is definitely one of them. With this work he established himself as one of the leading theorists in the domain of weak interactions.*

12. – Post-Cabibbo developments: a unified, renormalizable, electroweak theory

Eight Nobel Prizes (fig. 2) have been given for the theory of the unified electroweak interactions pioneerd by S. L. Glashow [24], S. Weinberg [25] and A. Salam [26]. The Cabibbo theory has been a crucial step towards this great achievement.

Post-Cabibbo developments are summarized in the following.

- The introduction of the charmed quark by S. Glashow, J. Iliopoulos and L. Maiani [27] made it possible to extend the Weinberg-Salam theory to hadrons, restoring lepton-quark symmetry and predicting hadronic weak neutral currents without strangeness change at about the same rate as charged currents; the suppression of the strangeness-changing neutral currents fixes the mass scale of charmed particles, in agreement with experimental observation;

- G. 't Hooft and M. Veltman, in 1972, proved the renormalizability of the spontaneously broken (via the Higgs mechanism) gauge theory [28];

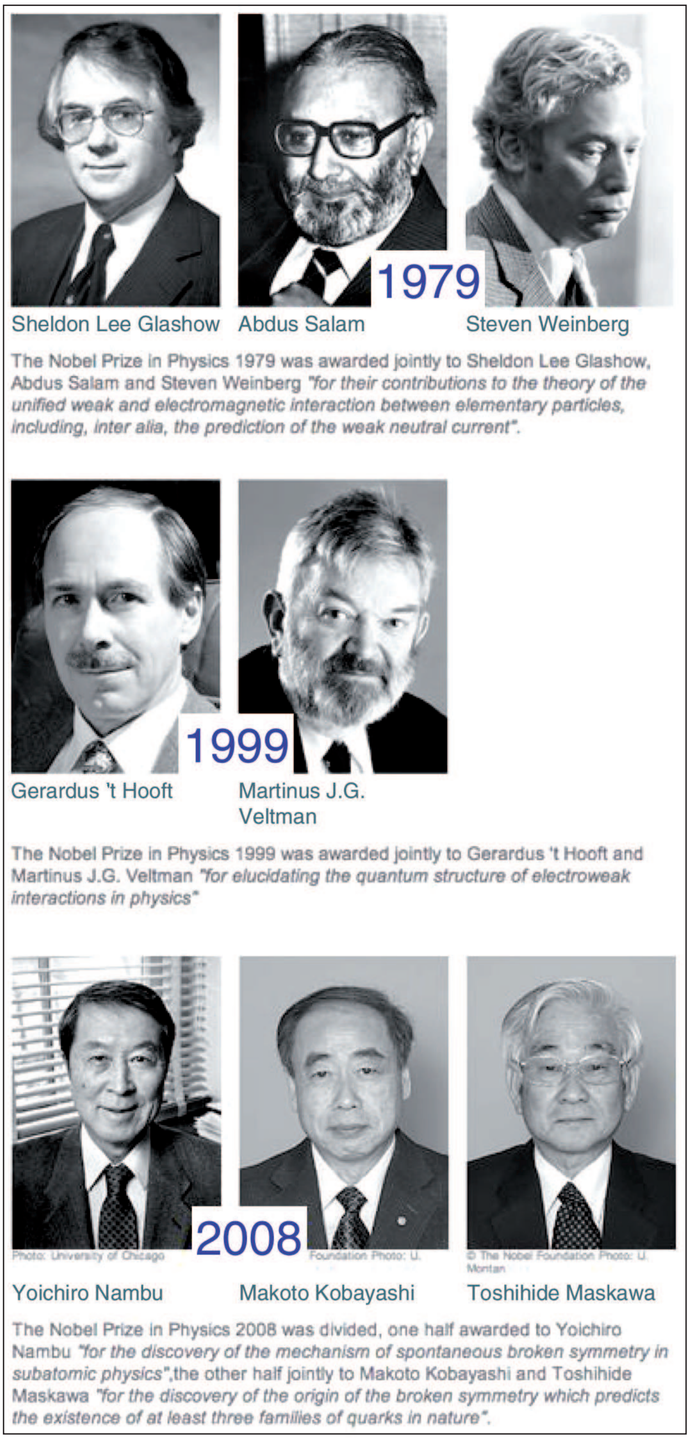

Fig. 2. – Nobel Prize winners who contributed to the theory of the unified electroweak interactions; Cabibbo theory has been a crucial step towards this great achievement.

– Adler anomalies in $SU(2) \times U(1)$ were the last obstacle towards a renormalizable electroweak theory and they were proven to cancel between quark (fractionally charged and in three colors) and lepton doublets, by C. Bouchiat, J. Iliopoulos and P. Meyer [29].

13. – CP violation

1973. Kobayashi and Maskawa discovery [30]: three left-handed quark doublets allow for one CP-violating phase in the quark mixing matrix, since known as the CKM matrix.

1976. S. Pakvasa and H. Sugawara [31] and L. Maiani [32], show that the phase agrees with the observed CP violation in K decays and (LM) leads to vanishing neutron electric dipole at one loop.

1986. I. Bigi and A. Sanda [33] predict direct CP violation in B decay.

2001. Belle [34] and BaBar [35] discover CP-violating mixing effects in B decays.

14. – New challenges

Problems which were on the table at the beginning of our story, the end of 1950s, have all been solved by an extraordinary mix of theoretical inventions and experimental results, illuminating each other . Some of the crucial steps have been described in this paper.

The proliferation of nuclear particles and resonances, initiated with the discovery of strange particles, has found an explanation in terms of more fundamental fermion fields, quarks coming in six flavours, each with three colours. The muon has found its place in the second quark-lepton generation. The fifth and sixth quarks neatly pair with the (ν_τ, τ) lepton doublet in a third generation, necessary to explain the CP violation initially observed with particles belonging to the first and second generations.

We understand the structure of the weak and electromagnetic currents, their renormalisation properties and the relation between leptonic, semi-leptonic and non-leptonic weak processes. The unified gauge theory of both interactions, electromagnetic and weak, has been experimentally confirmed in crucial instances, including existence and properties of the predicted, necessary, weak intermediaries. The mathematical consistency of the theory requires, by the way, precisely the lepton-quark simmetry which is so prominent in the spectrum of the elementary fermions.

Neutrino oscillations have been observed, in particular where they are required to support our understanding of the way the Sun works. We now know that neutrinos have masses, similarly to quarks and charged leptons, and that the phenomenon of fermion mixing, discovered by Cabibbo, is quite general, although we do not know yet how to predict its structure.

The description of the basic strong interactions with an asymptotically free gauge theory based on the colour symmetry is, perhaps, the most unexpected and most spectacular development of the second half of the last century. It has allowed for crucial *quantitative* tests of the strong interactions, in the short-distance region where we can apply perturbative methods. Non-perturbative calculations based on the numerical simulation of QCD in a space-time lattice, have produced highly non-trivial results in the large distance, strongly interacting, regime. One instance is the calculation of the axial couplings of the pseudo-scalar mesons, although, admittedly, we are still far from a systematic understanding of this domain. A gauge description of all fundamental interactions, including gravity, is a strong suggestion of a unified theory encompassing all interactions, realising the dream of Albert Einstein.

With the turn of the century, we have a new panorama of problems and challenges and a new machine, the Large Hadron Collider at CERN, to explore a new energy domain, ranging from 100 to above 1000 GeV = 1 TeV. I will list only a few of the challenges which may be attacked in the new round of experiments at the LHC. This is a personal list and may well turn out to be incomplete or even irrelevant: future will tell.

The first challenge is to find the Higgs boson [36]. The Higgs boson is needed for the unified electroweak theory to agree with Nature, validating the idea that symmetry-breaking particle masses arise from the spontaneous breaking of the gauge symmetry. At the same time, this mechanism gives a vision of the quantum vacuum which may help us to explain new phenomena in the universe at large: inflation, chaotic universe, etc.

Find the supersymmetric particles. The unification of forces requires a symmetry to relate different spins: this is *Supersymmetry*, a fermion-boson symmetry discovered in 1974 at CERN by J. Wess and B. Zumino [37] and in Russia by D. Akulov and V. Volkov [38].

There are arguments, related to the so-called hierarchy problem of fundamental scales, that suggest the presence of the supersymmetric partners of the known particles in the TeV range [39], possibly within reach of the LHC.

Indications for a form of stable matter other than we know, protons, electrons and neutrinos, come independently from the existence of non-luminous matter, gravitationally observed in the Universe. In fact, the data on the primordial abundance of helium and other light nuclei limit the abundance of baryonic matter to a few percent of the total mass and neutrinos are definitely too light. The origin of the *dark matter* is thus one of the most prominent puzzles of present physics. A neutral, very long-lived, supersymmetric partner surviving from the hot Big Bang could be a natural candidate to be the constituent of the dark matter in the Universe.

Finally, the search for extra space dimensions. String formulations of Quantum Gravity are not consistent in $3+1$ dimensions. Curved extra-dimensions are needed. How small is their radius? Can LHC high-energy particles get into and map for us the new dimensions?

15. – Cabibbo: leading the Roma school

Nicola settled in Roma La Sapienza in 1966, moved to Roma Tor Vergata for few years and came back to La Sapienza. Inspired by Nicola's physical intuition, mathematical skill and personal carisma, the Rome school significantly contributed to establishing what we call today the Standard Theory of particle physics, which Nicola had greatly helped to build. A few results of these wonderful years:

- the parton-model description of e^+e^- annihilation into hadrons [40];
- the first calculation of the electroweak contribution to the muon anomaly [41];
- the field-theoretic description of the parton densities in hadrons [42];
- the QCD prediction of a phase transition from hadrons into deconfined quarks and gluons starting from the limiting temperature introduced by R. Hagedorn [43];

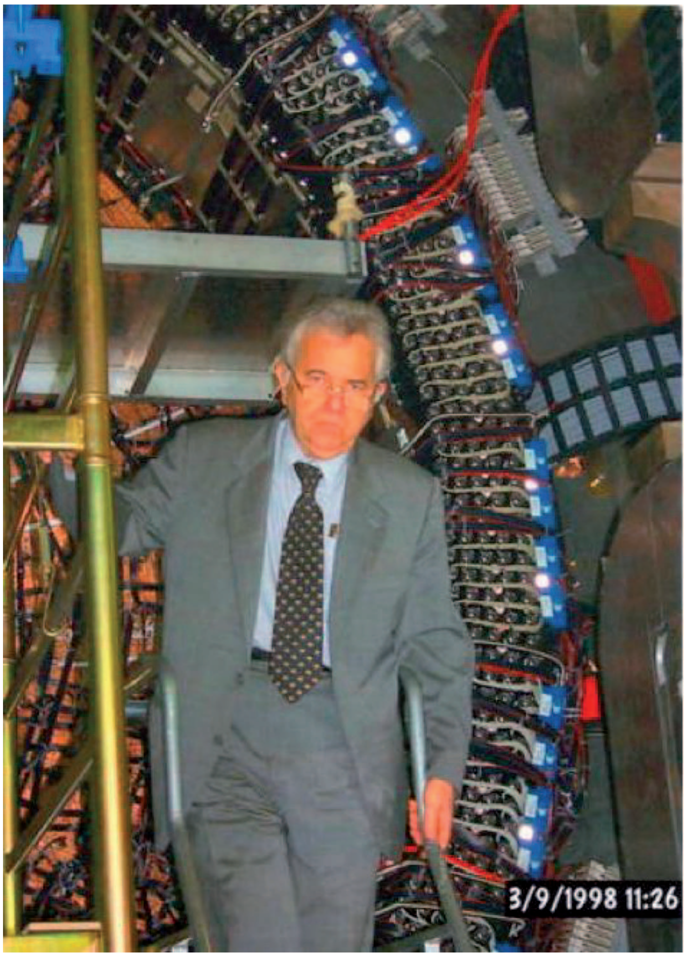

Fig. 3. – Nicola Cabibbo in 1998, visiting the KLOE detector in Frascati. Courtesy of Andrea Cabibbo.

– CP and T reversal violation in the oscillations of three flavored neutrinos [44];

– the upper and lower bounds to the Higgs boson and heavy-fermion masses in Grand Unified theories [45];

– the parton analysis of the heavy-quark β-decay spectrum (allowing one of the most precise determinations of the CKM mixing parameters) [46, 47];

– the lattice QCD calculation of weak parameters with lattice QCD [48];

– the proposal and realization with G. Parisi of a parallel supercomputer for lattice QCD calculations [49]; the APE supercomputers and their subsequent evolutions have played an important role in elucidating basic QCD in the non-perturbative regime.

16. – Nicola Cabibbo: science manager, teacher and friend

Nicola played an overall important role in the Italian scientific life of the turn of the century, as:

- Member of Academia Nazionale dei Lincei and of the American Academy of Science;
- President of Istituto Nazionale di Fisica Nucleare: 1983–1992;
- President of Ente Nazionale Energie Alternative: 1993–1998;
- President of the Pontifical Academy of Science: from 1993.

He held these important positions with vision, managerial skill and universally appreciated integrity.

Nicola liked to teach and he continued to do so until his very last months. Like all great minds, he could find simple arguments to explain the most difficult concepts. His students were fascinated by his simplicity, gentle modes and sense of humour. So were all of us, who had the privilege to be his collaborators and friends.

REFERENCES

[1] Feynman R. P., *Theory of Fundamental Processes* (Advanced Books Classics) 1962.
[2] Feynman R. P. and Gell-Mann M., *Phys. Rev.*, **109** (1958) 193.
[3] Gershtein S. S. and Zeldovich Ya. B., *Zh. Exp. Teor. Fiz.*, **29** (1955) 698 (*Sov. Phys. JETP*, **2** (1956) 576).
[4] Cabibbo N., *Phys. Rev. Lett.*, **10** 531 (1963).
[5] Cabibbo N. and Gatto R., *Phys. Rev.*, **124** (1961) 1577.
[6] Sudarshan E. C. G. and Marshak R. E., *Phys. Rev.*, **109** (1958) 1860.
[7] Feynman R. P. and Gell-Mann M., quoted under ref. [2].
[8] Gell-Mann M. and Levy M., *Nuovo Cimento*, **16** (1960) 705.
[9] Gatto R. and Cabibbo N., *Nuovo Cimento*, **21** (1961) 872.
[10] Coleman S. and Glashow S. L., *Phys. Rev. Lett.*, **6** (1961) 423.
[11] Barbaro-Galtieri A., Barkas W. H., Heckman H. H. and Smith Patrick, *Phys. Rev. Lett.*, **9** (1962) 26.
[12] Ademollo M. and Gatto R., *Phys. Rev. Lett.*, **13** (1964) 264.
[13] Wilson K. G., *Phys. Rev.*, **179** (1969) 1499.
[14] Gaillard M. K. and Lee B. W., *Phys. Rev. Lett.*, **33** (1974) 108; Altarelli G. and Maiani L., *Phys. Lett. B*, **52** (1974) 351.
[15] See, *e.g.*, Amsler C. *et al.* (Particle Data Group), *Phys. Lett. B*, **667** (2001) 1.
[16] An analysis of baryon decays including the most recent data is found in Cabibbo N., Swallow E. C. and Winston R., *Annu. Rev. Nucl. Part. Sci.*, **53** (2003) 39.
[17] Burgy M. T., Krohn V. E., Novey T. B., Ringo G. R. and Telegdi V. L., *Phys. Rev.*, **120** (1960) 1829.
[18] See, *e.g.*, Miscetti S. in *Electroweak interactions and Unified Theories, Rencontres de Moriond 2004, La Thuile, France, 21-28 March, 2004.*
[19] Michel L. and Radicati L. A., *Evolution of Particle Physics*, edited by Conversi M. (Academic Press, London) 1970, p. 191; Michel L., Radicati L. A., *Ann. Phys. (N.Y.)*, **66** (1971) 758.
[20] Cabibbo N. and Maiani L., *Evolution of Particle Physics*, edited by Conversi M. (Academic Press, London) 1970, p. 50.
[21] Fritzsch H., *Phys. Lett. B*, **70** (1977) 436.

[22] Pais A., *Inward Bound of Matter and Forces in the Physical World* (Clarendon, Oxford) 1986, (Oxford University Press, New York) 1986.
[23] Iliopoulos J., in *History of CERN*, Vol. **3** (Elsevier) 1996, p. 277.
[24] Glashow S. L., *Nucl. Phys.*, **22** (1961) 579.
[25] Weinberg S., *Phys. Rev. Lett.*, **19** (1967) 1264.
[26] Salam A., in *Elementary Particle Theory*, edited by Svartholm N., *Proceedings of the Nobel Symposium 1968* (Lerum, Sweden) 1968, pp. 367-377.
[27] Glashow S. L., Iliopoulos J. and Maiani L., *Phys. Rev. D*, **2** (1970) 1285.
[28] 't Hooft G. and Veltman M. J. G., *Nucl. Phys. B*, **44** (1972) 183.
[29] Bouchiat C., Iliopoulos J. and Meyer P., *Phys. Lett. B*, **38** (1972) 519.
[30] Kobayashi M. and Maskawa T., *Prog. Theor. Phys.*, **49** (1973) 652.
[31] Pakvasa S. and Sugawara H., *Phys. Rev. D*, **14** (1976) 305.
[32] Maiani L., *Phys. Lett. B*, **62** (1976) 183.
[33] Bigi I. I. Y. and Sanda A. I., *Nucl. Phys. B*, **193** (1981) 85; *Phys. Rev. D*, **29** (1984) 1393.
[34] Abe K. *et al.* (Belle Collaboration), *Phys. Rev. Lett.*, **87** (2001) 091802.
[35] Aubert B. *et al.* (BaBar Collaboration), *Phys. Rev. Lett.*, **86** (2001) 2515.
[36] Higgs P. W., *Phys. Rev.*, **145** (1966) 1156; Englert F. and Brout R., *Phys. Rev. Lett.*, **13** (1964) 321.
[37] Wess J. and Zumino B., *Nucl. Phys. B*, **70** (1974) 39.
[38] Akulov V. P. and Volkov D. V., *Theor. Math. Phys.*, **18** (1974) 28.
[39] Maiani L., in *Proceedings of the Summer School on Particle Physics, Gif-sur-Yvette, 3-7 September 1979*, edited by Davier M. *et al.* (IN2P3, Paris) 1979; Veltman M., *Acta Phys. Pol. B*, **12** (1981) 437; Witten E., *Nucl. Phys. B*, **188** (1981) 513; *Phys. Lett. B*, **105** (1981) 267.
[40] Cabibbo N., Parisi G. and Testa M., *Lett. Nuovo Cimento*, **4** (1970) 35; Cabibbo N., Parisi G., Testa M. and Verganelakis A., *Lett. Nuovo Cimento*, **4** (1970) 569.
[41] Altarelli G., Cabibbo N. and Maiani L., *Phys. Lett. B*, **40** (1972) 415.
[42] Altarelli G., Cabibbo N., Maiani L. and Petronzio R., *Nucl. Phys. B*, **69** (1974) 531.
[43] Cabibbo N. and Parisi G., *Phys. Lett. B*, **59** (1975) 67.
[44] Cabibbo N., *Phys. Lett. B*, **72** (1978) 333.
[45] Cabibbo N., Maiani L., Parisi G. and Petronzio R., *Nucl. Phys. B*, **158** (1979) 295.
[46] Cabibbo N. and Maiani L., *Phys. Lett. B*, **79** (1978) 109.
[47] Altarelli G., Cabibbo N., Corbo G., Maiani L. and Martinelli G., *Nucl. Phys. B*, **208** (1982) 365.
[48] Cabibbo N., Martinelli G. and Petronzio R., *Nucl. Phys. B*, **244** (1984) 381.
[49] Bacilieri P., Cabibbo N., Marinari E., Parisi G., Costantini F., Fiorentini G., Galeotti S., Passuello D. *et al.*, *Comput. Phys. Commun.*, **45** (1987) 345.

SELECTED PAPERS

A Theorem on the Elimination of Contact Muon-Electron Interactions.

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Summary. — A general theorem on the elimination of possible contact muon-electron interactions is given which includes as particular cases a theorem by Cabibbo and Gatto and a theorem by Feinberg, Kabir and Weinberg for particular types of interactions.

1. – Introduction.

GELL-MANN and FEYNMAN have remarked (1) that if one considers the expansion of a hypothetical muon-electron interaction in powers of momentum transfer, the decay rate inferred from the leading term of such an expansion by invoking gauge invariance is identically zero. Later, this result was shown (2) by two of us (N.C. and R.G.) to be contained in a general equivalence theorem. The theorem states that weak interactions of the form

$$\bar{\mu}(x)\gamma(\partial - ieA)(1+\gamma_5)e(x) + \text{h. c.}\,, \tag{1}$$

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(1) M. GELL-MANN and R. P. FEYNMAN: *Annual International Conference on High Energy Physics at CERN*, edited by B. FERRETTI (Geneva, 1958), p. 261.

(2) N. CABIBBO and R. GATTO: *Phys. Rev.*, **116**, 1134 (1959); see also R. GATTO: *Lectures at the International School of Physics in Varenna*, June 1959 (to appear in *Suppl. Nuovo Cimento*).

where $\mu(x)$ and $e(x)$ denote the muon and electron fields, respectively, can be removed by a canonical transformation. The argument demonstrated the existence of a unitary matrix which transforms the (eight component) spinor ψ,

$$\psi = \begin{vmatrix} e \\ \mu \end{vmatrix}, \tag{2}$$

in such a manner that the transformed Lagrangian no longer contains an interaction term of the type (1). Recently, FEINBERG, KABIR and WEINBERG [3] have noted the possibility of eliminating, by similar methods, hypothetical interactions of the form

$$-\varrho\bar{e}(x)\mu(x) - \xi\bar{e}(x)\gamma(\partial - ieA)\mu(x) + \text{H. c.} \tag{3}$$

or, alternatively, of the form

$$-\varrho\bar{e}(x)\gamma_5\mu(x) - \xi\bar{e}(x)\gamma\gamma_5(\partial - ieA)\mu(x) + \text{H. c.} \tag{3'}$$

to all orders in the coupling constants ϱ and ξ.

In this note, we present a generalization and unification of such arguments. The results described above appear as particular cases of a principle applicable to the most general renormalizable interactions of muons, electrons and photons. Our principal theorem is stated and proved in the next section. The conclusion does not depend on any perturbation assumption. We do assume that the energy operator deduced from the Lagrangian is positive definite. The argument utilizes a theorem on the diagonalization of finite dimensional matrices whose proof is given in Section 3.

2. – General form of the equivalence theorem.

We consider a Lagrangian of the type

$$\mathscr{L} = -\bar{\psi}[\gamma_\mu(\partial_\mu - ieA_\mu)(A + \gamma_5 B) + C + i\gamma_5 D]\psi - \tfrac{1}{4}F_{\mu\nu}F_{\mu\nu} + \mathscr{L}_s, \tag{4}$$

where ψ is defined in (2). The two-by-two matrices A, B, C and D operate in the $\mu - e$ space (which we shall call lepton space or, simply, L-space); $-(1/4)F_{\mu\nu}F_{\mu\nu}$ is the free photon Lagrangian, and $\mathscr{L}_s$ is the Lagrangian for strongly interacting particles. We adopt the customary assumption that $\mathscr{L}_s$ does not contain the electron or muon fields. In order that $\mathscr{L}$ be Hermitian, A, B, C and D must be Hermitian matrices. Then Eq. (4) represents the

([3]) G. FEINBERG, P. KABIR and S. WEINBERG: *Phys. Rev. Lett.*, **3**, 527 (1959).

most general Lagrangian containing renormalizable interactions and neglecting weak interactions.

We now assert that by means of a suitable non-singular matrix transformation in spin space and L-space,

$$\psi = \boldsymbol{T}\psi' , \tag{5}$$

the Lagrangian (4) may be brought into a form in which the electron and muon components of ψ' are not coupled, and such that the electron and muon terms in $\mathscr{L}$ are of the canonical type for spin 1/2 fields.

It is convenient to use the projections $a = \frac{1}{2}(1+\gamma_5)$ and $\bar{a} = \frac{1}{2}(1-\gamma_5)$. Then

$$A + B\gamma_5 = (A + B)a + (A - B)\bar{a} , \tag{6}$$

$$C + iD\gamma_5 = (C + iD)a + (C - iD)\bar{a} . \tag{6'}$$

We assume that the energy operator is positive definite. It follows ([4]) that $A+B\gamma_5$ is positive definite. Since a and $\bar{a}$ are orthogonal, we conclude from (6) that the two dimensional matrices $A+B$ and $A-B$ are positive definite.

From (4), we obtain for the canonical momentum conjugate to ψ,

$$\pi = i\psi^\dagger(A + \gamma_5 B) . \tag{7}$$

One may verify that the canonical anticommutation relations are consistent provided that

$$\det(A + \gamma_5 B) = \det(A - B)\det(A + B) \neq 0 . \tag{8}$$

Eq. (8) may also be regarded as the requirement that ψ does not obey an equation of constraint. Its validity is assured by the positive definite property.

The matrix $\boldsymbol{T}$ of eq. (5) may, if it exists, be written

$$\boldsymbol{T} = aR + \bar{a}S , \tag{9}$$

where R and S act in L-space. We require that in terms of ψ' and $\bar{\psi}'$, de-

([4]) One may adopt the procedure used, for example, by N. N. BOGOLJUBOV: *Introduction to the Theory of Quantized Fields*, (New York, 1959) p. 123, for free spinor fields. One obtains the energy as an integral over terms of the type $a^\dagger_\pm(\boldsymbol{k})(A+B\gamma_5)\cdot$ $\cdot a_\pm(\boldsymbol{k})|k_0|$ where the $a_\pm(\boldsymbol{k})$ are essentially Fourier components of the operators $\psi^\pm(x)$. The relationship between the positive definiteness of the energy and of $(A+B\gamma_5)$ then follows directly.

fined by

$$\psi = (aR + \bar{a}S)\psi' , \tag{10}$$

$$\overline{\psi} = \overline{\psi}'(\bar{a}R^{\dagger} + aS^{\dagger}) , \tag{10'}$$

the Lagrangian (4) assume the form

$$\mathscr{L} = -\overline{\psi}'[\gamma_\mu(\partial_\mu - ieA_\mu) + M]\psi' - \tfrac{1}{4}F_{\mu\nu}F_{\mu\nu} + \mathscr{L}_s , \tag{11}$$

where $\boldsymbol{M}$ is a diagonal matrix in L-space. We demand that $\boldsymbol{T}$ be chosen so that the diagonal elements of $\boldsymbol{M}$, which represent the electron and muon masses in the new representation, be real and non-negative.

Inserting (10) and (10′) into (4) and comparing with (11), we infer the following conditions on R and S:

$$R^{\dagger}(A+B)R = 1 , \qquad S^{\dagger}(A-B)S = 1 , \tag{12}$$

$$S^{\dagger}(C+i\boldsymbol{D})R = \boldsymbol{M} , \qquad R^{\dagger}(C-i\boldsymbol{D})S = \boldsymbol{M} . \tag{13}$$

The two equations of (13) are Hermitian conjugates of each other; therefore only one of them need be considered.

We first construct R and S so as to satisfy (12). Let V_+ and V_- be the unitary matrices which diagonalize the (Hermitian) matrices $A+B$ and $A-B$. Thus

$$V_+^{-1}(A+B)V_- = G_- , \qquad V_-^{-1}(A-B)V = G_- , \tag{14}$$

where G_+, G_- are diagonal and positive definite. We also introduce the two matrices

$$T_+ = (G_+)^{-\frac{1}{2}} , \qquad T_- = (G_-)^{-\frac{1}{2}} . \tag{15}$$

We then observe that if U_+ and U_- denote *arbitrary* unitary matrices, the choices

$$R = V_+T_+U_+ , \qquad S = V_-T_-U_- \tag{16}$$

satisfy eq. (12). With these choices, the first equation of (13) becomes

$$\boldsymbol{U}_-^{-1}[T_-V_-^{-1}(C+i\boldsymbol{D})V_+T_+]U_+ = \boldsymbol{M} . \tag{17}$$

The existence of unitary matrices U_+ and U_- which satisfy (17) is a direct consequence of a theorem we prove in Section **3**. Therefore, the existence of

R and S, and hence of $\boldsymbol{T}$ is also proved. Since $V_{\pm}$, $T_{\pm}$, and $U_{\pm}$ are non-singular, R, S and hence $\boldsymbol{T}$ are likewise non-singular and possess inverses. This establishes the theorem.

We now comment on the interpretation of the theory. We suppose that a non-singular operator T corresponds to the matrix $\boldsymbol{T}$ in the sense that

$$T^{-1}\psi' T = \psi = \boldsymbol{T}\psi' . \tag{18}$$

Let P_μ be the four-momentum operator constructed from (4) and let P'_μ be the corresponding operator constructed from (11). Then $P_\mu = T^{-1}P'_\mu T$. One sees that

$$P'_\mu P'_\mu |\alpha\rangle = -\mu^2 |\alpha\rangle \tag{19}$$

has solutions corresponding to the eigenvalues $\mu^2 = m_e^2$, $\mu^2 = m_\mu^2$, where m_e and m_μ are the diagonal elements of the non-negative matrix M. Hence $P_\mu P_\mu$ has eigenstates $T^{-1}|\alpha\rangle$ with the same eigenvalues $-\mu^2$. The theories defined by (4) and (11) are completely equivalent and both describe the same physical situation. The form (11) has the advantage of corresponding to the simplest limiting relation between the interpolating Heisenberg field ψ' and the asymptotic « in » fields $e^{(\text{in})}(x)$ and $\mu^{(\text{in})}(x)$. This remark aids in clarifying the question of the formal interpretation of symmetry operations in the theory (such as parity, time inversion, universality, etc.). For instance, the Lagrangian (4) is invariant under space inversion even if A, C and B, D are simultaneously non zero; it is also invariant under time reversal even if A, B, C, D are not all real, etc.: the point is that such symmetry operations have a direct interpretation on the asymptotic fields and they must be correspondingly redefined when the interpolating Heisenberg fields do not satisfy the proper limiting conditions.

3. – A diagonalization theorem for finite dimensional matrices.

We recall two simple facts about finite dimensional matrices:

(i) If the matrix Q commutes with its adjoint, it is diagonalizable by a unitary transformation;

(ii) If F is an arbitrary matrix, the matrices $FF^\dagger$ and $F^\dagger F$ have the same eigenvalues.

The equation $QQ^\dagger = Q^\dagger Q$ implies that the Hermitian and skew Hermitian parts of Q commute and hence can be diagonalized by the same unitary matrix. Then Q itself is diagonalized by this matrix, confirming (i). To verify (ii), we note that the eigenvalues are determined by the equations

$$\det(FF^\dagger - \lambda I) = 0 \tag{20}$$

and

$$\det(F^\dagger F - \lambda I) = 0 \,. \tag{20$'$}$$

If $\det F \neq 0$,

$$\det(FF^\dagger - \lambda I) = \det F \det(F^\dagger - \lambda F^{-1}) = \det(F^\dagger F - \lambda I)\,,$$

so that eq. (20) and (20′) are identical. If $\det F = 0$, we define $F(\varepsilon) = F - \varepsilon I$. For all $\varepsilon \neq 0$ in a small interval around $\varepsilon = 0$, $\det F(\varepsilon) \neq 0$ and hence $\det\big(F(\varepsilon)F^\dagger(\varepsilon) - \lambda I\big) = \det\big(F^\dagger(\varepsilon)F(\varepsilon) - \lambda I\big)$. This is a relation between two polynomials in ε and in the limit $\varepsilon \to 0$, we obtain the desired result.

We now prove the diagonalization theorem.

Theorem: For any matrix F there exist unitary matrices U_1 and U_2 such that

$$U_1^{-1} F U_2 = Z\,,$$

where Z is a non-negative definite diagonal matrix.

Proof: Let W_1 and W_2 be the unitary matrices which diagonalize the Hermitian matrices $FF^\dagger$ and $F^\dagger F$. Since these matrices have the same eigenvalues, one can choose W_1 and W_2 so that

$$W_1^{-1} F F^\dagger W_1 = W_2^{-1} F^\dagger F W_2 \,. \tag{21}$$

Let us define $Q = W_1^{-1} F W_2$. It follows that $Q^\dagger = W_2^{-1} F^\dagger W_1$ and (21) can be written

$$QQ^\dagger = Q^\dagger Q \,. \tag{22}$$

Therefore, by (i), there exists a unitary matrix V such that

$$V^{-1} Q V = V^{-1} W_1^{-1} F W_2 V = (W_1 V)^{-1} F (W_2 V) = Z'\,, \tag{23}$$

where Z' is diagonal. One can always write Z' in the form

$$Z' = Z V'\,, \tag{24}$$

where Z is the diagonal non-negative definite matrix obtained by replacing the elements of Z' by their absolute values, and V' is diagonal and unitary.

If now one takes

$$U_1 = W_1 V ,$$

$$U_2 = W_2 V (V')^{-1}$$

one finds

$$U_1^{-1} F U_2 = (W_1 V)^{-1} F (W_2 V)(V')^{-1} = Z'(V')^{-1} = Z$$

as desired.

RIASSUNTO

Viene dimostrato un teorema generale che permette l'eliminazione di eventuali interazioni locali μ-e e che contiene come casi particolari un teorema di Cabibbo e Gatto ed un teorema di Feinberg, Kabir e Weinberg.

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Consequences of Unitary Symmetry for Weak and Electromagnetic Transitions.

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Recent papers (1) have dealt with the introduction of unitary symmetry, *i.e.* invariance under the three-dimensional unitary group, as a convenient approximation in the theory of strong interaction. The strong-interaction Lagrangian is assumed to consist of a part invariant under the unitary group, plus a « correction » breaking the invariance.

In this note we shall examine the properties that follow, for weak and electromagnetic amplitudes, from this hypothesis, together with the violent assumption that the symmetry-breaking « corrections » can be neglected. We do not know under what conditions this last hypothesis can be applied. It might be applicable in the high-energy region — or, better, we know for sure that it is not generally applicable at low energies.

The consequences of violent assumptions are usually far-reaching. Thus we find that the K^0 (or $\bar{K}^0$) electromagnetic form factors must be zero, the charged K form factors must be equal to the charged pion form-factors; there are simple stringent relations betwen the form factors of the baryons (already given by Coleman and Glashow (2)), of the vector mesons, between the electromagnetic amplitudes from vector to pseudoscalar mesons, between the amplitudes of χ^0 decay and π^0 decay, and between weak interaction amplitudes. These relations depend in part on the particular representation adopted for the baryons.

For instance according to the « eightfold way » the Λ form-factors are one-half of the neutron form factors. They must instead be equal in the Sakata represen-

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(1) S. Ogawa: *Progr. Theor. Phys.*, **21**, 209 (1959); Y. Yamagouchi: *Progr. Theor. Phys. Suppl.*, **11**, 37 (1959); M. Ikada, S. Ogawa and Y. Ohnuki: *Progr. Theor. Phys.*, **22**, 715 (1959); **23**, 1073 (1960); I. E. Wess: *Nuovo Cimento*, **15**, 52 (1960); Z. Meki, M. Nakagawa, Y. Ohnuki and S. Sakata: *Progr. Theor. Phys.*, **23**, 1174 (1960); M. Ikeda, Y. Miyachi and S. Ogawa: *Progr. Theor. Phys.*, **24**, 569 (1960); M. Gell-Mann: to be published in *Phys. Rev.*; A. Salam and J. C. Ward: *Nuovo Cimento*, **20**, 419 (1961); Y. Neeman: preprint.

(2) S. Coleman and S. L. Glashow: *Phys. Rev. Lett.*, **6**, 1423 (1961).

tation. If this is true also at low energy a measurement of the magnetic moment would distinguish among the two cases.

1. – The unitary group in three dimensions has 8 generators F_m. Their commutation rules are given, in terms of the totally antisymmetric tensor (3) f_{mnl}, by

$$[F_m, F_n] = if_{mnl}F_l \,. \tag{1}$$

In correspondence to each F_m there is a current $j_m(x)$, conserved as long as unitary simmetry holds. The currents transform according to

$$[F_m, j_n(x)] = if_{mnl}j_l(x) \,. \tag{2}$$

In the theory of Gell-Mann and Neeman (eightfold way), where baryons transform according to the 8-dimensional rapresentation, the electromagnetic current, $j(x)$, is given by

$$j(x) = j_3(x) + \frac{1}{\sqrt{3}} j_8(x) \,. \tag{3}$$

From (2) and (3) one sees that F_3, F_8, F_6, and F_7 are the generators of the subgroup that leaves $j(x)$ invariant. Conservation of F_3 and F_8 expresses conservation of I_3 and of hypercharge. We thus limit our considerations to F_6 (F_7 gives the same results). From the identity

$$\langle A|[F_6, j(x_1) \cdots j(x_n)]|B\rangle = 0 \,;$$

where $j(x_1) \cdots j(x_n)$ is any product of n currents, we derive, if A and B are one-particle states

$$\langle 0|[\psi_A, F_6]j \cdots j|B\rangle = \langle A|j \cdots j[F_6, \psi_B^+]|0\rangle \,, \tag{4}$$

where ψ^+, ψ are the relevant creation or annihilation operators, and we have briefly denoted by $j \cdots j$ the product of the currents. In Gell-Mann's « eightfold way » both baryons and mesons are assumed to transform according to the 8-dimensional representation. Therefore, for a suitable choice of the ψ's,

$$[\psi_m, F_n] = if_{mnl}\psi_l \,, \tag{5}$$

and thus (4) relates directly different matrix elements between one particle states. If we apply it to the mesons we obtain, for instance, the following conclusions:

— the K^0 (or $\bar{K}^0$) form factor is always zero (we use also charge conjugation invariance);
— the form factor of K^+ (K^-) is equal to that of π^+ (π^-);
— the amplitude for Compton scattering on $K^\pm$ is equal to the same amplitude of a $\pi^\pm$. For the corresponding amplitudes on neutral mesons one has relations

of the kind:

$$\sqrt{3}\langle K^0|jj|K^0\rangle = \langle\chi^0|jj|\pi^0\rangle - \sqrt{3}\langle\chi^0|jj|\chi^0\rangle\,;$$

$$-\langle K^0|jj|K^0\rangle = \langle\pi^0|jj|\pi^0\rangle - \sqrt{3}\langle\pi^0|jj|\chi^0\rangle\,.$$

— The amplitude for $\chi^0\to\gamma+\gamma$ is $1/\sqrt{3}$ times the amplitude for $\pi^0\to\gamma+\gamma$ etc.
By applying (4) to the baryons one finds

$$(6)\qquad\left\{\begin{aligned}
&\langle\Sigma^+|j\cdots j|\Sigma^+\rangle = \langle p|j\cdots j|p\rangle\,,\\
&\langle\Sigma^-|j\cdots j|\Sigma^-\rangle = \langle\Xi^-|j\cdots j|\Xi^-\rangle\,,\\
&\langle\Xi^0|j\cdots j|\Xi^0\rangle = \langle n|j\cdots j|n\rangle -\\
&-\frac{1}{\sqrt{3}}\langle\Sigma^0|j\cdots j|\Lambda\rangle = \langle n|j\cdots j|n\rangle - \langle\Lambda|j\cdots j|\Lambda\rangle\\
&-\sqrt{3}\langle\Lambda|j\cdots j|\Sigma^0\rangle = \langle n|j\cdots j|n\rangle - \langle\Sigma^0|j\cdots j|\Sigma^0\rangle\,.
\end{aligned}\right.$$

A relation between the electromagnetic mass splittings,

$$\delta m_{\Xi^-} - \delta m_{\Xi^0} = \delta m_p - \delta m_n + \delta m_{\Sigma^-} - \delta m_{\Sigma^+}\,,$$

given by COLEMAN and GLASHOW (2), is contained in (6).

If the current product $j\cdots j$ reduces to a single j one can make further use of the transformation properties of j. A matrix element $\langle A|j_m|B\rangle$ where A, B, j_m all transform according to the eight-dimensional representation can be decomposed as

$$(7)\qquad \langle A|jm|B\rangle = if_{ABm}\,\mathcal{O} + d_{ABm}\mathcal{E}\,,$$

in terms of the totally antisymmetric tensor f, of the totally symmetric tensor d(3), and of the quantities $\mathcal{O}$ and $\mathcal{E}$.

The identity (7) is similar to the familiar Wigner-Eckart theorem for space rotations. The quantities $\mathcal{O}$ and $\mathcal{E}$ play the role of the so-called reduced matrix elements, and f and d of Clebsch-Gordon coefficients.

The reason why one has two reduced matrix elements in this case is that the reduction of the direct product $8\times8\times8$ contains the representation 1 twice.

Applying (7) to (3) one has

$$(8)\qquad \langle A|j|B\rangle = i\left(f_{AB3}+\frac{1}{\sqrt{3}}f_{AB8}\right)\mathcal{O} + \left(d_{AB3}+\frac{1}{\sqrt{3}}d_{AB8}\right)\mathcal{E}\,.$$

It is instructive to compare with the corresponding situation when only charge independence is assumed. In that case one has two independent matrix elements,

(3) A table of the elements of f_{mnl} as well as of d_{mnl} is given in the paper by GELL-MANN [see (1)].

usually called the scalar part and the vector part, which originate directly from the decomposition of j, analogous to (3), into an isovector and an isoscalar part. Here instead, j_3 and j_8 transform both in the same way, according to (2), but each of them originates two reduced matrix elements.

With (8) one fonds directly

$$(9)\quad \left\{\begin{array}{lll} (6) & \langle \Sigma^0 | j | \Sigma^0 \rangle = \frac{1}{3}\mathcal{E}\,, \\ (3) & \langle \Lambda^0 | j | \Lambda^0 \rangle = -\frac{1}{3}\mathcal{E}\,, \\ (4) & \langle \Sigma^0 | j | \Lambda^0 \rangle = \dfrac{1}{\sqrt{3}}\mathcal{E}\,, \\ (7) & \langle \Sigma^- | j | \Sigma^- \rangle = \frac{1}{3}\mathcal{E} - \mathcal{O}\,, \\ (9) & \langle \Xi^- | j | \Xi^- \rangle = \frac{1}{3}\mathcal{E} - \mathcal{O}\,, \\ (8) & \langle \Xi^0 | j | \Xi^0 \rangle = -\frac{2}{3}\mathcal{E}\,, \\ (1) & \langle p | j | p \rangle = \frac{1}{3}\mathcal{E} + \mathcal{O}\,, \\ (2) & \langle n | j | n \rangle = -\frac{2}{3}\mathcal{E}\,, \\ (5) & \langle \Sigma^+ | j | \Sigma^+ \rangle = \frac{1}{3}\mathcal{E} + \mathcal{O}\,. \end{array}\right.$$

From (9) one obtains the relations between the anomalous magnetic moments given by COLEMAN and GLASHOW [2,4]. We recall that one has (denoting explicitly the tensor indices)

$$\mathcal{O}^\mu = \bar{u}(p_f)\,\mathcal{O}_1(K^2)\gamma^\mu + \mathcal{O}_2(K^2)\sigma^{\mu\nu}K^\nu]u(p_i)\,,$$

with $K = p_f - p_i$, and similarly for $\mathcal{E}^\mu$.

Relations similar to (9) hold for the form factors of the postulated vector mesons or for the amplitudes of radiative transitions between vector mesons and pseudoscalar mesons. Thus one finds, for instance,

$$\langle \pi_0' | j | \chi_0 \rangle = \langle \chi_0' | j | \pi^0 \rangle = -\frac{1}{\sqrt{3}}\langle \chi_0' | j | \chi_0 \rangle = \frac{1}{3}\langle \chi_0' | j | \pi_0 \rangle = -\frac{2}{\sqrt{3}}\langle K_0' | j | K^0 \rangle\,,$$

$$\langle \pi'^+ | j | \pi^+ \rangle = \langle K'^+ | j | K^+ \rangle\,,$$

where, for instance, π_0' is the vector meson with the same isospin properties of π^0. Furthermore K'^+ (K'^-) has the same form factors as π'^+ (π'^-), and the form factors of neutral vector mesons are all zero.

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(4) Consistency with the relations of Coleman and Glashow requires an additional minus-sign in ther definition of the Σ-Λ transition moment.

2. – If one assumes that the weak currents are simply linear combinations of the currents j_m ([5]), one can apply (7) to derive relations between amplitudes for leptonic decays, always in the same spirit of neglecting that part of the strong Lagrangian that violates unitary simmetry. Thus, the $\Delta S=+1$, $\Delta Q=+1$ weak current could be of the form $g(j_4+ij_5)$ where g is a constant. One then has the decomposition

$$g\langle A|j_4+ij_5|B\rangle=(if_{AB4}-f_{AB5})\,\mathcal{O}'+(d_{AB4}+id_{AB5})\,\mathcal{E}', \tag{10}$$

where $\mathcal{O}'$ and $\mathcal{E}'$ are $g\mathcal{O}$ and $g\mathcal{E}$.

If one assumes universality in the coupling of the weak currents to the leptons the $\Delta S=0$, $\Delta Q=+1$ weak current would be $g(j_1+ij_2)$, with the same g as in (10). But then the rates for hyperon leptonic decays would be much larger than observed. Therefore the use of the universality hypothesis is, at least, inconvenient, in such a scheme; of course, the hypothesis may be true, but masked by strong renormalization effects. We do not therefore insist on the relations between matrix elements of different currents. From (10) one finds

$$(11)\quad \left\{\begin{aligned}
g\langle \Xi^-|j_4+ij_5|\Lambda\rangle &= \frac{1}{\sqrt{2}}\left(\sqrt{3}\,\mathcal{O}'-\frac{1}{\sqrt{3}}\,\mathcal{E}'\right),\\
g\langle \Sigma^- j_4+ij_5|n\rangle &= -\,\mathcal{O}'+\mathcal{E}',\\
g\langle \Sigma^0|j_4+ij_5|p\rangle &= \frac{1}{\sqrt{2}}\left(-\,\mathcal{O}'+\mathcal{E}'\right),\\
g\langle \Lambda|j_4+ij_5|p\rangle &= \frac{1}{\sqrt{2}}\left(-\sqrt{3}\,\mathcal{O}'-\frac{1}{\sqrt{3}}\,\mathcal{E}'\right),\\
g\langle \Xi^-|j_4+ij_5|\Sigma^0\rangle &= \frac{1}{\sqrt{2}}\left(\mathcal{O}'+\mathcal{E}'\right),\\
g\langle \Xi^0|j_4+ij_5|\Sigma^+\rangle &= \mathcal{O}'+\mathcal{E}'.
\end{aligned}\right.$$

3. – Still in the same spirit of neglecting violations of unitary simmetry we can easily establish that, in the limit of zero momentum transfer, $\mathcal{E}'\to 0$ (*i.e.* the form factor multiplying γ_μ in the expansion of $\mathcal{E}'$ is zero for $K^2=0$). In fact in the limit of zero momentum transfer the relevant matrix element is proportional to $\langle A|F_4+iF_5|B\rangle$, since the generators F_m are also the space integrals of the fourth component of j_m (and, of course, are conserved if j_m is divergenceless). However

$$\langle A|F_4+iF_5|B\rangle=\langle 0|[\psi_A,\,F_4+iF_5]|B\rangle=if_{AB4}-f_{AB5}, \tag{12}$$

showing that $\mathcal{E}'(0)=0$ and $\mathcal{O}'(0)=g$.

([5]) If strange currents violating $\Delta S=\Delta Q$ exist (preliminary report from the Padua-Wisconsin group) this possibility is lost, at least in the framework of the three dimensional unitary group.

4. – There are other versions of the models based on unitary symmetry differing mainly in the representation of the baryons. In the model by Gell-Mann and Neeman the eight known baryons are the basis of an eight-dimensional representation. In the original Sakata model [1] three baryons p, n, Λ, are the basis of a three dimensional representation, while other baryons belong to higher representations (to the 15-dimensional, or to both the 15 – and the 6 – dimensional representation). One can easily extend the considerations we have made in the previous sections to the Sakata model. One has however to be careful in identifying properly the currents. The electromagnetic current in the Sakata model is no longer given by (3) but it is given instead by

$$j(x) = j_3(x) + \frac{1}{\sqrt{3}}\, j_8(x) + \frac{1}{3}\, j_0(x) \,. \tag{13}$$

Note the addition of $\frac{1}{3}j_0(x)$, proportional to the baryonic current, which belongs to the one-dimensional representation, and therefore does not transform according to (2). This circumstance again brings two independent « reduced » matrix elements and we find the relation

$$\langle n|j|n\rangle = \langle \Lambda|j|\Lambda\rangle \,, \tag{14}$$

i.e., neutron and Λ have the same form factors and anomalous moment.

In contrast to this results, the eightfold way gives $\langle n|j|n\rangle = 2\langle \Lambda|j|\Lambda\rangle$. The Λ anomalous magnetic moment will presumably be measured rather soon by precession in a strong magnetic field [6].

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[6] This experiment is under development at Brookhaven and at CERN.

Electron-Positron Colliding Beam Experiments

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Possible experiments with high-energy colliding beams of electrons and positrons are discussed. The role of the proposed two-pion resonance and of the three-pion resonance or bound state is investigated in connection with electron-positron annihilation into pions. The existence of a three-pion bound state would give rise to a very large cross section for annihilation into $\pi^0+\gamma$. A discussion of the possible resonances is given based on consideration of the relevant widths as compared to the experimental energy resolution. Annihilation into baryon-antibaryon pairs is investigated and polarization effects arising from the nonreal character of the form factors on the absorptive cut are examined. The density matrix for annihilation into pairs of vector mesons is calculated. A discussion of the limits from unitarity to the annihilation cross sections is given for processes going through the one-photon channel. The cross section for annihilation into pairs of spin-one mesons is rather large. The typical angular correlations at the vector-meson decay are discussed.

A neutral weakly interacting vector meson would give rise to a strong resonant peak if it is coupled with lepton pairs. Effects of the local weak interactions are also examined. The explicit relation between the e^2 corrections to the photon propagator due to strong interactions and the cross section for annihilation into strongly interacting particles is given.

INTRODUCTION

A PROPOSAL for electron-electron colliding beams was made some time ago at Stanford by Barber, Gittelman, O'Neill, Panofsky, and Richter, and an experiment on electron-electron scattering based on such a proposal is being carried out.[1] Projects for electron-positron colliding beams are also under development at Stanford[1] and at Frascati.[2] The project at Frascati is intended to obtain high-energy (>1 Bev) electron-positron colliding beams.[3] We have already discussed possible experiments with e^+-e^- colliding beams.[4] In this note we shall present a more detailed discussion of possible electron-positron experiments and of the theoretical questions connected with them.

Like electron-electron experiments, electron-positron experiments can test the validity of quantum electrodynamics at small distances.[4a] They present, however, some very typical features that sufficiently justify the effort to produce electron-positron colliding beams. Most of the annihilation processes of e^+-e^- take place through the conversion of the pair into a virtual photon of mass equal to the total center-of-mass energy. The photon then converts into the final products. These reactions proceed through a state of well-defined quantum numbers, and as consequence the possible initial and final states are essentially limited. The interaction of the final particles with the virtual photon is directly measured in the experiment. The virtual photon four-momentum is timelike in these experiments, in contrast, for instance, to electron scattering on nucleons where the four-momentum of the transferred virtual photon is spacelike. Form factors of strongly interacting particles can thus be measured for timelike values of the momentum, in a region where they have, in general, an imaginary part. Electron-positron annihilations in flight offer the possibility of carrying out a Panofsky program, of a systematic exploration of the spectrum of elementary particles by observing their production by the intermediate virtual gammas. Unstable particles with the same quantum numbers as the intermediate photon can be produced singly as resonant states that soon after decay. At the appropriate energy there would appear resonance peaks in the production cross section for the final decay products.

1. GENERAL CONSIDERATIONS

1.1. We consider a reaction of the kind

$$e^+ + e^- \rightarrow a+b+\cdots+c, \tag{1}$$

where a, b, $\cdots$, c are strongly interacting particles. At the lowest electromagnetic order we assume that the reaction goes through the one-photon channel represented by Fig. 1. In the figure, q_+ and q_- are the positron and electron four-momenta, respectively, $k=q_+ + q_-$ is the time-like four-momentum of the virtual photon, and a, b, $\cdots$, c are the four-momenta of the produced particles. The element of the S matrix is given by

$$\langle a,b\cdots c|S|e^+e^-\rangle = \frac{2\pi e}{k^2}(\bar{v}\gamma_\nu u)\langle a,b\cdots c;\,\mathrm{out}|j_\nu(0)|0\rangle \times\delta(q_+ + q_- - a - b - \cdots c), \tag{2}$$

[1] W. Barber, B. Gittelman, G. K. O'Neill, W. K. H. Panofsky, and W. C. Richter (to be published); G. K. O'Neill, *Proceedings of the International Conference on High-Energy Accelerators and Instrumentation, CERN, 1959* (CERN, Geneva, 1959), p. 125; W. K. H. Panofsky, *Proceedings of the 1960 Annual International Conference on High-Energy Physics at Rochester* (Interscience Publishers, Inc., New York, 1960), p. 769; G. K. O'Neill and E. J. Woods, Phys. Rev. **115**, 659 (1959).

[2] F. Amman, C. Bernardini, R. Gatto, G. Ghigo, and B. Touschek (unpublished). A smaller storage ring for electrons and positrons for maximum energy of 250 Mev is already at an advanced state of construction; see C. Bernardini, G. F. Corazza, G. Ghigo, and B. Touschek, Nuovo cimento **18**, 1293 (1960).

[3] Electron-positron colliding beams are also being considered at CalTech, Cornell, and Paris.

[4] N. Cabibbo and R. Gatto, Phys. Rev. Letters **4**, 313 (1960); Nuovo cimento **20**, 184 (1961).

[4a] See R. Gatto, Proceedings of the Aix-en-Provence Conference (1961) (to be published) for a discussion of the possible tests of electrodynamics with electron-positron beams.

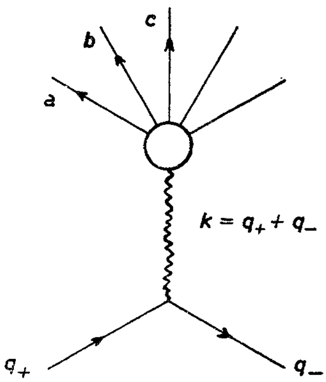

FIG. 1. Graph representing the one-photon channel. The symbols are defined in the text.

where v and u are Dirac spinors describing the positron and the electron, respectively, and $j_\nu(x)$ is the electromagnetic current operator. The relevant quantity is the matrix element of $j_\nu(0)$ between the vacuum and the final state of the produced particles. It will be convenient to define the four-vector

$$J_\nu = (2\pi)^{3n/2} \langle a,b,\cdots c;\,\text{out}\,|\,j_\nu(0)\,|\,0\rangle, \qquad (3)$$

where we have introduced for normalization purposes a factor $(2\pi)^{3n/2}$, where n is the number of the produced particles. From

$$\partial j_\nu(x)/\partial x_\nu = 0,$$

which holds for the charge current $j_\nu(x)$, it follows that

$$k_\nu J_\nu = 0. \qquad (4)$$

1.2. It will be convenient to refer all the quantities to the center-of-mass system for the reaction. In a colliding beam experiment the center-of-mass system is actually the laboratory system itself.

We shall in the following neglect the electron mass. We call E the energy of each incident particle in the center-of-mass system. It follows that

$$k^2 = (q_+ + q_-)^2 = -4E^2. \qquad (5)$$

Moreover, Eq. (4) in the center-of-mass system becomes

$$2iEJ_4 = 0. \qquad (6)$$

Therefore, J_ν has no time-like component in this system. We shall call $\mathbf{J}$ its space-like component.

The total cross section for unpolarized initial and final particles is given by

$$\sigma = \frac{(2\pi)^{5-3n}\alpha}{16E^4} \int d^3a\, d^3b \cdots d^3c\, \delta(E_a + E_b + \cdots + E_c - 2E)$$

$$\times \delta^3(\mathbf{a} + \mathbf{b} + \cdots + \mathbf{c}) T_{mn} \sum_{a,b\cdots} R_{mn}, \qquad (7)$$

where $\alpha = e^2/(4\pi) = (1/137)$; $\mathbf{a}$ and E_a are the momentum and energy of particle a, etc.; the tensor T_{mn} is given by

$$T_{mn} = \tfrac{1}{2}(i_m i_n - \delta_{mn}), \qquad (8)$$

where $\mathbf{i}$ is the unit vector pointing along the direction of, say, the incoming positron; the tensor R_{mn} is defined as

$$R_{mn} = -J_m J_n^*; \qquad (9)$$

and the summation $\sum_{a,b,\cdots c}$ is over the final spin states. Differential cross sections and cross sections for polarized final particles can be obtained from (7) by omitting the relevant integrations and spin summations.

1.3. For the production of two particles a, b of equal mass M, Eq. (7) gives

$$d\sigma = \frac{\alpha}{32} \frac{\beta}{E^2} (T_{mn} \sum_{a,b} R_{mn}) d(\cos\theta), \qquad (10)$$

where

$$\beta = [1 - (M/E)^2]^{\frac{1}{2}} \qquad (11)$$

is the velocity of the final particles. If the masses of a and b are different, (10) has to be replaced by

$$d\sigma = \frac{\alpha}{32} \left(\frac{p}{E}\right) \frac{1}{E^2} \frac{E_a E_b}{E^2} (T_{mn} \sum_{ab} R_{mn}) d(\cos\theta), \qquad (12)$$

where p is the final center-of-mass momentum and E_a and E_b are the energies of a and b. We have called θ the center-of-mass angle.

1.4. The inclusion of radiative corrections to the next electromagnetic order brings about (through its interference with the lowest-order term) the two-photon channel for which most of the general considerations valid for the one-photon channel (such as, for instance, angular momentum, parity, and charge conjugation rules) do not apply, at least in the same form. However, for experiments which do not distinguish between a final state and its charge conjugate (such as a total cross-section measurement, or any measurement that treats symmetrically the produced charged particles) such an interference term with the two-photon channel vanishes. Radiative corrections for such experiments are

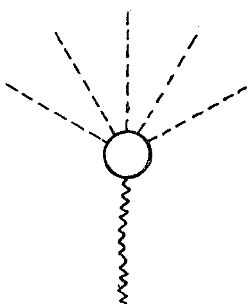

FIG. 2. Graph for the n-pion production reaction.

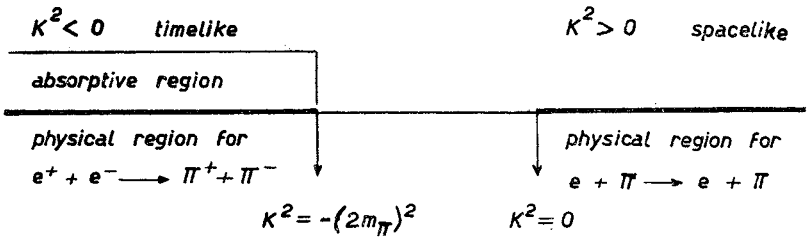

FIG. 3. The real k^2 axis for the pion form factor.

obtained by multiplying the expressions for the cross sections by an energy-dependent factor $\rho(E)$, and of course, by interpreting the matrix element J_ν as including the second-order radiative corrections.[5] The factor $\rho(E)$ is given by $1+\delta_{\text{s.e.}}+\delta_v+\delta_b$, where $\delta_{\text{s.e.}}$ is the percentage correction due to the photon self-energy graph, δ_v is the correction due to the vertex graph for the incoming electron and positron, and δ_b is the bremsstrahlung correction. The expressions for $\delta_{\text{s.e.}}$, δ_v, and δ_b can be found, for instance, in reference 5. These corrections take into account emission of soft photons. Before comparing with experiment one must, however, also add a correction for emission of hard photons under particular kinematical conditions that make them undetectable with the experimental apparatus employed (for an example see reference 6).

2. ANNIHILATION INTO PIONS AND K MESONS

2.1. Pion production in e^+-e^- collisions has already been discussed.[4,6] We shall here reproduce the main results and add some remarks. We consider the reaction

$$e^+ + e^- \longrightarrow n \text{ pions}, \tag{13}$$

occurring through a graph shown in Fig. 2. The relevant vertex is a γ-(n pions) vertex for a virtual γ of mass $k^2 = -4E^2$. Such vertices are important for the theory of the nucleon structure.[7,8] For n even, they contribute to the isotopic vector part of the nucleon structure; for n odd, to the isotopic scalar part.

We consider reaction (13) in its center-of-mass frame. The final n-pion state produced by the virtual γ, according to the graph of Fig. 2, must have parity -1, charge conjugation quantum number -1, total angular momentum 1, and total isotopic spin 1 for n even, 0 for n odd. In particular it follows that reaction (13) cannot occur at the lowest electromagnetic order if all final pions are neutral. The space-like part of J_ν in the center-of-mass system, $\mathbf{J}$, must be formed out of the final pion momenta and must have the character of a polar vector for n even and of an axial vector for n odd. For two final pions $\mathbf{J}$ will thus be proportional to the final relative momentum; for three final pions $\mathbf{J}$ will be proportional to the only available axial vector, namely, the normal to the production plane. Inserting (9) and (8) into (7), one finds uniquely the form of the dependence of the cross section on the angle between $\mathbf{J}$ and the initial electron-positron relative momentum:

$$T_{mn} \sum_{a,b,-c} R_{mn} = \tfrac{1}{2} |\mathbf{J}|^2 \sin^2\theta, \tag{14}$$

where θ is the angle between $\mathbf{J}$ and $\mathbf{i}$, the unit vector along the initial positron momentum. Therefore, for two pions the angular distribution is $\sim\sin^2\theta$; for three pions the angle between the normal to the production plane and the initial line of collision is also distributed $\sim\sin^2\theta$.

2.2. The simplest pion production process is

$$e^+ + e^- \longrightarrow \pi^+ + \pi^-. \tag{15}$$

The matrix element of the current J_ν is written as

$$J_\nu = e(4\omega_+\omega_-)^{-\frac{1}{2}} F(k^2)(p_\nu^{(+)} - p_\nu^{(-)}), \tag{16}$$

where ω_+ and ω_- are the pion energies, and $p^{(+)}$ and $p^{(-)}$ are the pion momenta. The form factor $F(k^2)$ is taken at $k^2 = -4E^2$. The cross section is given by

$$\frac{d\sigma}{d(\cos\theta)} = \frac{\pi}{16} \alpha^2 \frac{1}{E^2} \beta^3 |F(k^2)|^2 \sin^2\theta. \tag{17}$$

The dependence $\beta^3 \sin^2\theta$ is a direct consequence of angular momentum conservation that requires that the two final pions be produced in a p state and of our approximation of neglecting the electron mass. The total cross section is given by

$$\sigma_{\text{total}} = \frac{1}{m^2}(0.53\times 10^{-32}\ \text{cm}^2) b(x) |F(-4E^2)|^2, \tag{18}$$

where m (expressed in Bev) is the pion mass (or in general the mass of the produced boson) and

$$b(x) = (1/x^2)(1-1/x^2)^{\frac{3}{2}}, \tag{19}$$

with $x = E/m$.

To predict the absolute values of the cross section at

[5] G. Putzolu, Nuovo cimento **20**, 542 (1961).

[6] The same results as those of reference 4 have also been given by Yung Su Tsai, Phys. Rev. **120**, 269 (1960), and Proceedings of the 1960 Annual International Conference on High-Energy Physics at Rochester (Interscience Publishers, Inc., New York, 1960), p. 771.

[7] G. F. Chew, *Proceedings of the 1960 Annual International Conference on High-Energy Physics at Rochester* (Interscience Publishers, Inc., New York, 1960), p. 775.

[8] Chew, Karplus, Gasiorowicz, and Zachariasen, Phys. Rev. **110**, 265 (1958), Federbush, Goldberger, and Treiman, *ibid.* **112**, 643 (1958).

the different energies, one should know the values of $|F(k^2)|$ for $k^2<-4m_\pi^2$. These values of k^2 lie in the absorption cut on the k^2 plane. In the graph of Fig. 3 we indicate the real axis of k^2 with a specification of the different regions.

The form factor at $k^2=0$ takes the value 1. The physical region for space-like k^2 can in principle be explored by pion-electron scattering. In such experiments k has the character of a momentum transfer. The physical region for negative k^2 can be explored with pair production in electron-positron collisions. The absorptive region starts at $k^2=-(2m_\pi)^2$. The lowest-mass intermediate state contributing in the $\gamma-2\pi$ vertex is the 2π state itself. The next state consists of four pions and its contribution to the absorptive part starts at $k^2=-(4m_\pi)^2$. An interpretation of $e^++e^-\rightarrow\pi^++\pi^-$ in the vicinity of its threshold can reasonably be given in terms of the two-pion intermediate states only, and therefore directly in terms of pion-pion scattering. This situation is indeed a very fortunate one and does not occur in other cases of pair production in e^+-e^- collisions. A pion-pion resonance with $T=1$, $J=1$ has been proposed by Frazer and Fulco[9] as a simple way of explaining the isotopic vector part of the nucleon structure. The form factor proposed by Frazer and Fulco can be approximated near the resonance by a resonant shape of the form

$$|F(k^2)|^2=[\beta^2+(k_0^2)^2]/[\beta^2+(k^2-k_0^2)^2],$$

where $\beta\cong2.65m_\pi^2$ and $k_0^2\cong10.4m_\pi^2$. At an energy $E=230$ Mev (total center-of-mass energy $2E=460$ Mev), near the maximum of the form factor, one finds a total cross section of 8.35×10^{-31} cm^2 for $e^++e^-\rightarrow\pi^++\pi^-$. Bowcock, Cottingham, and Lurié[10] suggest a resonance with the same quantum numbers but with rather different parameters. They propose a resonant shape

$$F_\pi(t)=\frac{t_r+\gamma}{t_r-t-i\gamma(t/4-1)^{\frac{3}{2}}},$$

with $t_r=22.4m_\pi^2$ and $\gamma=0.4m_\pi^{-1}$.

At an energy $E\approx330$ Mev (total center-of-mass energy ~660 Mev) near the maximum of the form factor the total cross section for $e^++e^-\rightarrow\pi^++\pi^-$ reaches a value of 6.6×10^{-31} cm^2.

These cross sections are much higher than the cross section calculated for $|F|=1$ (a factor ~17 in the Frazer-Fulco case and ~33 according to Bowcock, Cottingham, and Lurié).

Interpretations of the proposed $T=1$, $J=1$ resonance in terms of an unstable meson with $J=1$, $T=1$, and negative parity, which decays rapidly into $\pi^++\pi^-$ have been proposed.[11] The neutral meson of such a triplet has charge conjugation number $C=-1$. Electron-positron collisions offer a good way for detecting systematically neutral mesonic resonant states with $J=1$, $C=-1$, and negative parity. The $T=1$ resonant state discussed here belongs to such a class of states.

The netural production process

$$e^++e^-\rightarrow\pi^0+\pi^0,$$

does not occur at the lowest electromagnetic order (it requires the exchange of at least two photons).

2.3. The three-pion production process

$$e^++e^-\rightarrow\pi^++\pi^-+\pi^0, \tag{18}$$

can occur by the lowest-order graph (Fig. 2). If we call l the relative $\pi^+\pi^-$ angular momentum and L the angular momentum of π^0 relative to the $\pi^+\pi^-$ center of mass, we find that only the states $l=L=1$, $l=L=3$, $l=L=5$, etc., can be produced at the lowest electromagnetic order. This follows directly from parity, charge conjugation, and angular momentum conservation.

The matrix element J_ν of the current operator for three pions can be written[8]

$$J_\nu=-i(8\omega_+\omega_-\omega_0)^{-\frac{1}{2}}H^*(E,\omega_+,\omega_-)\epsilon^{\nu\rho\sigma\tau}p_\rho^{(+)}p_\sigma^{(-)}p_\tau^{(0)}. \tag{19}$$

The form factor H^* depends on three independent scalars that we have chosen as E, $\omega_+=$energy of π^+, and $\omega_-=$energy of π^-, all in the center-of-mass frame. The final momenta of π^+, π^-, and π^0 are $p^{(+)}$, $p^{(-)}$, and $p^{(0)}$. The differential cross section can then be written

$$\frac{d^2\sigma}{d\omega_+d\omega_-d(\cos\theta)}=\frac{\alpha}{(2\pi)^2}\frac{1}{64E^2}|H|^2\sin^2\theta(\mathbf{p}^{(+)}\times\mathbf{p}^{(-)})^2. \tag{20}$$

Here θ, as already explained, is the angle between the initial line of collision and the normal to the production plane. The absolute value of the cross section depends entirely on the form factor $|H|^2$. Knowledge of this form factor is very important for the theory of the isotopic scalar part of the nucleon structure.[8]

At present there is not much information available on $|H|^2$. There have been proposals for the existence of a three-pion resonance or bound state with $T=0$, $J=1$.[12] If one assumes the $T=1$, $J=1$ two-pion resonance, a three-pion state with $T=0$, $J=1$ may be formed in which all pairs of pions interact in the resonant state. The possibility of such a saturated structure might lead to the existence of a bound three-pion state with $T=0$, $J=1$. It has in fact been proposed to identify such a bound state with a possible resonant behavior observed by Abashian, Booth, and Crowe.[13] In this case its mass would be as low as 2.2 pion masses. A preliminary fit to the scalar part of the

[9] W. R. Frazer and J. R. Fulco, Phys. Rev. **117**, 1609 (1960).
[10] T. Bowcock, W. N. Cottingham, and D. Lurié, Phys. Rev. Letters **5**, 386 (1960).
[11] J. J. Sakurai, Ann. Phys. **11**, 1 (1960); A. Salam, Revs. Modern Phys. **33**, 426 (1961); A. Salam and J. G. Ward, Nuovo cimento **19**, 167 (1961); M. Gell-Mann (to be published).
[12] G. F. Chew, Phys. Rev. Letters **4**, 142 (1960).
[13] A. Abashian, N. E. Booth, and K. M. Crowe, Phys. Rev. Letters **5**, 258 (1960).

nucleon structure, according to the latest data,[14] has been attempted by Bergia *et al.* assuming the existence of such a three-pion bound state.[15] The existence of such a bound state may strongly influence the behavior of $|H|^2$ near the production threshold. Or, if a three-pion resonance exists in the physical region, it would be directly exhibited in the cross section for (18). A three-pion bound state with $T=0$, $J=1$ would decay mostly into $\pi^0+\gamma$ and $2\pi+\gamma$. It would lead to spectacular peaks in $e^++e^-\rightarrow\pi^0+\gamma$, and $e^++e^-\rightarrow 2\pi+\gamma$. This will be examined in more detail in the next sections. It is difficult to calculate its rate of decay. Its dominant modes of decay would involve the emission of a photon and the resulting lifetime may be relatively long as compared to typical nuclear times. A lifetime of the order of 10^{-21} sec would correspond to width of the order of a fraction of a Mev. The form factor H in (19) could then be approximated near threshold as

$$H=(\text{constant})\times[1/(k^2+M^2)],$$

where M is the mass of the bound state, whose contribution is assumed to dominate the behavior of H near the threshold.

Gauge theories of elementary particles[11] also lead to the prediction of a $J=1$, $T=0$ meson with negative parity and negative charge conjugation number. We have already mentioned that e^+-e^- collisions provide a systematical way of searching for resonant states with $J=1$, $C=-1$, and negative parity.

2.1. Production of four pions, five pions, etc., will become important at high k^2, as strongly suggested by the high pion multiplicity in nucleon-antinucleon annihilation. The direction of the current matrix element $\mathbf{J}$ cannot be specified in terms of the final pion momenta from parity considerations alone, as it could for production of two or three pions. Gauge invariance alone does therefore not lead to any simple geometrical consequence. The presence of two-pion and three-pion resonances will strongly affect the final state and could suggest models for a simplified treatment. Methods used in the analysis of the nucleon-antinucleon annihilation into pions[16] can be applied to the present problem, with the substantial simplification of the complete knowledge of the initial quantum numbers. In particular, assuming the dominance of the $T=1$, $J=1$ pion-pion resonance, there is the possibility of a "saturated" $T=0$, $J=1$ three-pion state of negative parity and charge conjugation number, with all pairs coupled by the resonant interaction.[12] No such "saturated" states can be formed with more than three pions. In fact, already with four pions, two pions must have the same charge and therefore their relative angular momentum must be even.

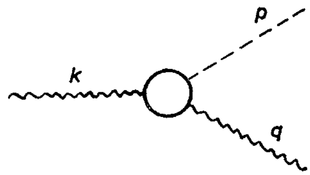

FIG. 4. Vertex for production of $\pi^0+\gamma$. The symbols are defined in the text.

2.5. Production of $K\bar{K}$ pairs can occur according to

$$e^++e^-\rightarrow K^++K^-,$$
$$e^++e^-\rightarrow K^0+\bar{K}^0,$$
$$e^++e^-\rightarrow K+\bar{K}+\pi, \quad \text{etc.}$$

Expression (17) applies for production of a $K-\bar{K}$ pair with $F(k^2)$ interpreted as the relevant K^+, or K^0, form factor. The charged K form factor is the sum of an isotopic vector form factor and an isotopic scalar form factor; the neutral K form factor is the difference of the isotopic vector and isotopic scalar form factors. Two-pion intermediate states and $K-\bar{K}$ states of isotopic spin one are among the contributors to the vector part, while three-pion states and zero isotopic spin $K-\bar{K}$ states are among the contributors to the scalar part. Presumably, $K-\bar{K}$ scattering will play a relevant role for production near threshold and the experiment will give information on its properties. Similarly $e^++e^-\rightarrow K+\bar{K}+\pi$ could give information on $K-\pi$ and $\bar{K}-\pi$ interactions.

In the $K^0-\bar{K}^0$ processes there is a very simple consequence of charge conjugation invariance that should be pointed out. The final $K^0\bar{K}^0$ pair must be produced in a state of $C=-1$. Therefore, the final amplitude is of the form

$$K^0\bar{K}^0-\bar{K}^0K^0.$$

In terms of the physical particles

$$K_1^0=(1/\sqrt{2})(K^0+\bar{K}^0), \quad K_2^0=(1/\sqrt{2})(K^0-\bar{K}^0),$$

the final amplitude can be written as

$$K_1^0K_2^0-K_2^0K_1^0.$$

It is now evident that only $K_1^0-K_2^0$ pairs can be produced (but not $K_1^0-K_1^0$ or $K_2^0-K_2^0$ pairs). This means that one particle must decay as a K_1^0 and the other as a K_2^0. It also follows that for a given configuration at production $K_1^0-K_2^0$ pairs are produced with the same probability as $K_2^0-K_1^0$ pairs. Note that the final amplitude maintains its form at any time in absence of interactions. The time development in fact is just given by

$$K_1^0\rightarrow K_1^0e^{-(\lambda_1+im_1)t}, \quad K_2^0\rightarrow K_2^0e^{-(\lambda_2+im_2)t},$$

if K_1^0 and K_2^0 propagate through vacuum. Analogous conclusions apply if additional π^0's are produced

[14] D. N. Olson, H. F. Schopper, and R. R. Wilson, Phys. Rev. Letters **6**, 286 (1961); R. Hofstadter, C. De Vries, and R. Herman, *ibid.* **6**, 290 (1961); R. Hofstadter and R. Herman, *ibid.* **6**, 293 (1961).
[15] S. Bergia, A. Stanghellini, S. Fubini, and C. Villi, Phys. Rev. Letters **6**, 367 (1961).
[16] A. Pais, Ann. Phys. **9**, 548 (1960).

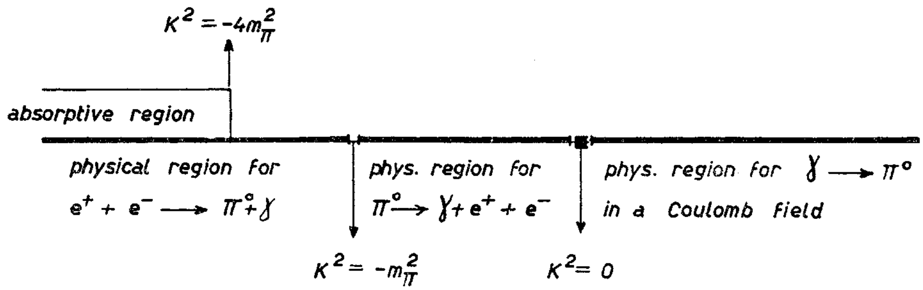

FIG. 5. The real k^2 axis for the π^0 form factor.

together with the $K^0\bar{K}^0$ pair. If, however, $\pi^+\pi^-$ pairs are also produced the $K^0\bar{K}^0$ pair can be produced in a $C=+1$ combination and the correlation would be different.

3. ANNIHILATION INTO $\pi^0+\gamma$

3.1. The process

$$e^+ + e^- \rightarrow \pi^0 + \gamma \tag{21}$$

is very interesting from a theoretical point of view as it is directly related to the properties of the vertex shown in Fig. 4. Here k and q are the photon momenta and p is the π^0 momentum. In the electron-positron annihilation process (21), the photon momentum k is off the mass shall, corresponding to a virtual photon mass of $(-k^2)^{\frac{1}{2}}=2E$. For $k^2=0$, the vertex describes π^0 decay into two photons and can be computed in terms of the π^0 lifetime. There exist various experimental possibilities for exploring the above vertex for the different ranges of values of k^2, as illustrated in Fig. 5. In the figure we have exhibited the real k^2 axis and indicated the various physical regions. We have also indicated the absorptive region, whose threshold starts at two pion masses.

The decay of a free π^0 into two photons occurs at $k^2=0$. The region at the right of $k^2=0$ can in principle be explored through the so called Primakoff effect.[17] In the Primakoff effect an incident real photon produces a π^0 through the interaction with the Coulomb field of a nucleus. The vertex of Fig. 4 can be held responsible for such a process with q taken as the incident photon momentum and k as the virtual photon momentum. The Dalitz process $\pi^0 \rightarrow \gamma + e^+ + e^-$ may be used to investigate the $(\pi^0\gamma\gamma)$ vertex for small negative values of k^2.[18]

The physical region for (21) starts at $k^2=-m_\pi^2$. The absorptive region starts only at $k^2=-4m_\pi^2$, with the possibility of two-pion intermediate states. The $T=1$, $J=1$ two-pion resonance would produce a strong resonant-like behavior of the vertex in the physical region for (21). The next absorptive threshold due to three-pion continuum starts at $k^2=-9m_\pi^2$. If there is a bound $T=0$, $J=1$, 3π state at some $k^2>-9m_\pi^2$, which decays only through electromagnetic interaction, its presence would lead to a pole contribution to the vertex at the relevant value of $k^2=-M^2$, where M is the mass of the bound state. Of course, the pole would occur strictly at a complex value of k^2, due to the finite lifetime of the bound state. The associated width is, however, presumably only a fraction of a Mev, and the description by a pole may be safely applied except for the immediate vicinity of the resonance peak. The contribution from the peak to the cross section may turn out to be effectively very big, possibly of the order of $10^{-29}-10^{-30}$ cm^2. This can be seen from simple considerations based on a Breit-Wigner description of the resonance. The $T=0$, $J=1$ three-pion bound state would presumably decay into $\pi^0+\gamma$, or $2\pi+\gamma$, with a lifetime $\sim 10^{-20}$ sec. The corresponding width Γ is then between 0.06 Mev and 0.6 Mev. The experimental energy resolution is given by $2\Delta E$, where ΔE is the energy resolution for each colliding beam (positrons or electrons). If the energy spread of the incident beams is larger than a few Mev, as it will presumably be, the measured quantity will be the integral of the cross section in a region comprising the peak. We therefore estimate the average cross section as

$$\bar{\sigma}=\frac{1}{2\Delta E}\int_{2E=M-\Delta E}^{2E=M+\Delta E}\sigma d(2E)=\frac{1}{\Delta E}\int_{E=\frac{1}{2}M-\frac{1}{2}\Delta E}^{E=\frac{1}{2}M+\frac{1}{2}\Delta E}\sigma dE. \tag{22}$$

We use a Breit-Wigner formula for the cross section near the resonance. For production through a $J=1$ resonant state of mass M, total disintegration rate Γ and partial rates Γ_i and Γ_f for decay, respectively, into the initial $e^+ + e^-$ channel and final $\pi^0+\gamma$ (or $2\pi+\gamma$) channel, we have

$$\sigma=\tfrac{3}{4}\pi\lambda^2\Gamma_i\Gamma_f/[(2E-M)^2+\Gamma^2/4]. \tag{23}$$

The contribution to $\bar{\sigma}$ from the peak is then given by

$$\bar{\sigma}=\tfrac{3}{2}\pi^2\lambda^2B_iB_f(\Gamma/2\Delta E), \tag{24}$$

[17] H. Primakoff, Phys. Rev. 81, 899 (1951); C. Chiuderi and G. Morpurgo, Nuovo cimento 19, 497 (1961); V. Glaser and R. A. Ferrell, Phys. Rev. 121, 886 (1961); S. Berman (to be published).

[18] S. M. Berman and D. A. Geffen, Nuovo cimento 18, 1192 (1960); How Sen Wong, Phys. Rev. 121, 289 (1961).

where B_i and B_f are the branching ratios Γ_i/Γ and Γ_f/Γ, respectively. In performing the integration we have assumed $\Gamma \ll 2\Delta E$ but the result can be applied for an estimate if $\Gamma \lesssim 2\Delta E$. Let us assume $M \lesssim 3m_\pi$ and a branching ratio into $e^+ + e^-$, B_i, of the order of 10^{-3}. One obtains $\bar{\sigma} = 1.3\times10^{-28}(\Gamma/2\Delta E)$ cm^2, or, with $\Gamma \sim 10^{20}$ sec^{-1}, $\bar{\sigma} = 0.8\times10^{-29}(2\Delta E$ in Mev$)^{-1}$ cm^2.

3.2. In the following we shall discuss in detail the process $e^+ + e^- \to \pi^0 + \gamma$ assuming the dominance of the resonant 2π state and of the 3π bound state. From what we have seen in the previous paragraphs, if the 3π $T=0$, $J=1$ bound state exists, its contribution is likely to be very important.

Presumably also a $T=0$, $J=1$ three-pion resonance (as opposed to a bound state) would have a very important effect in $e^+ + e^- \to \pi^0 + \gamma$. A limitation of the theory to the $T=1$, $J=1$ 2π resonance[19] would be rather artificial also in view of the remark by Chew[12] that a $T=1$, $J=1$ 2π resonance could generate a $T=0$, $J=1$ 3π resonance or bound state. The experimental result by Samios[20] on internal conversion of gamma rays in π^0 decay indicates a negative value for the derivative of the π^0 form factor at the origin. This result would be difficult to understand if only the 2π resonant state is kept in the calculations. A possible explanation could be to include contributions from the intermediate nucleon-antinucleon pairs. A theory of π^0 decay based on keeping only contributions from nucleon-antinucleon pairs has been published by Goldberger and Treiman.[21] The negative coefficient of the Samios experiment could be possibly understood through the coherent contribution of the 2π resonant state and of the nucleon-antinucleon states. Such a point of view has been proposed by Berman and Geffen in their discussion of internally converted pairs.[18] The point of view that we follow here by including two- and three-pion states is closer to that of Wong[18] in his discussion of π^0 decay.

The general form for the $\pi^0\gamma\gamma$ vertex as determined from invariance requirements is

$$\tfrac{1}{4}G(-k^2,-q^2,-p^2)\epsilon_{\mu\nu\lambda\rho}F_{\mu\nu}(q)F_{\lambda\rho}(k), \tag{25}$$

where k and q are the four-momenta associated to the photon lines, $F_{\mu\nu}(q)$ and $F_{\lambda\rho}(k)$ are Fourier components the electromagnetic tensor, p is the π^0 four-momentum, $\epsilon_{\mu\nu\lambda\rho}$ is the isotropic antisymmetric tensor, and G is a form factor. In our case one photon and the π^0 are on the mass shell. We shall denote by $G(-k^2)$ the value of the form factor for $-p^2 = m_\pi^2$ and $q^2=0$. The π^0 lifetime, τ, depends on $G(0)$ according to the relation

$$1/\tau = (m_\pi^3/64\pi)|G(0)|^2. \tag{26}$$

From (7), (8), and (9) we can derive an expression for the cross section for $e^+ + e^- \to \pi^0 + \gamma$. The matrix element of the current is given in the center-of-mass system for the reaction by

$$\mathbf{J} = -[E/(\omega|\mathbf{p}|)^{\frac{1}{2}}]G(-k^2)\boldsymbol{\varepsilon}\times\mathbf{p},$$

where ω is the π^0 energy, $\mathbf{p}$ its momentum, and $\boldsymbol{\varepsilon}$ the polarization vector of the emitted γ ray. The tensor R_{mn} is then given by

$$(E^2/\omega|\mathbf{p}|)|G(-k^2)|^2(p_n p_m - p^2\delta_{mn}),$$

and the differential cross section is given by

$$d\sigma = (\alpha/64)\beta^3|G(-k^2)|^2(1+\cos^2\theta)d(\cos\theta).$$

It will be convenient to express the cross section in terms of the π^0 lifetime through (26):

$$d\sigma = \frac{\pi\alpha}{m_\pi^3}\frac{1}{\tau}\beta^3(1+\cos^2\theta)\left|\frac{G(-k^2)}{G(0)}\right|^2 d(\cos\theta). \tag{27}$$

The total cross section is given by

$$\sigma = \frac{8\pi}{3}\frac{\alpha}{m_\pi^3}\frac{1}{\tau}\beta^3\left|\frac{G(-k^2)}{G(0)}\right|^2$$

$$\cong 2.75\times10^{-35}\ \text{cm}^2\left|\frac{G(-k^2)}{G(0)}\right|^2(1-x^{-2})^3, \tag{28}$$

where $x=2E/m_\pi$, and the numerical factor, inversely proportional to the π^0 lifetime, has been calculated for a lifetime of 2.2×10^{-16} sec.

The form factor $G(k^2)$ is assumed to satisfy a dispersion relation of the form

$$G(-k^2) = \frac{1}{\pi}\int_4^\infty \frac{\mathrm{Im}G(t)dt}{t+k^2-i\epsilon} \tag{29}$$

(we are using $m_\pi = 1$). The imaginary part has contributions from the absorptive region as indicated in Fig. 5. If a three-pion bound state is present an additional pole contribution should be added to (29). We first evaluate the contribution to the dispersion integral from the $T=1$, $J=1$ two-pion intermediate state, assuming a resonant pion-pion amplitude as proposed by Frazer and Fulco[9] and by Bowcock, Cottingham, and Lurié.[10] The contribution from the resonant $T=1$, $J=1$ two-pion intermediate state can be expressed, following Wong,[18] as

$$\frac{e}{48\pi^2}\int_4^\infty \frac{(t-4)^{\frac{3}{2}}}{t^{\frac{1}{2}}(t+k^2-i\epsilon)}F_\pi^*(t)M_1(t)dt, \tag{30}$$

where $F_\pi(t)$ is the pion electromagnetic form-factor and $M_1(t)$ is the amplitude for pion photoproduction on pions $(\gamma+\pi\to\pi+\pi)$ in p wave. This amplitude has been studied by Wong[22] who has shown that under the assumption of a resonant $T=1$, $J=1$ π-π interaction,

$$M_1(t) = \Lambda(1+a)D_1(1)/(t+a)D_1(t),$$

[19] G. Furlan, Nuovo cimento **19**, 840 (1961).
[20] N. P. Samios, Phys. Rev. **121**, 265 (1961).
[21] M. L. Goldberger and S. Treiman, Nuovo cimento **9**, 451 (1958).
[22] H. Wong, Phys. Rev. Letters **5**, 70 (1960).

where $D_1(t)$ is the denominator function, as defined by Chew,[23] a is a positive constant, and Λ is an unknown parameter. In this scheme also $F_\pi(t)$ is related to $D(t)$ by

$$F_\pi(t)=D_1(0)/D_1(t).$$

One sees that $F_\pi^*(t)M_1(t)$ is essentially proportional to the square of the absolute value of the pion form factor. The two-pion contribution $G_2(-k^2)$ is therefore proportional to the integral

$$\frac{1}{\pi}\int_4^\infty \frac{(t-4)^{\frac{3}{2}}|F_\pi(t)|^2dt}{t^{\frac{1}{2}}(t+a)(t+k^2-i\epsilon)}. \tag{31}$$

We shall evaluate the above integral by assuming for $|F_\pi(t)|^2$ the form proposed by Bowcock, Cottingham, and Lurié[10]

$$F_\pi(t)=(t_2+\gamma)/[t_2-t-i\gamma(t/4-1)^{\frac{3}{2}}], \tag{32}$$

where t_2 and γ are the parameters that characterize the resonance. The form factor $F_\pi(t)$ satisfies the nonsubstracted dispersion relation

$$F_\pi(-k^2)=\frac{1}{\pi}\int_4^\infty \frac{\mathrm{Im}F_\pi(t)dt}{t+k^2-i\epsilon}. \tag{33}$$

Normally a subtracted dispersion relation is used; however, with the form (32) for $F_\pi(t)$ also a nonsubtracted relation like (33) is convergent. The absorptive term in (33) as derived from (32) can be written in the form

$$\mathrm{Im}F_\pi(t)=\frac{\gamma}{4(t_2+\gamma)}(t-4)^{\frac{3}{2}}|F_\pi(t)|^2\theta(t-4). \tag{34}$$

The comparison with (33) and (34) will permit a direct evaluation of the integral (31). The factor $|F_\pi(t)|^2$ in the integral (31) represents a very sharp resonance. It will therefore be possible to neglect the energy variation of the slowly varying terms in the denominator of (31) and obtain

$$G_2(-k^2)\propto\frac{1}{\pi}\int_4^\infty \frac{(t-4)^{\frac{3}{2}}|F_\pi(t)|^2dt}{t+k^2-i\epsilon}.$$

By comparing with (33) and (34) we then find directly

$$G_2(-k^2)=c_2/[t_2+k^2-i\gamma(-\tfrac{1}{4}k^2-1)^{\frac{3}{2}}], \tag{35}$$

where c_2 is a constant.

We shall now approximate $G(-k^2)$ for not very large values of $|k^2|$ as

$$G(-k^2)=\frac{c_2}{t_2+k^2-i\gamma(-\frac{1}{4}k^2-1)^{\frac{3}{2}}}+\frac{c_3}{t_3+k^2-i\Gamma}. \tag{36}$$

The first term in the right-hand side of (36) represents the contribution from the 2π resonant state. The second term represents the "pole" contribution from the proposed 3π bound state. The mass M of the bound state is given by $t_3^{\frac{1}{2}}$ and Γ is its total decay rate, corresponding to a lifetime presumably of the order of 10^{-20} sec. The imaginary part $-i\Gamma$ is very small, producing a very narrow peak in $G(-k^2)$ in the neighborhood of $k^2=-t_3$. The constant c_3 is the residuum of the pole corresponding to the 3π bound state. We shall not try any theoretical determination of c_2 and c_3 but rather try to evaluate them on the basis of experimental information. $G(0)$ is related to the π^0 lifetime through (26). This gives approximately

$$|c_2/t_2+c_3/t_3|^2=(64\pi/m_\pi^3)\tau^{-1}. \tag{37}$$

A second piece of information is given by the Samios experiment on internally converted electron pairs from π^0 decay.[20] The experiment gives some information on the derivative of $G(-k^2)$ at $k^2=0$. With the definition

$$-\frac{1}{G(0)}\left(\frac{\partial G(-k^2)}{\partial k^2}\right)_0=\alpha m_\pi^2, \tag{38}$$

Samios finds $\alpha=-0.24\pm0.16$. From (36) and (38) one finds directly ($m_\pi=1$)

$$\frac{c_3}{c_2}=-\left(\frac{t_3}{t_2}\right)^2\frac{1-\alpha t_2}{1-\alpha t_3}. \tag{39}$$

We now use for t_2 the value suggested by Bowcock, Cottingham, and Lurié, $t_2=22.4$, and for t_3 the value suggested from the identification of the bound state with the resonance observed by Abashian, Booth, and Crowe,[13] namely $t_3=5$. From (39) and from the experimental value of α there follows a value for c_3/c_2 between -0.22 and -0.13. Expressed in terms of τ and α, the π^0 form factor becomes

$$|G(-k^2)|^2=\frac{64\pi}{m_\pi^3}\tau^{-1}\frac{1}{(t_2-t_3)^2}\left|\frac{t_2^2(1-\alpha t_3)}{t_2+k^2-i\gamma(-\frac{1}{4}k^2-1)^{\frac{3}{2}}}-\frac{t_3^2(1-\alpha t_2)}{t_3+k^2-i\Gamma}\right|^2. \tag{40}$$

The form factor (40) is composed by two resonant terms. The first term due to the resonant $T=1$, $J=1$ π-π interactions reaches its maximum around $k^2=-t_2$ and has a width given by $\gamma(-\frac{1}{4}t_2-1)^{\frac{3}{2}}t_2^{-\frac{1}{2}}$. With the Bowcock, Cottingham, and Lurié values for the pion-pion form factor, $\gamma\cong0.4m_\pi^{-1}$, and the resonance width is then about $0.8m_\pi$. The second term, due to the proposed 3π bound state, reaches its maximum at $k^2=-t_3=-(\text{mass of the bound state})^2$. Its width, presumably determined by the decay rate of the bound state into $\pi^0+\gamma$, is expected to be only a fraction of a Mev. According to (40) with the values $t_2=22.4$ and $t_3=5$, the contribution from the 3π resonant term is negligible for values of k^2 about -22.4, in comparison with the 2π resonant term that reaches its maximum

[23] G. F. Chew and S. Mandelstam, Phys. Rev. **119**, 467 (1960).

in that region. The enhancement factor we find at $k^2=-t_2$ is

$$|G(t_2)/G(0)|^2 \cong 250. \tag{41}$$

The reason for the enormous enhancement can be traced back in the required compensation of the two resonant contributions near $k^2=0$ to produce a small negative derivative. The enhancement factor (41) multiplies the perturbation-theory (i.e., constant form factor) cross section of 2.75×10^{-35} cm$^2\times(1-t_2^{-1})^3 \cong 2.4\times10^{-35}$ cm^2. The resulting cross section may thus become observable in the region of the 2π resonance maximum.

The resonance peak due to the 3π resonance is very narrow and only an average cross section, integrating the contributions from the peak, will be measurable. On the basis of the expression (40) for the form factor we can again estimate the value of the average cross section $\bar{\sigma}$, as defined by (22). Near the 3π resonance the cross section can be approximated as

$$\sigma = 2.75\times10^{-35}\ \mathrm{cm}^2(1-t_3^{-1})^3\left|\frac{t_3-\alpha t_2 t_3}{t_3-t_2}\right|^2\left|\frac{t_3}{t_3+k^2+i\Gamma}\right|^2.$$

The resulting average $\bar{\sigma}$ is

$$\bar{\sigma} \cong 3.5\times10^{-29}\left(\frac{t_3-\alpha t_2 t_3}{t_3-t_2}\right)^2\frac{1}{(2\Delta E)}\ \mathrm{cm}^2,$$

with ΔE expressed in Mev. With the proposed values for the parameters, $\bar{\sigma}$ can become $\cong 10^{-28}/(2\Delta E)$ cm^2, a value about a factor of 10 higher than what we obtained before on the basis of assumptions on the decay rates. The considerations that we have developed strictly apply to the explicit case of a 3π bound state. In the case of a 3π resonance there might occur an important change in the conclusion if the decay rate of the resonant state into three pions is strong enough. As no selection rules would be expected to prevent such a decay mode, its rate is expected to be rather big and might become much bigger than the rate for decay into $\pi^0+\gamma$ as soon as the energy release is large enough to overcome the effects of the smaller statistical weight and of the centrifugal barrier. In this case the resonance width would be much larger and a better dispersion theoretical approach would be required to obtain reliable estimates. Apart from the difficulties met in obtaining a precise estimate one can see that a very big cross section for $e^+ + e^- \rightarrow \pi^0+\gamma$ could be obtained if there is an intermediate bound state with spin one and charge conjugation number -1, which mainly decays into $\pi^0+\gamma$. We obtained the first indication for such a big cross section on the basis of a Breit-Wigner formula for the resonance with suitable choices of the relevant partial widths. The second indication is based on a theory for the π^0 form factor on the assumption that only the 2π resonant amplitude and the three-pion bound state contribute, with a relative weight determined in such a way as to give the value for the derivative at $k^2=0$ required by the distribution of internal converted electrons, in π^0 decay. Of course both approaches are rather tentative and subject to criticisms. For an energy resolution $\Delta E\sim 5$ Mev one expects a cross section, on the resonance peak, of the order of $10^{-30}-10^{-29}$ cm^2, tremendously big if compared to the perturbation-theory (i.e., constant form factor) values, always smaller than 2.75×10^{-35} cm^2 for a π^0 lifetime of 2.2×10^{-16} sec. Verification of the existence or nonexistence of such big cross section should be a feasible though probably very delicate task.

3.3. In general the reaction (21) would be observed as an annihilation into three gammas of the initial electron-positron pair. A close examination of the competing electromagnetic process,

$$e^+ + e^- \rightarrow 3\gamma, \tag{42}$$

will therefore be necessary. The process (42) occurs at the same order in the fine structure constant as the $\pi^0+\gamma$ process.

A relevant contribution to π^0 production in $e^+ + e^-$ collisions will also come from a process first discussed by Low,

$$e^+ + e^- \rightarrow e^+ + e^- + \pi^0. \tag{43}$$

Low calculates the leading term of the cross section using a Weizsäcker-Williams method.[24] Such a leading term corresponds to a forward scattering pole in (43) and its value depends only on the value of the π^0 form factor at $k^2=0$. For $E=150$ Mev, Low finds a total cross section for (43) of about 10^{-33} cm^2 with a π^0 lifetime 10^{-18} sec. With the value for the lifetime that we have used above, 2.2×10^{-16} sec, the cross section would be of the order 10^{-35} cm^2. A recent re-evaluation by Chilton[25] has lead to essentially similar results. The cross section, as calculated from the pole term, increases linearly with energy already at $E \gtrsim m_\pi$ and, for $\tau \cong 10^{-16}$ sec, can be approximated as $\sigma = 2.2\times10^{-35}(E/m_\pi)$. At the same electromagnetic order of (43) a double bremsstrahlung process can occur, and the two emitted photons may simulate the photons from π^0 decay; Low suggests discrimination between the two processes by the different spread of the photon angular distributions.[24] However, a detailed calculation of the double bremsstrahlung process should be carried out for an accurate discrimination.

4. GENERAL DISCUSSION OF THE POSSIBLE RESONANCES

4.1. In the previous sections we have discussed the possibility of resonances due to the contribution of pion-pion real or virtual bound states. In particular, we have examined the role of the proposed $T=1$, $J=1$ pion-pion resonance and of the proposed $T=0$, $J=1$

[24] F. E. Low, Phys. Rev. **120**, 582 (1960).
[25] F. Chilton (to be published).

3π bound state in reactions such as $e^++e^- \to 2\pi$, or 3π, or $\pi^0+\gamma$. Both the π-π resonant state and the 3π bound state or resonant state that we have considered have angular momentum $J=1$, and charge conjugation number, $C=-1$. Electron-positron collisions offer indeed a very suitable means for exploring the properties of intermediate neutral states with $J=1$, $C=-1$, $P=-1$, zero nucleonic number, and zero strangeness. Such states can transform into a single virtual gamma and this is in fact what selects them among all the other states accessible only through the exchange of more virtual gammas. However other quantities, such as the experimental energy resolution ΔE, and the partial decay rates from the intermediate state, are relevant to the discussion, and a more detailed examination is necessary. In the following we shall illustrate our statement by employing a simplified description of the resonant reaction based on a Breit-Wigner formula. We consider a resonant channel of the type

$$e^++e^- \to B_J \to \text{(final state)}, \qquad (44)$$

where B_J represents an intermediate state of zero strangeness and nucleonic number, spin J, and mass M. In the vicinity of the resonance we assume that a Breit-Wigner description holds. The resonance cross section for (44) at a total energy $2E$ around M will then be approximated by

$$\sigma_R(E)=\pi\lambda^2\frac{2J+1}{4}\frac{\Gamma_i\Gamma_f}{(2E-M)^2+\Gamma/4}, \qquad (45)$$

where Γ_i is the rate for $B_J \to e^++e^-$ and Γ_f the rate for $B_J \to$ (final state). The total rate is given by Γ. In any actual measurement the measured quantity is the integrated product of $\sigma(E)$ with the experimental resolution curve. For our purposes it will be enough to approximate the resolution curve with a rectangle of width $2\Delta E$. It will be necessary to distinguish among three cases: case (a): The resonance is very narrow, with a width much smaller than the experimental energy resolution, $\Gamma \ll 2\Delta E$; case (b): The resonance is wide, with a width much larger than the experimental energy resolution, $\Gamma \gg 2\Delta E$; and case (c): The resonance has a width comparable to the experimental energy resolution, $\Gamma \sim 2\Delta E$. The contribution to $e^+ + e^- \to$ (final state), from the resonance, in an experiment carried at an energy $2E=M$ with an energy spread given by $2\Delta E$ will be given by

$$\bar{\sigma}_R=\frac{1}{\Delta E}\int_{\frac{1}{2}(M-\Delta E)}^{\frac{1}{2}(M+\Delta E)} \sigma(E)dE.$$

This quantity is given in case (a) by

$$\bar{\sigma}_R=2\pi\lambda^2(\pi/4)(2J+1)B_iB_f\Gamma/(2\Delta E), \qquad (46)$$

with $B_i=\Gamma_i/\Gamma$ and $B_f=\Gamma_f/\Gamma$. In case (b) it is simply given by

$$\sigma(2M)=\pi\lambda^2(2J+1)B_iB_f. \qquad (47)$$

For the intermediate case (c) both (46) or (47) can be applied for order-of-magnitude estimates, since they only differ by a factor $\pi/2$ if $\Gamma \cong 2\Delta E$.

4.2. For purposes of comparison, one can consider a typical cross section, such as that for $e^++e^- \to \mu^++\mu^-$, which for $E \gg m_\mu$ is given by $\frac{1}{3}\pi\alpha^2\lambda^2$. It will also be sufficient to limit the discussion to the most important final channels, for which B_f is of the order unity. The important quantity is then $\Gamma_i/\Delta E$ in case (a); Γ_i/Γ in case (b); and any of these two quantities in case (c). We can then examine what important factors will appear in Γ_i, the rate for the transition $B_J \to e^++e^-$. We make use of gauge invariance and of the charge conjugation selection rules. For $J=0$, $C=1$, Γ_i is proportional to $\alpha^4 m_e^2$, and for $J=0$, $C=-1$, it will be proportional to $\alpha^6 m_e^2$. The rates vanish for $m_e=0$ because the final electron and positron should be emitted in configurations with parallel spiralities thus violating angular momentum conservation. For $J=1$, $C=1$, Γ is proportional to α^4, but for $J=1$, $C=-1$ it is proportional to α^2. For $J=2$, $C=1$, Γ_i is proportional to α^4, for $J=2$, $C=-1$ it is proportional to α^6; and, similarly, higher powers of α appear when J is increased.

It is now important to state that the energy resolution ΔE will presumably not be smaller than ~ 1 Mev. An energy spread of this order corresponds to rates Γ of the order of 10^{21} sec^{-1}. Therefore, in case (a) only intermediate states with $J=1$, $C=-1$ will produce comparatively large effects. In fact, rather large effects will be expected if $\Gamma_i/\Delta E \gg \alpha^2$. Next in importance are states with $J=1$, $C=+1$, and $J=2$, $C=1$ with an additional factor α^2. In case (b) the relevant quantity is Γ_i/Γ, and Γ is supposed to be much bigger than $2\Delta E$. Therefore the same conclusions apply as for case (a), and, of course, they hold also in case (c). If B_J can decay through strong interactions, Γ is expected to be of the order of 10^{23}–10^{22} sec^{-1} and thus much bigger than $2\Delta E$. The $T=1$, $J=1$ pion-pion resonance belongs to this class of resonances, case (b). For the 3π bound state, which decays slowly into $\pi^0+\gamma$ or $2\pi+\gamma$, Γ is presumably of the order 10^{20} sec^{-1}, rather smaller than $2\Delta E$, case (a) or case (c). For a narrow energy resolution, the factor $\Gamma_i/\Delta E$ is expected to be big enough to give large resonance peaks. In general, the occurrence of case (a) or (c) requires some inhibition of a fast decay via strong interactions and would in fact correspond to a rather exceptional situation (such as a bound state of very low mass).

5. ANNIHILATION INTO BARYON PAIRS

5.1. We shall discuss in this section electron-positron annihilation in flight into a fermion-antifermion pair according to

$$e^++e^- \to f+\bar{f}. \qquad (48)$$

Pairs of strong interacting fermions can be produced according to

$$e^{+}+e \rightarrow p+\bar{p},\; n+\bar{n}, \quad e^{+}+e^{-} \rightarrow \Sigma+\bar{\Sigma}, \tag{49}$$
$$e^{+}+e^{-} \rightarrow \Lambda+\bar{\Lambda}, \qquad e^{+}+e^{-} \rightarrow \Xi+\bar{\Xi},$$

all of the type (48). The final pair is produced in the states ${}^3S_1$ and ${}^3D_1$ as follows directly from angular momentum and parity considerations (charge conjugation does not add anything new to this case). The cross section near the threshold is thus expected to grow up proportional to the velocity of the final particles in the centers-of-mass system, and the threshold angular dependence is expected to be isotropic. The matrix element (3) of the electromagnetic current operator between the vacuum and the state containing the fermion-antifermion pair can be written in the form

$$J_\mu = e\bar{u}(p)[F_1(k^2)\gamma_\mu - (\mu/2m)F_2(k^2)\sigma_{\mu\nu}k_\nu]v(\bar{p}), \tag{50}$$

where p_ν and $\bar{p}_\nu$ are the four-momenta of the produced fermion and antifermion, respectively; $\bar{u}(p)$ and $v(\bar{p})$ are their Dirac spinors; and $F_1(k^2)$, $F_2(k^2)$ are the analytic continuations of the electric and magnetic form factors of the fermion for the values of k^2 relevant in (48), namely $k^2 < -4m^2$, where m is the mass of the produced fermion. In (50), μ is the static anomalous magnetic moment of the produced fermion. The form (50) for J_ν follows from Lorentz and gauge invariance. The form factors are normalized in such a way that $F_1(0)=1$ and $F_2(0)=1$ if the fermion is charged; and that $F_1(0)=0$ and $F_2(0)=1$ if it is neutral. The current matrix element J_ν can be decomposed, as usual, as the sum of an isotopic vector part and of an isotopic scalar part (for Λ and Σ^0 there is only the scalar part). This decomposition brings about a considerable simplification for the three processes leading to Σ-$\bar{\Sigma}$ production which are described in terms of four independent form factors. On the basis of the presently assumed mass spectrum, we expect for the isotopic vector form factors the absorptive cut in the k^2 plane to start at $k^2=-4m_\pi^2$ and for the isotopic scalar form factors to start at $k^2=-9m_\pi^2$, except for the possible presence of a 3π bound state, producing a pole contribution at a lower $|k^2|$. The above consideration does not hold for the Σ which can transform into an intermediate Λ by pion emission, giving rise to a lowering for the absorptive cut[26] (for instance, for the charged Σ's the isotopic vector amplitude has a threshold at

$$-4m_\pi^2\left[1-\left(\frac{m_\Sigma^2-(m_\Lambda^2+m_\pi^2)}{2m_\pi m_\Lambda}\right)^2\right]\Bigg).$$

5.2. The form factors are in general complex along the absorptive cut. In particular, they are complex for the physical values of k^2 in reaction (48), $k^2<-4m^2$. Thus in $e^{+}+e^{-} \rightarrow f+\bar{f}$ there can be a polarization of f normal to the production plane, already at the lowest electromagnetic order. The polarization will be proportional to the sine of the phase difference between the electric and the magnetic form factors. The situation here is different from that of the scattering process $e+f\rightarrow e+f$, occurring at positive k^2, where the form factors are real. In the scattering process there can be no polarization of f normal to the scattering plane, except for higher order electromagnetic corrections. This follows from usual time-reversal arguments.

In calculating $\sum R_{mn}$ according to (7) and (9) we sum over the polarization states of $\bar{f}$, but we introduce a spin projection operator before summing on the polarization states of f. The spin projection operator is $\frac{1}{2}(1+i\gamma_5\gamma_\mu s_\mu)$ where s_μ is the covariant polarization vector for f. We know that the polarization of f will be transverse and normal to the production plane; therefore s_μ will be of the form $(\boldsymbol{\zeta},0)$ where $\boldsymbol{\zeta}$ is a unit vector normal to the production plane.

The cross section can be expressed in the form

$$\frac{d\sigma}{d(\cos\theta)}=\frac{\pi}{8}\alpha^2\lambda^2\beta\left[|F_1(k^2)+\mu F_2(k^2)|^2(1+\cos^2\theta)+\left|\frac{m}{E}F_1(k^2)+\frac{E}{m}\mu F_2(k^2)\right|^2\sin^2\theta\right], \tag{51}$$

where $\lambda = E^{-1}$.

The form factors are taken at $k^2=-4E^2$. Near the threshold $E\cong m$ the cross section (51) is proportional to β, the velocity of the final fermion, and is isotropic, in accordance with production in the ${}^3S_1$ state.

As we have already remarked, the fact that the form factors have an imaginary part for k^2 in the physical region for reaction (48) implies the possibility of a polarization of f normal to the plane of production and proportional to the sine of the phase difference between the electric and the magnetic form factors. The polarization of f along the normal to the production plane is given by $p(\theta)$, defined from

$$\frac{d\sigma}{d(\cos\theta)}p(\theta)=-\frac{\pi}{8}\alpha^2\lambda^2\beta^3\mu\frac{E}{m}\times\mathrm{Im}[F_1(k^2)F_2^*(k^2)]\sin(2\theta). \tag{52}$$

The normal to the plane here has been defined as the unit vector pointing in the direction of $\mathbf{p}\times\mathbf{q}_+$, where $\mathbf{p}$ is the momentum of the final fermion and $\mathbf{q}_+$ that of the incoming positron. We note that, by direct application of the TCP theorem, the polarization of the produced antifermion, $\bar{f}$, is equal but opposite in sign to that of the produced f.

With $F_1=1$ and $F_2=0$ the total cross section, as obtained from (51), is

$$\sigma=m^{-2}(2.1\times10^{-32}\ \mathrm{cm}^2)u(1-u)^{\frac{1}{2}}(1+\tfrac{1}{2}u), \tag{53}$$

with m in Bev and $u=(m/E)^2$. Of course, there is no reason whatsoever why the position $F_1=1$, $F_2=0$ should

[26] R. Karplus, C. M. Sommerfield, and E. H. Wichmann, Phys. Rev. **111**, 1187 (1958).

have any reliability in the physical region for the production process, which is far away from the limit $k^2=0$.

5.3. It is at present difficult to decide whether the form factor in (56) should strongly decrease or increase the value of the cross sections over the perturbation theory value given by (53). At present there is no information available on the form factors of the hyperons. Electron scattering on nucleons has provided reliable information on the nucleon form factors for positive values of k^2. It will not be easy, however, to extract information from the form factor at positive k^2, as determined from electron scattering experiments, about the values for large negative k^2 relevant to the production experiments. The recent indication of a core term in the nucleon structure[14] could eventually be related to the presence of singularities for large negative values of k^2, but the location and the nature of such singularities cannot be determined at present. To show a kind of science-fiction argument that one can use to relate the information from the scattering experiments to possible guesses on the pair production reactions, we shall make some (completely arbitrary) hypotheses on the origin of the core term in the nucleon structure and see what consequences it leads to for pair production. Suppose, for instance, that the core terms in the form factors given by Hofstadter and Herman[14] come from a big absorptive term concentrated around, say, $k^2=-(3m)^2$. This choice is quite arbitrary and, as far as the experiments tell us, there is no reason why the core term should not originate from singularities at much lower values of k^2, say, $k^2\cong -m^2$, and furthermore, it is very likely that it merely results from contributions of singularities extending all over the absorptive region. Let us also, for definiteness, assign some small width Γ to the states originating the singularities. For $k^2\cong -(3m)^2$, the nucleon form factors should then be approximated, using the Hofstadter[14] results, as $F_{1p}=1.2/D$, $F_{2p}=-3.4/D$, $F_{1n}=3.2/D$, and $F_{2n}=0$, where the common denominator is given by $D=20-2E+i(\Gamma/2)$, and we have expressed all energies in units of the pion mass. Inserting into (54) we find for the cross sections near the singularity $\sigma\cong(\pi/3)\alpha^2\lambda^2\beta \times 3.6\times(m_\pi/\Gamma)^2$ for p-$\bar{p}$ production and $\sigma\cong(\pi/3)\alpha^2\lambda^2\beta \times 50\times(m_\pi/\Gamma)^2$ for n-$\bar{n}$ production. If, for instance, $\Gamma\cong m_\pi$, these values are about 3.6 and 50 times bigger than the perturbation theory value for $e^+ + e^- \rightarrow f^+ + f^-$ (the p-$\bar{p}$ cross section is smaller because of an accidental cancellation). The above considerations have admittedly little value, except that they may serve to illustrate the hope that the cross sections, at least in same energy intervals, might come out rather bigger than what expected an the basis of (53).

5.4. Besides the reactions (49) one should also list

$$e^+ + e^- \rightarrow \Sigma^0 + \bar{\Lambda},\ \Lambda + \bar{\Sigma}^0, \tag{54}$$

which involve a fermion-antifermion pair, but not charge conjugate of each other. The expression for the cross section of (54) depends on the relative Σ-Λ parity and, actually, if an experiment like (48) could be carried out, it would provide a good mean for measuring the relative Σ-Λ parity. That the cross section for (54) has a strong dependence on the relative Σ-Λ parity can also be seen directly by examining the threshold behavior. If the relative Σ-Λ parity is positive, the final $\Sigma\bar{\Lambda}$ (or $\bar{\Sigma}\Lambda$) will be produced in ${}^3S_1$, and ${}^3D_1$, as follows from parity and angular momentum conservation. If the relative Σ-Λ parity is negative, the accessible final states are instead ${}^1P_1$ and ${}^3P_1$. Therefore, the cross section near the threshold increases linearly with the final momentum p in the center-of-mass system for even parity, and it is also isotropic. For odd parity it increases as p^3 and will contain in general a $\cos^2\theta$ term.

The general form of the matrix element J derived from the requirements of Lorentz and gauge invariance is different from the case considered in the preceding section of a self-conjugate fermion-antifermion pair. For even relative parity we can write

$$J_\nu = \bar{u}_\Lambda[f_1(k^2)\gamma_\nu + f_2(k^2)\sigma_{\nu\mu}k_\mu + f_3(k^2)k_\nu]v_\Sigma, \tag{55}$$

subject to the condition $k_\nu J_\nu=0$ which gives

$$f_3(k^2)=i\frac{f_1(k^2)}{k^2}(m_\Sigma - m_\Lambda). \tag{56}$$

For odd relative parity

$$J_\nu = \bar{u}_\Lambda[f_1(k^2)\gamma_\nu + f_2(k^2)\sigma_{\nu\mu}k_\mu + f_3(k^2)k_\nu]\gamma_5 v_\Sigma, \tag{57}$$

and $k_\nu J_\nu=0$ gives

$$f_3(k^2)=-i\frac{f_1(k^2)}{k^2}(m_\Sigma + m_\Lambda). \tag{58}$$

The form factors $f_1(k^2)$, $f_2(k^2)$, $f_3(k^2)$ are the analytic continuations of the form factors describing, for positive k^2, a virtual transition $\Sigma\rightarrow\Lambda+\gamma$. The correspondence is correct, provided k_μ is defined in the $\Sigma\rightarrow\Lambda+\gamma$ transition as $k_\mu=\Sigma_\mu-\Lambda_\mu$, where Σ_μ and Λ_μ are the Σ and Λ four-momenta. One notices that $f_3(k^2)$ will not enter in the description of the production process (54), as can be seen by specializing (55) or (57) in the center-of-mass system where k_ν has only the time-like component, but J_4 is zero because of $k_4 J_4=0$.

The cross section is given by

$$\frac{d\sigma}{d(\cos\theta)}=\frac{\pi}{8}\alpha^2\lambda^2\beta\bigg\{\beta^2\cos^2\theta[|f_1(k^2)|^2+k^2|f_2(k^2)|^2] + [|f_1(k^2)|^2-k^2|f_2(k^2)|^2]\frac{E_\Lambda E_\Sigma \pm m_\Lambda m_\Sigma}{E^2} - 4\frac{m_\Lambda E_\Sigma \pm m_\Sigma E_\Lambda}{E}\operatorname{Re}[f_1(k^2)f_2^*(k^2)]\bigg\}, \tag{59}$$

where the plus sign refers to even relative Σ-Λ parity and the minus sign to odd relative parity, and $\beta = p/E$. For production near the threshold, the cross sections become

$$\frac{d\sigma}{d(\cos\theta)} = \frac{\pi}{4}\alpha^2\lambda^2\beta\frac{m_\Lambda m_\Sigma}{E^2}|f_1 - 2Ef_2|^2 \tag{60}$$

for even relative parity, and

$$\frac{d\sigma}{d(\cos\theta)} = \frac{\pi}{8}\alpha^2\lambda^2\beta^3\{A + B\cos^2\theta\} \tag{61}$$

for odd relative parity, with

$$A = (|f_1|^2 - k^2|f_2|^2)\frac{1}{2}\left(\frac{m_\Lambda}{m_\Sigma} + \frac{m_\Sigma}{m_\Lambda}\right) + 2E\left(\frac{m_\Sigma}{m_\Lambda} - \frac{m_\Lambda}{m_\Sigma}\right)\mathrm{Re}[f_1 f_2^*]$$

$$B = |f_1|^2 + k^2|f_2|^2.$$

The decay process,

$$\Sigma^0 \to \Lambda^0 + \gamma,$$

is expected to be essentially determined by $f_2(0)$ (proportional to the so-called transition magnetic moment between Σ and Λ), for each case of relative parity. In fact, for a real γ, terms proportional to k_ν in both (55) and (57) do not contribute because of the transversality condition $k_\nu\epsilon_\nu = 0$. Similarly, $f_1(k^2)$ should presumably vanish at $k^2 = 0$ as suggested from (56) or (58). The same should apply to the quasi-real gammas in $\Sigma^0 \to \Lambda^0 + e^+ + e^-$. The physical values of k^2 in the production process $e^+ + e^- \to \Sigma^0 + \Lambda^0$ lie very far from $k^2 \cong 0$ so that a direct connection with Σ^0 decay seems unjustified.

6. ANNIHILATION INTO POSSIBLE VECTOR MESONS

6.1. Vector mesons have been discussed recently[11] because of their formal connection with local conservation laws.[27] We have already discussed in some detail the possibility of detecting neutral unstable vector mesons with charge conjugation number -1 through their resonant effect in reactions

$$e^+ + e^- \to B^0 \to \text{(final state)}, \tag{62}$$

where B^0 is the unstable meson. In this section we shall discuss reactions of the type

$$e^+ + e^- \to B + \bar{B}, \tag{63}$$

where B is a (charged or neutral) spin-one meson. Reactions of the kind (62) will be very suitable to detect vector mesons B^0 with $C = -1$ and zero strangeness. However, a vector meson K' with nonzero strangeness would not appear as intermediate state in (62), but it could be produced according to (63) or to reactions of the kind

$$e^+ + e^- \to K' + K, \tag{64}$$

conserving the total strangeness. The suggested strongly interacting vector mesons are all expected to be eminently unstable. Reactions like (63) would therefore be observed as many-body reactions, and the possibility of separating the over-all process into two stages, of which the first is the production process of the vector mesons, relies essentially on the hypothesis of a sufficiently long lifetime for the intermediate vector meson. When this hypothesis is not satisfied the separation of the process into two stages is less justified and can only lead to approximate results.

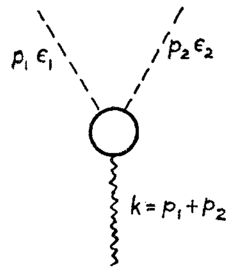

FIG. 6. Electromagnetic vertex for production of vector boson pair. The symbols are defined in the text.

6.2. We shall here examine in detail reaction (63), including also a discussion of the angular correlation at the decay of B, also in view of applications that we will consider in the next section to the verification of the intermediate meson theory of weak interactions. We shall first give the general form for the electromagnetic vertex of a vector boson on the basis of Lorentz-invariance, gauge invariance, and charge conjugation invariance. The vertex is described by three form factors. In the static limit they correspond to the charge, the magnetic moment, and the electric quadrupole moment.

In the electromagnetic vertex shown in Fig. 6, we call p_1^μ, ϵ_1^μ and p_2^μ, ϵ_2^μ the four-momenta and polarization four-vectors of the (physical) particles B and $\bar{B}$. They satisfy $p_1^2 = p_2^2 = -m_B^2$, $\epsilon_1^2 = \epsilon_2^2 = 1$, and $(p_1\epsilon_1) = (p_2\epsilon_2) = 0$. The matrix element J^μ of the electromagnetic current must be constructed out of p_1^μ, p_2^μ, ϵ_1^μ, and ϵ_2^μ. We take as independent vectors: $k^\mu = p_1^\mu + p_2^\mu$, $p^\mu = p_1^\mu - p_2^\mu$, ϵ_1^μ, and ϵ_2^μ. We note that $p^2 = -k^2 - 4m_B^2$, $(kp) = 0$, $(\epsilon_1 p) = -(\epsilon_1 k)$, $(\epsilon_2 p) = -(\epsilon_2 k)$. The only independent scalars are therefore: $k^2(\epsilon_1 k)$, $(\epsilon_2 k)$, and $(\epsilon_1\epsilon_2)$. The matrix element J_μ must transform like a vector and must depend linearly on each ϵ. We thus write

$$J^\mu = k^\mu[(\epsilon_1\epsilon_2)a(k^2) + (\epsilon_1 k)(\epsilon_2 k)b(k^2)] + p^\mu[(\epsilon_1\epsilon_2)c(k^2) + (\epsilon_1 k)(\epsilon_2 k)d(k^2)] + \epsilon_1^\mu(\epsilon_2 k)e(k^2) + \epsilon_2^\mu(\epsilon_1 k)f(k^2). \tag{65}$$

From the condition $(kJ) = 0$ we obtain $a(k^2) = 0$ and $-k^2 b(k^2) = e(k^2) + f(k^2)$. We then make use of invariance

[27] C. N. Yang and R. Mills, Phys. Rev. **96**, 191 (1954).

under charge conjugation. The electromagnetic current operator j^μ transforms into $-j^\mu$ under charge conjugation. Such a condition requires that the matrix element J^μ transforms into $-J^\mu$ when $k^\mu \to k^\mu$, $p^\mu \to -p^\mu$ and $\epsilon_1^\mu \leftrightarrow \epsilon_2^\mu$. It follows that $e(k^2)=-f(k^2)$. The general form of J^μ is thus

$$J^\mu = p^\mu[(\epsilon_1\epsilon_2)c(k^2)+(\epsilon_1 k)(\epsilon_2 k)d(k^2)] + [\epsilon_1^\mu(\epsilon_2 k)-\epsilon_2^\mu(\epsilon_1 k)]e(k^2). \quad (66)$$

It will be convenient to introduce form factors $G_1(k^2)$, $G_2(k^2)$, and $G_3(k^2)$ such that $eG_1(k^2)$, $\mu G_2(k^2)$, and $\epsilon G_3(k^2)$ describe in suitable linear combinations (for small spacelike k^2) the charge distribution, the magnetic moment distribution, and the electric quadrupole moment distribution. The new form factors are linearly related to $c(k^2)$, $d(k^2)$, and $e(k^2)$. We will thus write

$$J^\mu = (2\pi)^3\langle B\bar{B};\mathrm{out}|j^\mu(0)|0\rangle = \frac{e}{(4\omega_1\omega_2)^{\frac{1}{2}}}\{G_1(k^2)(\epsilon_1\epsilon_2)p^\mu + [G_1(k^2)+\mu G_2(k^2)+\epsilon G_3(k^2)][(\epsilon_1 k)\epsilon_2^\mu-(\epsilon_2 k)\epsilon_1^\mu] + \epsilon G_3(k^2)m_B^{-2}[(k\epsilon_1)(k\epsilon_2)-\tfrac{1}{2}k^2(\epsilon_1\epsilon_2)]p^\mu\}, \quad (67)$$

where ω_1 and ω_2 are the center-of-mass energies of B and $\bar{B}$. The static anomalous magnetic moment is $\mu+\epsilon$; the static anomalous electric quadrupole moment is 2ϵ.

6.3. In a Lagrangian theory of vector mesons one would assume a Lagrangian

$$\mathfrak{L} = -\tfrac{1}{2}B_{\mu\nu}^\dagger B_{\mu\nu} - m_B^2 U_\mu^\dagger U_\mu, \quad (68)$$

where U_μ is the vector field, $B_{\mu\nu}=\partial_\mu U_\nu - \partial_\nu U_\mu$, with $\partial_\mu = (\partial/\partial x)_\mu - ieA_\mu$, and m_B the mass of the meson. The supplementary condition,

$$\partial_\mu U_\mu = (ie/2)F_{\mu\nu}B_{\mu\nu}, \quad (69)$$

follows from the field equations (if $m_B \neq 0$). The minimal electromagnetic current is thus

$$j_\mu = -ie[U_\nu^\dagger B_{\mu\nu} - U_\nu B_{\mu\nu}^\dagger]. \quad (70)$$

To such a current one can add nonminimal terms

$$j_\mu' = -ie\mu(\partial/\partial x_\nu)(U_\mu^\dagger U_\nu - U_\nu^\dagger U_\mu), \quad (71)$$

$$j_\mu'' = ie(\epsilon/m_B^2)(\partial/\partial x_\nu)(B_{\mu\lambda}^\dagger B_{\nu\lambda} - B_{\nu\lambda}^\dagger B_{\mu\lambda}), \quad (72)$$

The total current is then of the form (67) with $G_1(k^2) = G_2(k^2) = G_3(k^2) = 1$.

6.4. The cross section formula (7) reduces to

$$\sigma = \frac{\alpha}{(2\pi)}\frac{1}{16E^4}\int d^3p_1 d^3p_2\,\delta(\omega_1+\omega_2-2E) \times \delta^3(\mathbf{p}_1+\mathbf{p}_2)T_{mn}\sum_{1,2} R_{mn}, \quad (73)$$

where T_{mn} is given by (8) and R_{mn} by (9) and (67). Differential cross sections and cross sections for polarized final particles can be obtained from (73) by omitting the relevant integrations and spin summations.

We note that $T_{mn}\sum R_{mn}$ is a Lorentz invariant quantity. We want a complete description of one of the produced bosons, say of B, after averaging over the polarizations of the other. We first sum over the polarizations of $\bar{B}$, using

$$\textstyle\sum_2 \epsilon_2^\mu \epsilon_2^\mu = \delta_{\mu\nu} + p_{2\mu}p_{2\nu}/m_B^2,$$

and we write

$$T_{mn}\sum R_{mn} = R^{\rho\sigma}\epsilon_1^\rho\epsilon_1^\sigma. \quad (74)$$

Equation (74) defines the tensor $R_{\rho\sigma}$. The density matrix will be described in terms of the tensor

$$\tilde{R}_{\rho\sigma} = \Lambda_{\rho\tau}R_{\tau\omega}\Lambda_{\omega\sigma}, \quad (75)$$

where

$$\Lambda_{\mu\nu} = \delta_{\mu\nu} + p_{1\mu}p_{1\nu}/m_B^2 \quad (76)$$

is a projection operator such that $\tilde{\epsilon}_\mu^{(1)} = \Lambda_{\mu\nu}\epsilon_\nu^{(1)}$ always satisfies $(p^{(1)}\tilde{\epsilon}^{(1)})=0$.

The differential cross section is given from (73), (74) and (75), by

$$\frac{d\sigma}{d(\cos\theta)} = \frac{\alpha}{32}\frac{1}{E^2}\beta\,\mathrm{Tr}[\tilde{R}], \quad (77)$$

where

$$\beta = (1-m_B^2/E^2)^{\frac{1}{2}}$$

is the velocity of the produced bosons. The differential cross section can be evaluated directly from (77), (76), (75), and (74), or using the expression for $\tilde{R}$ that we give in the next section. Its expression is given by

$$\frac{d\sigma}{d(\cos\theta)} = \frac{\pi}{16}\alpha^2\lambda^2\beta^3\left\{2\left(\frac{E}{m_B}\right)^2|G_1(k^2) + \mu G_2(k^2)+\epsilon G_3(k^2)|^2(1+\cos^2\theta) + \sin^2\theta\left[2\left|G_1(k^2)+2\left(\frac{E}{m_B}\right)^2\epsilon G_3(k^2)\right|^2 + \left|G_1(k^2)+2\left(\frac{E}{m_B}\right)^2\mu G_2(k^2)\right|^2\right]\right\}. \quad (78)$$

The β^3 dependence in (78) for production near threshold is typical of P-state production. In our approximation of neglecting higher-order electromagnetic terms, the final mesons must be produced in a state of total angular momentum $J=1$, parity $P=-1$, and charge conjugation number $C=-1$. From angular momentum and parity conservation it follows that the final mesons can only be in ${}^1P_1$, ${}^3P_1$, ${}^5P_1$, and ${}^5F_1$. However, triplet states of odd orbital parity cannot be produced because they have $C=+1$, so we are left with ${}^1P_1$, ${}^5P_1$, and ${}^5F_1$ as the only permitted final states.

With $G_1=1$, $G_2=G_3=0$, the total cross section is

$$\sigma = m_B^{-2}(2.1\times10^{-32}\ \mathrm{cm}^2)\tfrac{3}{4}(1-u)^{\frac{3}{2}}(\tfrac{4}{3}+u), \quad (79)$$

where m_B is expressed in Bev and $u=(m/E)^2$. Therefore, e^+-e^- collisions may turn out to be very efficient for detecting possible unstable vector mesons.

6.5. The above cross section obtained with $G_1=1$, $G_2=G_3=0$ formally violates unitarity at high energies. For high energies (79) goes to a constant whereas it can be shown, on the basis of unitarity arguments, that the total reaction cross section must decrease proportionally to $\lambda\!\!\!^{-2}$.

Unitarity arguments are not very informative usually at relativistic energies. Electron-positron collisions present, however, an exceptional circumstance, that they go through one specified channel, the one-photon channel, as long as one neglects higher order electromagnetic terms. We shall present here a derivation of the upper limit to the reaction cross section required from unitarity for electron-positron collisions at the lowest electromagnetic order. For the derivation we shall employ the Jacob-Wick notation. Let us consider a process

$$a+b \rightarrow \text{(final state)}. \tag{80}$$

We shall denote by F a set of final states specified by the nature of the final products. The initial state is defined, for a given center-of-mass momentum of the colliding particles a and b, by their helicities λ_a and λ_b. We shall write

$$|i\rangle = |\lambda_a,\lambda_b\rangle. \tag{81}$$

The total cross section from such a state summing over the set F is then given, in the notations of Jacob and Wick, by

$$\sigma(\lambda_a\lambda_b;F)=(2\pi)^2\lambda\!\!\!^{-2}\langle\lambda_a\lambda_b|T(E)^\dagger P_F(E)T(E)|\lambda_a\lambda_b\rangle, \tag{82}$$

where $T(E)$ is the T matrix at energy E of each of the colliding particles and $P_F(E)$ is the projection operator into the states of total energy $2E$ of the set F. Both $T(E)$ and $P(E)$ are rotation invariant and therefore they commute with the total angular momentum J. The cross section σ can thus be written as a sum of σ_J belonging to the different J's,

$$\begin{aligned}\sigma_J(\lambda_a,\lambda_b;F)=&\pi\lambda\!\!\!^{-2}(2J+1)\\ &\times\langle J,\lambda_a,\lambda_b|T_J^\dagger(E)P_J(E)T_J(E)|J,\lambda_a\lambda_b\rangle,\end{aligned} \tag{83}$$

where $\langle J,\lambda_a,\lambda_b|$ is the J component of $\langle i|$. Now for a reaction (as opposed to scattering) we can substitute S for T, and using $S_J^\dagger(E)S_J(E)=1$, we obtain an upper limit for (77):

$$\sigma_J(\lambda_a,\lambda_b;F)\leq\pi\lambda\!\!\!^{-2}(2J+1). \tag{84}$$

We can apply this result to our reactions,

$$e^++e^-\rightarrow(\gamma)\rightarrow\text{(final state)}. \tag{85}$$

The initial e^+-e^- states must have $J=1$, $C=-1$, and $P=-1$. Two linear combinations of states (81) exist that have such quantum numbers, namely,

$$(1/\sqrt{2})(|1,1\rangle+|-1,-1\rangle),$$
$$(1/\sqrt{2})(|1,-1\rangle+|-1,+1\rangle),$$

both for $J=1$. Helicity $+1$ for a particle means that the spin is pointing in the direction of the momentum. However, only the second of such states participates to (85) in the limit when the electron mass can be neglected. In fact, the initial electron and positron appear in the combination $\bar{v}\gamma_\mu u$, which can be written $\bar{v}(\bar{a}\gamma_\mu a+a\gamma_\mu\bar{a})u$ where $a=\frac{1}{2}(1+\gamma_5)$ and $\bar{a}=\frac{1}{2}(1-\gamma_5)$ are the projection operators for negative and positive helicity. By averaging (84) over the initial polarizations we then find the upper limit

$$\tfrac{3}{4}\pi\lambda\!\!\!^{-2}$$

for the cross section of a reaction (85), neglecting the electron mass. The cross section (79), derived from (78) with the position $G_1=1$, $G_2=G_3=0$, is the same as would be given by the lowest order perturbation contribution to $e^++e^-\rightarrow B^++B^-$, ignoring any structure of B. The expression (79) violates unitarity at high energies. The violation, however, occurs at very high energies, of the order of 10^2m_B. At these high energies it is certainly inaccurate to neglect higher-order electromagnetic terms, and also structure effects due to other interactions of B, if they exist, would anyway be important.

6.6. The matrix

$$\rho=\tilde{R}/\mathrm{Tr}[\tilde{R}], \tag{86}$$

gives complete information on the produced B, and has the transformation properties of a tensor. Its calculation is long but straightforward, using (9), (67), (74), (75), and (76). We give here the result:

$$\begin{aligned}\tilde{R}^{\mu\nu}=\frac{e^2}{8}\bigg\{&B_1\delta_{\mu\nu}+B_2\frac{k^\mu k^\nu}{m_B^2}+B_3\frac{q^\mu q^\nu}{m_B^2}+B_4\frac{p^\mu p^\nu}{m_B^2}\\ &+B_5\frac{k^\mu q^\nu+k^\nu q^\mu}{m_B^2}+B_6\frac{k^\mu p^\nu+k^\nu p^\mu}{m_B^2}+B_7\frac{q^\mu p^\nu+q^\nu p^\mu}{m_B^2}\\ &+iB_8\beta x^2\cos\theta\bigg[\frac{p^\mu q^\nu-p^\nu q^\mu}{m_B^2}+\beta^2\frac{k^\mu q^\nu-k^\nu q^\mu}{m_B^2}\\ &\qquad-\beta\cos\theta\frac{k^\mu p^\nu-k^\nu p^\mu}{m_B^2}\bigg]\bigg\},\end{aligned} \tag{87}$$

where $q=q_+-q_-$, $x=E/m_B$, and the B's are given by

$$B_1=4\beta^2\sin^2\theta[|G_1+2x^2\epsilon G_3|^2+4x^2\beta^2|G_1+\mu G_2+\epsilon G_3|^2,$$

$$\begin{aligned}B_2=&x^2\beta^2(1+\beta^2\cos^2\theta)|G_1+\mu G_2+\epsilon G_3|^2\\ &+\beta^2\sin^2\theta\{|G_1+2x^2\mu G_2|^2\\ &+4\,\mathrm{Re}[(G_1+x^2\mu G_2+x^2\epsilon G_3)(\epsilon G_3-\mu G_2)^*],\end{aligned}$$

$$B_3=-\beta^2|G_1+\mu G_2+\epsilon G_3|^2,$$

$$\begin{aligned}B_4=&x^2\beta^2(1+\cos^2\theta)|G_1+\mu G_2+\epsilon G_3|^2\\ &+4x^2\,\mathrm{Re}[(G_1+\mu G_2+\epsilon G_3)(\mu G_2-\epsilon G_3)^*]\\ &+\beta^2\sin^2\theta(|G_1|^2+4x^2\epsilon\,\mathrm{Re}G_1G_3^*+4x^4\mu^2|G_2|^2),\end{aligned}$$

$$B_5=-2\beta^3x^2\cos\theta\,\mathrm{Re}[(G_1+\mu G_2+\epsilon G_3)(\mu G_2-\epsilon G_3)^*],$$

$$B_6 = x^2\beta^2(1+\cos^2\theta)|G_1+\mu G_2+\epsilon G_3|^2 + \beta^2\sin^2\theta|G_1+2x^2\mu G_2|^2 + 2x^2\beta^2\cos^2\theta\,\mathrm{Re}[(G_1+\mu G_2+\epsilon G_3)(\mu G_2-\epsilon G_3)^*],$$
$$B_7 = \beta\cos\theta|G_1+\mu G_2+\epsilon G_3|^2 - 2\beta x^2\cos\theta\,\mathrm{Re}[(G_1+\mu G_2+\epsilon G_3)(\mu G_2-\epsilon G_3)^*],$$
$$B_8 = 2\beta^2\,\mathrm{Im}G_1(\mu G_2+\epsilon G_3)^*. \tag{88}$$

The form factors are all taken at $k^2=-4E^2$.

6.7. The density matrix ρ contains a complete description of the produced B. If one knows the amplitude for a mode of decay of B, the angular correlations of the decay products with respect to the incident and final momenta in the production process can be calculated. Consider, for instance, a two-body decay of B. The decay amplitude will be of the form

$$\epsilon_1{}^\mu \mathfrak{A}^\mu, \tag{89}$$

where $\mathfrak{A}^\mu$ is a vector (or pseudovector, or a sum of both). The angular distribution of the secondaries in the rest system of the decaying B is then given by

$$\sum_{\text{spin}} (\mathcal{A}^\mu \rho^{\mu\nu} \mathfrak{A}^\nu) d\Omega, \tag{90}$$

where $d\Omega$ is the solid angle in the B rest frame and $\mathcal{A}^\mu = (\mathfrak{A}^{i*} - \mathfrak{A}^{4*})$. The summation is extended over the final spin states. The quantity $\mathcal{A}^\mu\rho^{\mu\nu}\mathfrak{A}^\nu$ is a scalar invariant and can be evaluated in the production center-of-mass system (system of the laboratory in a colliding beam experiment) using the expressions (87) and (89) that are valid in that system. The distribution in the laboratory system of the colliding beam experiment is thus given directly by

$$\sum_{\text{spin}} (\mathcal{A}^\mu \rho^{\mu\nu} \mathfrak{A}^\nu) \frac{d\Omega}{d\Omega'} d\Omega', \tag{91}$$

where $d\Omega'$ is the decay solid angle in the laboratory system and $d\Omega/d\Omega'$ only depends on the decay angle with respect to the line of flight of B and on the velocity of B.

As an application we consider the decays $B\to\pi+\pi$ and $B\to\mu+\nu$, $B\to e+\nu$. The amplitude $\mathfrak{A}^\mu$ for

$$B\to\pi+\pi,$$

has the general form $\mathfrak{A}^\mu = a(s^2)p_1{}^\mu + b(s^2)s^\mu$ where s^μ is the difference of the two final four-momenta; $p_1{}^\mu$, the momentum of B, is their sum; and $a(s^2)$ and $b(s^2)$ are form factors. However, the first term in the above expression for $\mathfrak{A}^\mu$ does not contribute in the decay of a physical B, because of $p_1{}^\mu\epsilon_1{}^\mu=0$. So we take the amplitude in the form

$$\mathfrak{A}^\mu = b(s^2)s^\mu. \tag{92}$$

The decay correlation is thus given by

$$\rho^{\mu\nu}s^\mu s^\nu d\Omega. \tag{93}$$

We have calculated the angular correlation for B mesons produced close to threshold and assuming $\mu=0$, $\epsilon=0$, that is, neglecting any anomalous magnetic dipole or electric quadrupole moment. The angular correlation is given by

$$2-(\mathbf{i}\cdot\mathbf{f})^2-(\mathbf{i}\cdot\mathbf{d})^2+2(\mathbf{i}\cdot\mathbf{d})(\mathbf{d}\cdot\mathbf{f})(\mathbf{f}\cdot\mathbf{i}), \tag{94}$$

where $\mathbf{i}$, $\mathbf{f}$, and $\mathbf{d}$ are unit vectors in the direction, respectively, of the incoming momentum in the collision process, of the outgoing momentum in the collision process, and of the relative final momentum in the decay.

The amplitude $\mathfrak{A}^\mu$ for $B\to\mu+\nu$ and $B\to e+\nu$, assuming that the leptons are produced locally in the $1+\gamma_5$ projection, is given by in general by

$$[\bar{l}\gamma_\mu(1+\gamma_5)\nu]c(s^2), \tag{95}$$

where l denotes either μ or e, and ν denotes the neutrino, and $c(s^2)$ is a form factor depending on the relative final four-momentum in the decay. The angular correlation can be obtained from (90) and is given by

$$\rho^{\mu\nu}[p_1{}^\mu p_1{}^\nu - s^\mu s^\nu + \delta^{\mu\nu}(m_B{}^2-m_l{}^2) + \tfrac{1}{2}\epsilon^{\mu\nu\rho\sigma}(s^\rho p_1{}^\sigma - s^\sigma p_1{}^\rho)]. \tag{96}$$

Again we specialize to B production near the threshold and neglect μ and ϵ. The angular correlation is then given by

$$3+(\mathbf{i}\cdot\mathbf{d})^2-2(\mathbf{i}\cdot\mathbf{f})(\mathbf{f}\cdot\mathbf{d})(\mathbf{d}\cdot\mathbf{i}), \tag{97}$$

in terms of the same vectors defined before. We have neglected the mass of the final lepton m_l in comparison to the mass of B. General formulas can be easily derived from (93), (95) and their analogs, and the general expression for $\bar{R}^{\mu\nu}$ reported in (87) and (88), to cover all interesting cases.

7. EXPERIMENTS ON WEAK INTERACTIONS

7.1. Semiweakly interacting bosons have been suggested as intermediary agents of weak interactions.[28] A simplest scheme of weak interactions is based on charged weak currents only and can be reproduced by postulating only charged vector mesons. It is known that the absence of $\mu\to e+\gamma$ leads to a difficulty in a theory with intermediate vector bosons, and the usual suggestion to overcome such a difficulty is that there are two different neutrinos ν_e and ν_μ. Charged currents alone do not allow a simple incorporation of the $\Delta T=\frac{1}{2}$ rule in the theory of weak interactions. However, a coupling of neutral intermediate vector mesons to both the neutral strangeness nonconserving current and the neutral lepton current leads to contradictions with experimental data. Therefore it is probable that even if intermediate neutral vector mesons exist they do not couple to the neutral lepton currents and, in particular, to the initial electron-positron state of the reactions that we are discussing. A check of this supposition

[28] R. P. Feynman and M. Gell-Mann, Phys. Rev. **109**, 193 (1958); T. D. Lee and C. N. Yang, *ibid.* **119**, 1410 (1960).

could be carried out experimentally on the basis of the following remarks. If a B^0 exists which couples to e^+-e^-, $\mu^+-\mu^-$, etc., it would give rise to resonances in reactions of the kind

$$e^+ + e^- \rightarrow B^0 \rightarrow e^+ + e^-, \tag{99}$$

$$e^+ + e^- \rightarrow B^0 \rightarrow \mu^+ + \mu^-, \tag{100}$$

etc. It is remarkable that such resonances could lead to large observable effects in spite of the fact that two semiweak couplings are involved in reactions like (99) and (100). The mass of B^0 must be $>M_K$ in order to avoid a semiweak decay of K. We assume a width Γ appropriate to the semiweak decay couplings of B^0 of the order of 5×10^{17} sec^{-1}. We also assume for B^0 a mass of the order of the K mass, and we suppose that the branching ratio for its decay into $e^+ + e^-$ (and similarly into $\mu^+ + \mu^-$) is about one fifth. The width $\Gamma \cong 5\times 10^{17}$ sec^{-1} corresponds to a very sharp resonance extending over a few hundreds of ev and what will be actually measured is $\bar{\sigma}_R$ defined as in (22). For $\bar{\sigma}_R$ we find a value of $2.6\times 10^{-5}(2\pi\lambda^2)$ which is about three times bigger than the cross section for $e^+ + e^- \rightarrow \mu^+ + \mu^-$ at any energy $E \gg m_\mu$.

7.2. Intermediate charged vector mesons can be produced according to the reaction

$$e^+ + e^- \rightarrow B^+ + B^- \tag{63}$$

that we have discussed in the previous section. Of course, it seems perfectly consistent in this case of semiweakly interacting mesons to deal separately with their production processes and with their decay. Experimentally, reaction (63) would still appear as a many-body reaction, like for instance

$$e^+ + e^- \rightarrow (\mu^+ + \nu) + (e^- + \bar{\nu}). \tag{101}$$

An electromagnetic process like $e^+ + e^- \rightarrow \mu^+ + \mu^- + e^+ + e^-$, which could also originate a final μ^+ and e^-, is of higher order and would have a much smaller probability than (63) followed by the successive decay of B^+ and B^- into the final particles. The decay products of B^+ and B^- would exhibit specific angular correlations as we have already discussed in the previous sections.

In the absence of structure effects for B, the expression for the cross section obtained from (78) would violate unitarity at high energy. Inclusion of a point magnetic moment or of a point electric quadrupole moment does not change this situation. For instance, if a point magnetic moment μ_B is introduced, the cross section derived from (78) increases quadratically with E, making the unitarity violation worse.

Of course, the considerations that make possible the existence of the intermediate boson B, having no strong interactions, would also apply to a possible fermion with mass bigger than the K mass, which had no strong interactions. Such a fermion would hardly have been detected, if it existed, and $e^+ + e^-$ collisions may allow one to definitely exclude its existence. The cross section for production of a fermion-antifermion pair is given by (53) in the absence of structure effects.

One can also ask about the contribution of the known local weak interactions to electron-positron processes. If, for instance, a weak lepton interaction of the type $(\mu^+\mu^-)(e^+e^-)$ exists, there could be a weak amplitude of the form

$$(2\pi)^{-2}\sqrt{8}G[\bar{u}(\mu^-)\gamma_\mu \tfrac{1}{2}(1+\gamma_5)v(\mu^+)] \times [\bar{v}(e^+)\gamma_\mu \tfrac{1}{2}(1+\gamma_5)u(e^-)], \tag{102}$$

adding coherently to the electromagnetic amplitude for $e^+ + e^- \rightarrow \mu^+ + \mu^-$. The contribution from (102) is, however, very small though increasing very rapidly with energy. The cross-section for $e^+ + e^- \rightarrow \mu^+ + \mu^-$ obtained by adding the contribution from (102) to the lowest order electromagnetic amplitude is

$$\frac{d\sigma}{d(\cos\theta)} = \frac{\pi}{8}\alpha^2\lambda^2[(1+\cos^2\theta)(1+\epsilon+\epsilon^2) + 2(\epsilon+\epsilon^2)\cos\theta], \tag{103}$$

where $\epsilon = 6.2\times 10^{-4}(E/M_N)^2$, with M_N = nucleon mass. The numerical coefficient in the expression for ϵ has been calculated by taking for G the value of the β-decay coupling constant. The appearance of the $\cos\theta$ term is entirely due in (103) to the weak interaction (102). However, a $\cos\theta$ term in the differential cross section for $e^+ + e^- \rightarrow \mu^+ + \mu^-$ would also occur from the high order electromagnetic graphs (for instance, from a diagram with two gammas exchanged). The parity-nonconserving effects of (102) would constitute a more unique test of its presence. For instance, to the differential cross section (103) would be associated a longitudinal polarization

$$P^{\pm} = \pm(\epsilon+\epsilon^2)\frac{(1+\cos\theta)^2}{(1+\cos^2\theta)+(\epsilon+\epsilon^2)(1+\cos^2\theta)} \tag{104}$$

of the final $\mu^{\pm}$. For energies $E \sim 30$ Gev, ϵ becomes of the order unity and the polarization should be quite large. For colliding beam energies of the order of 1–2 Bev, effects of local weak interactions should be negligible. On the other hand, if intermediate mesons exist they would show off in various ways and electron-positron collisions would in fact constitute a good experimental means for their detection.

8. EXPRESSION FOR THE VACUUM POLARIZATION DUE TO STRONG INTERACTING PARTICLES

The quantity

$$\Pi(k^2) = -\frac{(2\pi)^3}{3k^2}\sum_{p^{(z)}=k}\langle 0|j_\nu(0)|z\rangle\langle z|j_\nu(0)|0\rangle \tag{105}$$

is known to be of fundamental importance in quantum electrodynamics.[29] In (105), j_ν is the current operator and the sum is extended over all the physical states

[29] G. Källén, Helv. Phys. Acta **25**, 417 (1952).

with total four-momentum $p^{(z)}=k$. The Fourier transform of the photon propagator

$$D_{\mu\nu}{}^{F'}(x-x')=i\langle 0|P(A_\mu(x')A_\nu(x))|0\rangle,$$

where P is the chronological product and A_μ is the electromagnetic field, can be expressed in terms of $\Pi(k^2)$ as[29]

$$D_{\mu\nu}{}^{F'}(k)=\frac{\delta_{\mu\nu}}{k^2-i\epsilon}+\frac{k^2\delta_{\mu\nu}-k_\mu k_\nu}{k^2}\times\frac{\bar{\Pi}(0)-\bar{\Pi}(k^2)-i\pi\Pi(k^2)}{k^2-i\epsilon}. \quad (106)$$

In (106) $\bar{\Pi}(k^2)$ is defined as

$$\bar{\Pi}(k^2)=P\int_0^\infty \frac{\Pi(-a)}{k^2+a}da. \quad (107)$$

We show in this section that the experimentally measured cross sections for processes $e^+ + e^- \rightarrow \gamma \rightarrow F$, where F denotes a group of final states, is directly related to the contribution to (105) from the group of states F in the summation over the intermediate states z. This result will permit, for instance, calculation of the modifications of the photon propagator due to virtual strong interacting particles, directly from the measured cross sections.

A problem of this sort has been considered by Brown and Calogero,[30] who calculated the modifications to the photon propagator expected from intermediate two-pion states with resonant interaction. Here we shall determine the general relation between the modification to the photon propagator and the measured total cross sections for the annihilation processes.

We note that the matrix elements $\langle 0|j_\nu(0)|z\rangle$ occurring in (105) are proportional to the corresponding J_ν defined in (3). Therefore the total cross section for annihilations leading to the final states F in the center-of-mass system can be written, according to (7), as

$$\sigma_F(E)=-\frac{(2\pi)^5\alpha}{16E^4}T_{mn}\sum_{p_z=k}{}_F\,\langle 0|j_m(0)|z\rangle\langle z|j_n(0)|z\rangle. \quad (108)$$

Now we use gauge invariance to relate the sum in (108) to the analogous sum in (105). We have

$$(2\pi)^3\sum_{p_z=k}{}_F\langle 0|j_\mu(0)|z\rangle\langle z|j_\nu(0)|0\rangle=\Pi_F(k^2)(k_\mu k_\nu-k^2\delta_{\mu\nu}). \quad (109)$$

In (109) we have indicated by $\Pi_F(k^2)$ the contribution to $\Pi(k^2)$ from the group of intermediate states F. Substituting into (108) we obtain

$$\sigma_F(E)=(\pi^2\alpha/E^2)\Pi_F(-4E^2), \quad (110)$$

[30] L. M. Brown and F. Calogero, Phys. Rev. **120**, 653 (1960).

which gives the desidered connection. Note that integrals of the type

$$\int^\infty \frac{\Pi(-a)}{a^2}da, \quad (111)$$

must be convergent, as noted by Källén,[29] otherwise observable expressions would not be finite. It follows that for any group of states F, $\sigma_F(E)$ must be such that

$$\int^\infty \frac{\sigma_F(E)}{E}dE$$

converges. Such a condition is weaker than the one we derived in Sec. 6 from the unitarity requirement for the cross sections $\sigma_F(E)$. The integral

$$\bar{\Pi}(0)=P\int_0^\infty \frac{\Pi(-a)}{a}da$$

is connected to charge renormalization. If one wants it finite, $\int^\infty E\sigma_F(E)dE$ must be finite for any group of states F. If the cross sections decrease as λ^2, (111) is logarithmically divergent. Note that all the above statements about convergence only refer to the one-photon channel and they are not vigorous at all orders.

9. CONCLUSIONS

In high-energy electron-positron colliding beam experiments we see a possible field of spectacular developments for high-energy physics. Electron-positron experiments offer a unique possibility for a consistent and direct exploration of the electromagnetic properties of elementary particles. At the lowest electromagnetic order the annihilation proceeds through a virtual intermediate photon of timelike four-momentum which then disintegrates into the final products. The form factors of strongly interacting particles produced in the reaction are thus explored for negative values of the invariant four-momentum squared, k^2, inside the absorption cut in k^2 plane. The coupling to the one-photon intermediate state selects out of the incoming states a particular state with total angular momentum one, negative parity, and opposite helicity for the colliding relativistic particles. Pairs of spin-zero bosons, of positive relative parity, are produced in P state. Fermion-antifermion pairs are produced in ${}^3S_1$ and ${}^3D_1$ (or in ${}^1P_1$ and ${}^3P_1$ if the relative parity is negative). Pairs of spin-one bosons, of positive relative parity, are produced in ${}^1P_1$, ${}^5P_1$, and ${}^5F_1$. In Sec. 1 we have reported some general considerations relative to the most probable annihilations, occurring through one single photon. Radiative corrections do not substantially alter the single-photon picture as long as the experimental arrangements are symmetrical with respect to the produced charges. Annihilation into pions, $\pi^0+\gamma$, and K mesons should be the most important annihilation processes producing strongly interacting particles for not very high energies. Pion form factors can be directly explored along the absorptive cut on the k^2 plane and, as already discussed many times,[4,6,7] their values are directly related to the nature of forces among pions. A $T=1$, $J=1$ pion-pion resonance would be directly exhibited in the two-pion annihilation mode, and a $T=0$, $J=1$ three-pion bound state (or resonance) could dominate the amplitude for annihilation into three pions. Depending on the magnitude of the K^0, electromagnetic form factors for values of k^2 inside the physical region, pairs of neutral K mesons, in the combination $K_1^0+K_2^0$, could be produced. The electromagnetic form factor of the neutral pion can be explored, through the mode of annihilation into $\pi^0+\gamma$, for values of k^2 larger than one pion mass; two-pion and three-pion resonances (or bound states) may produce very large effects on the annihilation amplitude. A three-pion bound state would mostly decay into $\pi^0+\gamma$, or $2\pi+\gamma$, and give rise to a very sharp resonance, with a width presumably of a fraction of a Mev, in the $\pi^0+\gamma$ annihilation reaction. The annihilation cross section, averaged around the resonance, may possibly reach values of the order of 10^{-30} cm^2. In a theory of the π^0 electromagnetic form factor, one can tentatively assume the dominance of a two-pion resonance and a three-pion bound state, and introduce the suggested values for the π^0 lifetime and for the derivative of the form factor at the origin. Also these estimates lead to a very big annihilation cross section at the energy of the assumed bound state. From the assumed values of the derivative of the form factor near the origin one would also estimate a very big enhancement of the cross section at an energy corresponding to that of the assumed two-pion resonance. A discussion of the possible resonances is given in Sec. 4, based on general considerations of the relevant partial and total widths as compared to the experimental energy resolution. It is concluded that electron-positron collisions offer a very suitable mean for detecting intermediate neutral resonant states of total angular momentum one, negative charge conjugation quantum number and parity, and zero nucleonic number and strangeness. Other intermediate states are not expected to lead to observable effects. Annihilation into baryon-antibaryon pairs would allow exploration of the baryon form factors for the relevant negative values of k^2. Near the threshold the cross section is isotropic and rises proportionally to the final velocity. The form factors are complex in the physical region for the process and, as a consequence, the produced fermions are expected to have a polarization normal to the plane of production and proportional to the sine of the phase difference between the electric and the magnetic form factor (in contrast, for instance, to electron-nucleon scattering in which the final nucleon is unpolarized, excluding radiative correction terms). There is at present no information available on the form factors for the large negative values of k^2 of the experiment. If one assumes, quite arbitrarily, that the recently found core terms in the nucleon structure originate from contributions in the absorptive region above the nucleon-antinucleon threshold, one can then roughly expect cross sections for annihilation into nucleon plus antinucleon well above the perturbation theory estimates. The $\Sigma-\Lambda$ electromagnetic vertex is measured in annihilation into $\bar{\Sigma}+\Lambda$ and the processes show a strong dependence on the relative $\Sigma-\Lambda$ parity. Vector mesons have been suggested recently and shown formally to be connected to local conservation laws.[27] Pair production of spin-one mesons is discussed in Sec. 6, on the assumption that their lifetime is sufficiently long to allow a separation of the over-all process into a first stage of production of the vector mesons and a second stage in which they decay. Three form factors are needed to specify the electromagnetic interaction of a vector boson, corresponding to its charge, magnetic moment, and electric quadrupole moment. The perturbation theory cross section for annihilation into a pair of spin-one bosons increases to a value of the order (m_B in Bev)$^{-2}$(2.1×10^{-32} cm^2) at energies much larger than the boson mass m_B. The perturbation theory increase is certainly not valid at very high energies because it would lead to a direct violation of unitarity. For electron-positron annihilation through the one-photon channel, one can strictly state the unitarity limitation in the form of an upper limit to the reaction cross section, that must decrease not slower than $\lambdabar^2$. In Sec. 6 we also discuss the angular correlations that would be observed at the decay of vector bosons from electron-positron annihilations into their final products.

Vector bosons have also been suggested as intermediary agents of weak interactions.[28] Their production in pairs in electron-positron annihilation would be a convenient test for their existence. Neutral intermediary vector bosons can only be coupled to neutral lepton pairs provided they do not couple to the weak strangeness-nonconserving currents. If they existed and were coupled to leptons they would produce an evident resonance-like behavior in annihilation reactions. Particular effects, such as those arising from parity nonconservation, would most directly inform on the presence of weak interactions in a high-energy annihilation process. However, for a local weak interactions, such effects become large only at colliding beam energies greater than 10 Gev.

Quantum electrodynamics vacuum polarization is known to be affected by strong interactions. The effect is insignificant at the lower energies but its analysis is important for an examination of an eventual high-energy breakdown of the theory. In the last section of this paper we give the explicit relation between the strong interaction corrections to vacuum polarization (or, equivalently, modification of the photon propagator) and the cross section for electron-positron annihilation into strongly interacting particles.

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Quantum Electrodynamics with Dirac Monopoles.

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1. – In a classical paper [1] Dirac has shown that quantum mechanics allows the existence of particles (monopoles) bearing a magnetic charge. The strength of the magnetic charge is not arbitrary: if monopoles must coexist with electrons, the allowed values are [2]

$$G = 2\pi n/e\,, \qquad (n \text{ integer}). \tag{1}$$

If different kinds of charged particles exist, eq. (1) must still be satisfied if we substitute their charge for the electron charge (possibly with different values of n). This means that the existence of monopoles would explain the empirical fact that the charges of elementary particles are all multiples of the electron charge e.

In this paper we discuss the extension of quantum electrodynamics to the case in which both fields with electric charge and monopole fields are present.

Previous theoretical treatments [3] made use of the usual representation of the e.m. field in terms of a vector potential A_μ:

$$F_{\mu\nu}(x) = \partial_\mu A_\nu(x) - \partial_\nu A_\mu(x)\,. \tag{2}$$

The field produced by a magnetic point charge can be described in this way only if A_μ is allowed to be singular along an arbitrary line (string) starting from the pole and going to infinity.

This is clearly an unphysical feature, since the singularity in A_μ does not correspond to a singularity in the e.m. field $F_{\mu\nu}$.

(1) P. A. M. Dirac: *Proc. Roy. Soc.*, A **133**, 60 (1931); *Phys. Rev.*, **74**, 817 (1948).
(2) W use rationalized units with $\hbar = c = 1$, and a metric with $p^2 = |\boldsymbol{p}|^2 - p_0^2$.
(3) An extensive bibliography on the subject can be found in the paper by Bradner and Isbell and in the paper by Amaldi *et al.*, see footnote (5).

As we show in the following section, a non pathological description of the e.m. field produced by a given distribution of electric and magnetic sources can be obtained in terms of two vector potentials. The introduction of a second potential is compensated by an enlargement of the group of gauge transformations.

In the next two sections we build a quantized theory for the interactions of monopoles and charged particles, with the e.m. field without making use of potentials. This theory is an extension of the treatment recently given by S. MANDELSTAM (4) for the ordinary electrodynamics. Monopoles and charged particles are treated in a symmetrical way: the internal consistency of the theory requires the Dirac condition (eq. (1)).

In the last section we give a brief discussion of the symmetry properties of the theory. We show that, although parity is not conserved, parity non conservation effects can only appear if physical monopoles are present. The existence of monopoles (5) is therefore not contraddicted by the conservation of parity in ordinary electromagnetic processes, in which monopoles might take part as virtual particles.

2. – The Maxwell equations in vacuo can be written as (6):

$$\partial_\nu F_{\mu\nu}(x) = j_\mu(x)\,, \tag{3}$$

$$\partial_\nu \tilde{F}_{\mu\nu}(x) = 0\,. \tag{4}$$

If sources of the magnetic field are allowed, eq. (4) should be substituted by:

$$\partial_\nu \tilde{F}_{\mu\nu}(x) = g_\mu(x)\,, \tag{4'}$$

where the four-vector g_μ represents the magnetic current and the density of magnetic charge. Equations (3) and (4′) can be solved by means of two vector potentials, instead of one:

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + \varepsilon^{\mu\nu\varrho\sigma}\partial_\varrho B_\sigma\,, \tag{5}$$

A_μ and B_μ are determined by $F_{\mu\nu}$ up to a group of gauge transformations; this contains individual gauge transformations, like

$$\begin{cases} A_\mu \to A_\mu + \partial_\mu \Lambda\,, \\ B_\mu \to B_\mu + \partial_\mu \Gamma\,, \end{cases} \tag{6}$$

as well as mixing transformations

$$\begin{cases} A_\mu \to A_\mu + A'_\mu\,, \\ B_\mu \to B_\mu + B'_\mu\,, \end{cases} \tag{7}$$

---

(4) S. MANDELSTAM: *Quantum Electrodynamics without Potentials*, preprint.

(5) Different experiments made up to now seem to exclude the existence of monopoles of mass $M \leqslant 2.5$ GeV: H. BRADNER and W. M. ISBELL: *Phys. Rev.*, **114**, 603 (1959); M. FIDECARO, G. FINOCCHIARO and G. GIACOMELLI: *Nuovo Cimento*, **22**, 657 (1961); E. AMALDI, G. BARONI, H. BRADNER, H. G. DE CARVALHO, L. HOFFMANN, A. MANFREDINI and G. VANDERHAEGHE: *1961 Conference on Elementary Particles at Aix-en-Provence*, vol. 1 (Saclay, 1962), p. 155. CERN Report, to be published.

(6) The symmetry between the electric field and the magnetic field is expressed by the duality operation $\tilde{F}_{\mu\nu} = -\frac{1}{2}\varepsilon^{\mu\nu\varrho\sigma}F_{\varrho\sigma}$ ($\varepsilon^{\mu\nu\varrho\sigma}$ the completely antisymmetric Ricci tensor, $\varepsilon^{1234} = i$). This operation is equivalent to the substitutions $\boldsymbol{E} \to \boldsymbol{H}$, $\boldsymbol{H} \to -\boldsymbol{E}$. Note that $\tilde{\tilde{F}}_{\mu\nu} = -F_{\mu\nu}$.

A' and B' shall satisfy the « zero field conditions »

$$\partial_\mu A'_\nu - \partial_\nu A'_\mu - \varepsilon^{\mu\nu\varrho\sigma}\partial_\varrho B'_\sigma = 0 \,. \tag{7'}$$

Note that transformations (6) are particular cases of (7).

We can use (6) to impose Lorentz conditions on A_μ and B_μ so that they will satisfy the following set of equations:

$$\begin{cases} \partial_\mu A_\mu = \partial_\mu B_\mu = 0 \,, \\ \Box A_\mu = j_\mu \,, \\ \Box B_\mu = g_\mu \,, \end{cases} \tag{8}$$

gauge transformations of the kind (6), with $\Box A = \Box \Gamma = 0$, or (7), with $\partial_\mu A'_\mu = \partial_\mu B'_\mu = 0$ are still allowed in the Lorentz gauge.

In the absence of sources ($g_\mu = j_\mu = 0$) we can adopt the usual gauge in which $B_\mu = 0$, but we could equally well adopt a gauge in which $A_\mu = 0$, or a general one as in eq. (5). The introduction of a second potential does not, due to the mixing transformations, cause an increase of the number of the independent variables which describe a free field. If we analise the free field in terms of photons we shall still have only two photons for each value of the linear momentum. The wave function of a given photon will however depend on the gauge adopted.

Any theory based on the general description (5) for the e.m. field, should be invariant under the whole of gauge transformations (6) and (7).

We note that if only monopoles and no charged particles were present one could adopt a description in terms of B_μ only. The resulting theory will be similar to ordinary electrodynamics, the only difference being in the higher value of the coupling constant (the minimum value allowed by eq. (1) is $g^2/4\pi = 34.25$). This treatment could be adequate for some problems like the annihilation of a monopole-anti-monopole pair into photons.

3. – MANDELSTAM has recently given a treatment of quantum electrodynamics in which no use is made of potentials [4]. We extend this approach to the case in which both charged particles and monopoles are in interaction with the electromagnetic field. The Dirac condition (eq. (1)) for the electric and magnetic charges is necessary for the consistency of the theory; the theory is Lorentz-invariant and symmetrical between charges and monopoles.

To proceed by steps, we shall first consider the case of a charged scalar field, $\varphi(x)$, representing particles of electric charge e, in interaction with the electromagnetic field. Suppose, for the moment, that no magnetic sources exist, so that eqs. (3) and (4) hold, and the e.m. field can be described by a vector potential $A_\mu(x)$ (eq. (2)). Following MANDELSTAM, we introduce a new field quantity

$$\Phi(x; P) = \varphi(x) \exp\left[-ie\int_{(P)}^{x} A_\mu(\xi)\,\mathrm{d}\xi_\mu\right]. \tag{9}$$

The integral is evaluated on a spacelike path P ending at the point x. The new quantity $\Phi(x; P)$ does not depend on the gauge selected for A_μ but depends on the path P.

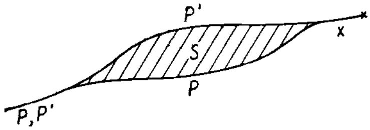

Fig. 1.

If we change the path from P to P' as in Fig. 1, Φ changes; from eq. (9):

$$(9') \qquad \Phi(x, P') = \Phi(x, P) \exp\left[- ie \oint A_\mu \, \mathrm{d}\xi_\mu\right],$$

the integral is now evaluated on the closed path P'—P; using the relativistic generalization of Stokes theorem:

$$(10) \qquad \Phi(x, P') = \Phi(x, P) \exp\left[- \frac{ie}{2} \int_S F_{\mu\nu} \, \mathrm{d}\sigma_{\mu\nu}\right],$$

where S is a surface delimited by the path P'-P. Derivatives of $\Phi(x, P)$ correspond to the « gauge invariant derivatives » of $\varphi(x)$

$$(11) \qquad \partial_\mu \Phi(x; P) = [(\partial_\mu - ieA_\mu)\varphi(x)] \exp\left[- ie \int_{(P)}^{x} A_\mu \, \mathrm{d}\xi_\mu\right].$$

The derivatives of $\Phi(x, P)$ do not commute:

$$(12) \qquad [\partial_\mu, \partial_\nu] \Phi(x, P) = \Phi(x, P)[- ie F_{\mu\nu}(x)] .$$

At this point we can forget eq. (9) and consider $\Phi(x, P)$ as defined by its path-dependence, which can be expressed directly in terms of the e.m. field $F_{\mu\nu}$ (eq. (10)) (7). Equation (10) can be used in the general case in which magnetic sources are present and the e.m. field cannot be described (eq. (2)) in terms of a single potential. Its consistence requires however that the change in Φ does not depend on the particular choice of the 2-dimensional surface S. If S_1 and S_2 are two such surfaces:

$$\Phi(x, P) \exp\left[- \frac{ie}{2} \int_{S_1} F_{\mu\nu} \, \mathrm{d}\sigma_{\mu\nu}\right] = \Phi(x, P) \exp\left[- \frac{ie}{2} \int_{S_2} F_{\mu\nu} \, \mathrm{d}\sigma_{\mu\nu}\right],$$

(7) Equation (12) can be derived directly from eq. (10), as shown in (4).

so that for the closed 2-dimensional surface $S = S_1 - S_2$ (8):

$$\exp\left[-\frac{ie}{2}\int_S F_{\mu\nu}\,\mathrm{d}\sigma_{\mu\nu}\right] = 1\,.$$

We can change the surface integral to an integral over the volume V enclosed by S and, using eq. (4′), obtain:

$$\exp\left[-ie\int_V g_\mu\,\mathrm{d}V_\mu\right] = 1\,. \tag{13}$$

Equation (13) should hold for any volume V. Apart from the trivial case $g_\mu = 0$ considered by MANDELSTAM, other solutions exist, which correspond to the existence of Dirac monopoles:

(i) If g_μ is a classical (c number) source, eq. (13), requires that

$$Q_V = \int_V g_\mu\,\mathrm{d}V_\mu = \frac{2\pi n}{e}\,, \tag{14}$$

since V is completely arbitrary, eq. (14) can only be satisfied if g_μ is due to one or more pointlike sources, each with a magnetic charge multiple of $g = 2\pi/e$.

(ii) If g_μ is a quantum operator, eq. (13) is satisfied in operator form if all the eigenvalues of Q_V are multiple of g. This is true if g_μ represents the current of one or more quantized fields, each of them bearing a magnetic charge which is a multiple of g. These fields would then be associated with monopoles.

The Mandelstam scheme for the interaction of a charged field with the e.m. field can therefore be extended to the case in which monopoles exist, as long as their magnetic charges satisfy the Dirac condition (eq. (1)).

The extension of the scheme to the monopole fields is straightforward: a scalar monopole of magnetic charge g will be described by a path-dependent field quantity $\Psi(x, P)$. The path-dependence will be assumed to be given by (9):

$$\Psi(x, P') = \Psi(x, P)\exp\left[-i\frac{g}{2}\int_S \tilde{F}_{\mu\nu}\,\mathrm{d}\sigma_{\mu\nu}\right]. \tag{15}$$

(8) The ordering of non commuting operators and the algebraic manipulations need some justification when the various quantities are quantized. In particular, it is sufficient to choose a particular ordering criterion and to follow it throughout the mathematical developments. In order to avoid the problems of commutativity between quantities calculated at points with a non-spacelike separation, one can restrict oneself to variations of the path P on a spacelike 3-dimensional surface Σ which contains x (*e.g.*, t = const) and to the choice of the 2-dimensional surface S lying on Σ. In this case we have only to deal with spacelike paths, surfaces, volumes. [It must be remarked, however, that eqs. (10) to (17) are not restricted to this case, but are valid in general. In this case all the commutation relations (20) to (25) for which the commutator is vanishing can be generalized to the form *e.g.* $[\Phi(x, P), \Phi(y, P')] = 0$ where it is understood that both x, y as well as the paths P, P' lie on a spacelike surface. For P, P' not satisfying this condition the path-dependence laws (10) and (15) have to be used explicitly.

(9) In a theory in which only monopoles are present we can describe the e.m. field $F_{\mu\nu}$ in terms of a potential B_μ (see Section 2). In this case eq. (15) can be derived from a definition of $\psi(x, P)$ similar to eq. (9):

$$\Psi(x, P) = \psi(x)\exp\left[-ig\int_{(P)}^{x}\mathrm{d}\xi_\mu B_\mu(\xi)\right].$$

The derivatives will obey the commutation relations:

$$[\partial_\mu, \partial_\nu]\,\Psi(x, P) = \Psi(x, P)\left[-i\,\frac{g}{2}\,\tilde{F}_{\mu\nu}(x)\right]. \tag{16}$$

In parallel to the case of a field with electric charge, the consistence of eq. (15) requires some constraint on the *electric* current j_μ:

$$\exp\left[-ig\int_V j_\mu\,\mathrm{d}V_\mu\right] = 1. \qquad (\text{any } V): \tag{17}$$

This condition can be satisfied if the electric charge is quantized.

4. – The scheme introduced in the last section for the description of charges and monopoles in interaction with the e.m. field does not contain pathological elements, like the string singularities. The path-dependence of the field variables is due to the fact that the space, in presence of an e.m. field, appears to a charged particle as curved (4).

In this section we will complete the scheme by postulating a set of equations of motion and commutation relations. We will proceed in three steps, considering cases in which: (i) only charged particles are present; (ii) only monopoles are present; (iii) both charges and monopoles are present. We note that the use of Lagrangians should be considered here only as an heuristic procedure. The problem of thed erivation of the equations given here from an action principle will be treated in a forthcoming paper.

For case (i) we follow the procedure given by MANDELSTAM (4): from a Lagrangian (m is the mass of the charged particle)

$$\mathscr{L} = -(\partial_\mu \Phi^*)(\partial_\mu \Phi) - m^2\Phi^*\Phi - \tfrac{1}{4}F_{\mu\nu}F_{\mu\nu}, \tag{18}$$

we get the following equations of motion

$$\Box\Phi - m^2\Phi = 0, \qquad \partial_\nu F_{\mu\nu} = j_\mu = -ie[\Phi^*(\partial_\mu\Phi) - (\partial\Phi^*)\Phi]. \tag{19}$$

This set should be completed by eq. (4). As we have seen, eq. (4) cannot be considered as a necessary constraint (as Mandelstam does). We can nevertheless justify eq. (4) since we have shown that the only admissible inhomogeneous terms g_μ in eq. (4′) represent Dirac monopoles. From the Lagrangian (18) one can derive the following commutation relations (4): (for equal times)

$$\begin{cases} [\Phi(x, P), \Phi(y, P)] = [\Phi(x, P), \Phi^*(y, P)] = 0, \\ [\dot{\Phi}(x, P), \dot{\Phi}(y, P)] = [\dot{\Phi}(x, P), \dot{\Phi}^*(y, P)] = 0, \\ [\Phi(x, P), \dot{\Phi}(y, P)] = [\Phi^*(x, P), \dot{\Phi}^*(y, P)] = 0. \end{cases} \tag{20}$$

$$
(20')\qquad
\begin{cases}
[\dot{\Phi}^*(x, P),\ \Phi(y, P)] = [\dot{\Phi}(x, P),\ \Phi^*(y, P)] = -i\,\delta^3(x-y)\,, \\[1ex]
[\overset{(\cdot)}{\Phi}(x, P),\ F_{ij}(y)] = [\overset{(\cdot)}{\Phi}{}^*(x, P),\ F_{ij}(y)] = 0\,, \\[1ex]
[\overset{(\cdot)}{\Phi}(x, P),\ F_{0i}(y)] = -e\displaystyle\int\limits_{(P)}^{x} \mathrm{d}\xi_i\,\delta^3(y-\xi)\,\overset{(\cdot)}{\Phi}(x, P)\,, \\[1ex]
[\overset{(\cdot)}{\Phi}{}^*(x, P),\ F_{0i}(y)] = e\displaystyle\int\limits_{(P)}^{x} \mathrm{d}\xi_i\,\delta^3(y-\xi)\,\overset{(\cdot)}{\Phi}{}^*(x, P)\,.
\end{cases}
$$

A dot denotes differentiation in respect to t; when the dot is enclosed in brackets, as in eqs. (20′), the equation holds whether or not it is present.

The commutation relations for the components of F are the same as in the free field case:

$$
(21)\qquad
\begin{cases}
[F_{ij}(x),\ F_{i'j'}(y)] = [F_{0i}(x),\ F_{0i'}(y)] = 0\,, \\[1ex]
[F_{0i}(x),\ F_{jk}(y)] = -i\left\{\delta_{ij}\dfrac{\partial}{\partial y_k} - \delta_{ik}\dfrac{\partial}{\partial y_j}\right\}\delta^3(x-y)\,.
\end{cases}
$$

Case (ii) is related to case (i) by the duality operation ([6]), so that in this case the equations of motion will be (μ is the mass of the monopole)

$$
(22)\qquad \Box\Psi + \mu^2\Psi = 0\,, \qquad \partial_\nu \widetilde{F}_{\mu\nu} = g_\mu = ig[\Psi^*(\partial_\mu\Psi) - (\partial_\mu\Psi^*)\Psi]\,,
$$

plus eq. (3) with $j_\mu = 0$. In analogy with eqs. (20′) we will have:

$$
(23)\qquad
\begin{cases}
[\overset{(\cdot)}{\Psi}(x, P),\ \widetilde{F}_{ij}(y)] = [\overset{(\cdot)}{\Psi}{}^*(x, P),\ \widetilde{F}_{ij}(y)] = 0\,, \\[1ex]
[\overset{(\cdot)}{\Psi}(x, P),\ \widetilde{F}_{0i}(y)] = -g\displaystyle\int\limits_{(P)}^{x} \mathrm{d}\xi_i\,\delta^3(y-\xi)\,\overset{(\cdot)}{\Psi}(x, P)\,, \\[1ex]
[\overset{(\cdot)}{\Psi}{}^*(x, P),\ \widetilde{F}_{0i}(y)] = g\displaystyle\int\limits_{(P)}^{x} \mathrm{d}\xi_i\,\delta^3(y-\xi)\,\overset{(\cdot)}{\Psi}{}^*(x, P)\,.
\end{cases}
$$

The commutation relations among Ψ and Ψ^* can be obtained by eq. (20) substituting Ψ for Φ, Ψ^* for Φ^*.

The commutation relations among different components of $F_{\mu\nu}$ will still be given by eq. (21), since these are easily seen to be invariant under the duality operation.

We come now to the general case (iii) in which we have a field Φ bearing an electric charge e and a monopole field Ψ with magnetic charge g. In this case we see that a complete and coherent scheme is given by the path-dependence of the two fields, specified by eqs. (10) and (15), together with the equations of motion (19) and (22), and the commutation relations (20), (20′), (21) and (23).

It is easily seen that the consistence conditions expressed by eqs. (13) and (16) are automatically satisfied if (and only if) the charges e and g satisfy the Dirac condition (eq. (1)).

We will also assume that for equal times [10].

$$[\Psi(x, P),\ \Phi(y, P')] = 0 . \tag{25}$$

5. – In the theory presented here the parity is not conserved; in fact if $F_{\mu\nu}$ is a tensor, $\partial_\nu \tilde{F}_{\mu\nu}$ is an axial vector, while g_μ is a vector. This is not surprising, since monopoles violate parity also in a classical theory: for instance in a magnetic field a monopole accelerates in the direction of the magnetic field. Also C, the conjugation of the electric charge, is not conserved. We find however new symmetries by combining the usual operations P and C with the reflection of the magnetic charge [11], M. Both $C' = CM$ and $P' = PM$ are conserved. In processes in which monopoles are not present as physical particles, P' and C' are equivalent to the usual operations P and C, and no parity (or C) violation is expected. The existence of monopoles is therefore not contraddicted by the observed parity conservation (and invariance under charge conjugation) in ordinary electromagnetic processes.

* * *

We are indebted to Prof. E. AMALDI who stimulated our interest in this subject, and to Professors R. GATTO, V. GLASER and L. VAN HOVE for discussions.

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(10) This commutation relation is not changed by displacing any of the two paths on a space like surface [see footnote (8)] only if the Dirac condition (1) is satisfied.

(11) N. F. RAMSEY: *Phys. Rev.*, **109**, 225 (1958).

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Proton-Antiproton Annihilation into Electrons, Muons and Vector Bosons.

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Summary. — The possibility of achieving relatively high intensity antiproton beams has prompted some considerations on the rather rare annihilation channels of the proton-antiproton system. We propose i) to study the two-electron mode as a means of investigating the electromagnetic structure of the proton for time like momentum transfers; ii) to study the two-muon mode and compare with the two-electron mode to investigate whether the muon behaves like a heavy electron for large time like momentum transfers; iii) to investigate the existence of weak vector bosons by the modes $p+\bar{p}\rightarrow B+\bar{B}$ and $p+\bar{p}\rightarrow B+\pi$. Although no precise theoretical predictions can be made, crude estimates indicate that the cross-section for these four channels could be roughly of the same order of magnitude.

1. – The electromagnetic annihilation $p+\bar{p}\rightarrow e^{+}+e^{-}$, $p+\bar{p}\rightarrow\mu^{-}+\mu^{+}$.

One of the significant programmes in high-energy physics has been the systematic study of the electromagnetic structure of nucleons carried out by HOFSTADTER (1) and co-workers, and by WILSON (2) and co-workers. The theo-

(*) Now at Stanford Linear Accelerator Center, Stanford, Cal.

(1) For example: R. HOFSTADTER and R. HERMAN: *Phys. Rev. Lett.*, **6**, 293 (1961).

(2) R. M. LITTAUER, H. F. SCHOPPER and R. R. WILSON: *Phys. Rev. Lett.*, **7**, 141 (1961).

retical explanation of these experiments has been one of the outstanding problems in the theory of strong interactions and has led to many new and interesting ideas (³). These experiments measure the form factors of the nucleon for spacelike momentum transfers where the form factors are real and apparently decreasing with increasing momentum transfers up to highest values thus far measured of order $q^2 \approx 2(M)^2$ (M = nucleon mass).

The advent of antiproton beams of relatively high intensity ($\approx 10^4$ particles per pulse) allows the possibility of further investigation of the electromagnetic structure of the proton in a region thus far completely unexplored. This is accomplished by the study of the reaction

$$\text{(1)} \qquad \mathrm{p} + \bar{\mathrm{p}} \to \mathrm{e}^- + \mathrm{e}^+ .$$

Reaction (1) is the inverse of proton-antiproton pair production from electron-positron clashing beams (⁴).

Figures 1-*a*) and 1-*b*) show the diagrams for proton-electron scattering and proton-antiproton annihilation into an electron pair, respectively, in the one-photon channel.

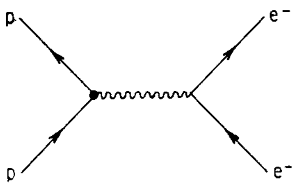

Fig. 1-*a*.

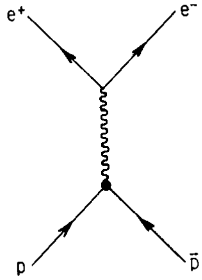

Fig. 1-*b*.

For the proton-electron scattering experiment the four-momentum carried by the photon is purely spacelike, *i.e.* $q^2 > 0$, whereas in the annihilation the photon four-momentum is purely timelike, $q^2 < -4M^2$. This is clearly demonstrated in the c.m. of target and projectile, in which case the four-momentum transfer has only space components for the scattering experiment and only a time component for the annihilation.

The momentum transfer for process (1) is determined uniquely by the antiproton energy $\mathscr{E}$ in the laboratory system, $q^2 = -2M(\mathscr{E} + M)$. Beginning at $q^2 = -4M^2$, when the antiproton is at rest, the momentum transfer continues to as negative a value of q^2 as can be achieved with the highest possible antiproton energy.

(³) S. D. Drell and F. Zachariasen: *Electromagnetic Structure of Nucleons* (Oxford, 1961).

(⁴) N. Cabibbo and R. Gatto: *Phys. Rev.*, **124**, 1577 (1961).

At the present time there exist no reliable theories for the behaviour of the form factors for timelike momentum transfers. Nevertheless we would like to propose the study of reaction (1). This programme will allow the investigation (just as in the spacelike experiments) of whether the proton has a corelike structure for large momentum transfers, or whether it has a broad and complex structure.

Whereas in the spacelike experiments the form factors are given the physical interpretation of the Fourier transforms of the spacial charge and magnetic structure of the proton, the timelike momentum transfers yield information about the frequency structure of the proton. For $q^2 < 0$ the « cloud » around the proton could have various kinds of resonance structure such as the ρ and ω^0 mesons. It would be of great interest to explore this region to see if this kind of structure is simple, *i.e.* one or two resonances with a more or less constant continuum, or whether more structure appears as the momentum transfer continues to larger negative values.

It appears that with existing machines such as the P.S. at CERN an antiproton beam of 3 GeV/c can be readily achieved. With an antiproton beam of this momentum it is possible to look at momentum transfers as negative as $(-8.7\,M^2)$ which is much larger in absolute value than presently possible in the spacelike experiments. An experimental investigation in these directions is being undertaken at CERN (5).

There is also the process

$$\bar{\mathrm{p}} + \mathrm{p} \to \mu^- + \mu^+ \tag{2}$$

which occurs with the same differential cross-section, in the one-photon channel, as the electron pair, providing we neglect terms of order $(M_e/M)^2$, $(M_\mu/M)^2$ and treat the muon simply as a heavy electron. An accurate measurement of the ratio of muon pairs to electron pairs would give information on either the muon or electrodynamics in a region which has never been explored by any kind of muon experiment.

The general form for the matrix element of *one* photon interacting with a proton and an antiproton is written in the usual manner as

$$\bar{u}_{\bar{\mathrm{p}}}\left[F_1\gamma_\mu + \frac{F_2}{2M}\sigma_{\mu\nu}q_\nu\right]u_{\mathrm{p}}\,,$$

where the form factors are functions of the momentum transfer q^2, and where $\sigma_{\mu\nu}=(\gamma_\mu\gamma_\nu-\gamma_\nu\gamma_\mu)/2$; $F_1(0)=e$; $F_2(0)=e\mu_{\mathrm{p}}$; and $q=P_{\bar{\mathrm{p}}}+P_{\mathrm{p}}$ [P = four-momen-

(5) M. Conversi, L. di Lella, F. J. M. Farley, Th. Muller and A. Zichichi.

tum]. In the timelike region both F_1 and F_2 can become complex, whereas they are real for spacelike momentum transfers. With the above expression for the $p\bar{p}$ matrix element the differential cross-section for the two-electron annihilation channel can be written in the one-photon exchange approximation in the following forms:

a) Center of mass system

$$\frac{d\sigma(p\bar{p}\to ee)}{d(\cos\theta_c)} = \frac{\pi}{8}\frac{\alpha^2}{EP}\left[|F_1+F_2|^2(1+\cos^2\theta_c) + \left|\frac{M}{E}F_1+\frac{E}{M}F_2\right|^2\sin^2\theta_c\right], \tag{3}$$

where E = c.m. energy of $\bar{p}$,

P = c.m. momentum of $\bar{p}$,

and θ_c = angle between e^- and $\bar{p}$ in c.m.

b) Laboratory system

$$\frac{d\sigma(p\bar{p}\to ee)}{d\Omega} = \left(\frac{\alpha^2}{\mathscr{E}(\mathscr{E}+M)}\right)\left(\frac{1}{4}\right)\left(\frac{\mathscr{E}}{\mathscr{P}}\right)\frac{\mathscr{E}_e^2}{M^2}\cdot \tag{4}$$

$$\cdot\left[2|F_1+F_2|^2+\operatorname{cotg}^2(\theta/2)\left\{|F_2|^2\mathscr{P}^2/\mathscr{E}^2-\frac{2M}{(\mathscr{E}+M)}(|F_1|^2-|F_2|^2)\right\}\right],$$

$$\frac{d\sigma(p\bar{p}\to ee)}{d\Omega_0} = \left(\frac{\alpha^2}{4M\mathscr{E}}\right)\left(\frac{\mathscr{E}_e}{\mathscr{E}+M-\mathscr{P}\cos\theta_0}\right)\cdot \tag{5}$$

$$\cdot\left[2|F_1+F_2|^2-\left(\frac{\mathscr{P}}{\mathscr{E}+M}\right)^2 g(\theta_0)\left\{|F_1|^2-|F_2|^2-\left(\frac{\mathscr{E}-M}{2M}\right)|F_2|^2\right\}\right],$$

where $\mathscr{E}$ = antiproton laboratory energy, $\mathscr{P}$ = antiproton laboratory momentum,

and $\mathscr{E}_e$ = electron energy $= \dfrac{M}{1-(\mathscr{P}\cos\theta_0)/(\mathscr{E}+M)} =$

$$= \frac{\mathscr{E}+M}{2}\left[1\pm\sqrt{1-\frac{4M}{(\mathscr{E}+M)(1-\cos\theta)}}\right],$$

$$g(\theta_0) = \frac{2M(\mathscr{E}+M)\sin^2\theta_0}{[\mathscr{E}+M-\mathscr{P}\cos\theta_0]^2}.$$

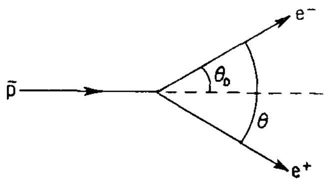

Fig. 2.

The angles θ_0 and θ are shown in Fig. 2.

c) Total cross-section

$$\sigma_T(p\bar{p}\to ee) = \left(\frac{\pi\alpha^2}{2M^2}\right)\left(\frac{\mathscr{E}+M}{\mathscr{E}-M}\right)^{\frac{1}{2}}\cdot \tag{6}$$

$$\cdot\left[\left(\frac{2M}{\mathscr{E}+M}\right)|F_1+F_2|^2+\frac{1}{3}\left(\frac{\mathscr{E}-M}{\mathscr{E}+M}\right)\left\{|F_2|^2-\left(\frac{2M}{\mathscr{E}+M}\right)|F_1|^2\right\}\right],$$

where $\mathscr{E}$ = antiproton laboratory energy.

In the above expression for the cross-section terms of the order $(M_e/M)^2 \approx \approx 2\cdot 10^{-7}$ have been neglected. For the muon pair channel we give the exact expression (not neglecting the muon mass) which in the c.m. system takes the form (6)

$$\text{(7)} \qquad \frac{d\sigma(p\bar{p}\to\mu\mu)}{d(\cos\theta)} = \frac{\pi\alpha^2}{8EP}\beta_\mu\left[|F_1+F_2|^2(2-\beta_\mu^2\sin^2\theta_c)+ \right.$$
$$\left. + \left|\frac{M}{E}F_1+\frac{E}{M}F_2\right|^2(1-\beta_\mu^2\cos^2\theta_c)\right],$$

where E and P are the c.m. energy and momentum of the antiproton and β_μ is the velocity of the muon in the c.m. system [6]. For $\beta_\mu = 1$ (7) reduces to (3).

The total cross-section from (7) is

$$\sigma_T(p\bar{p}\to\mu\mu) = \tfrac{1}{2}\beta_\mu(3-\beta_\mu^2)\,\sigma_T(p\bar{p}\to ee)\,.$$

We see from this equation that

$$\text{(8)} \qquad \frac{\sigma_T(p\bar{p}\to\mu\mu)}{\sigma_T(p\bar{p}\to ee)} = 1-\left(\frac{3}{8}\right)\left(\frac{M_\mu}{E}\right)^4+0\left[\left(\frac{M_\mu}{E}\right)^6\right],$$

and that no terms of order M_μ^2 appear in the total cross-section. We have neglected in eqs. (7) and (8) the radiative corrections which could be appreciable in this case because of the large momentum transfers involved.

By plotting the differential cross-section as a function of $\cot^2\theta/2$ we see by eq. (4) that one does not determine the complex form factors F_1 and F_2 separately but only the combinations

$$2|F_1+F_2|^2 \qquad \text{and} \qquad |F_2|^2\frac{\mathscr{P}^2}{\mathscr{E}^2}-\frac{2M}{\mathscr{E}+M}(|F_1|^2-|F_2|^2)\,.$$

The fact that the form factors are complex introduces an azimuthal dependence in the differential cross-section for polarized proton target or for polarized antiproton beam. If $\boldsymbol{p}$ is the polarization vector of the proton for polarized target, or antiproton for polarized beam, and $\boldsymbol{n}$ a unit vector in the direction $\boldsymbol{p}\times\boldsymbol{e}^-$ the differential cross-section takes the form in the c.m. system

$$\text{(9)} \qquad \frac{d\sigma}{d(\cos\theta_c)} = \left[\frac{d\sigma}{d(\cos\theta_c)}\right]_{\text{unpol}} \pm \frac{E}{M}\left(\frac{P}{E}\right)^2 I^m(F_2^*F_2)|\sin 2\theta_c|(\boldsymbol{pn})\,,$$

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[6] For a point proton with an anomalous magnetic moment eqs. (3) and (7) reduce to the cross-sections given by L. M. Brown and M. Peshkin: *Phys. Rev.*, **103**, 756 (1956).

where the upper sign is for polarized antiprotons and the lower sign for polarized target protons.

A numerical estimate of the cross-section depends very sensitively on the values of the form factors F_1 and F_2. Since there exists no reliable theory of these quantities in the timelike region, we can only give a very rough idea of what the cross-section might be. For example, we might choose the value

i) point proton

$$F_1 = e\,, \qquad F_2 = 1.79\,e\,;$$

ii) extrapolation of resonance fits of spacelike experiments to timelike region (7)

$$F_1 = \left(1 - \frac{1.18q^2}{q^2 + 30m_\pi^2}\right)e\,, \qquad F_2 = 1.79\left(1 - \frac{1.59q^2}{q^2 + 30m_\pi^2}\right)e\,.$$

In these examples we have assumed F_1 and F_2 real. Since the peak of the pion resonance fits to the spacelike form factors occurs far from the region of interest in this experiment, the imaginary parts in choice ii) give very small contributions. On the other hand, it is not known whether there are other resonances for larger timelike momentum transfers than the two-pion resonance, say, near $q^2 = -6M^2$. Should this be the case, there could be very large contributions to the cross-section from both the real and the imaginary parts of the form factors.

If the form factors decrease fairly rapidly in the timelike region, then, just as in the spacelike region, it is possible that the two-photon exchange might become important. However, if the form factors do not decrease rapidly for timelike momentum transfer, then the one-photon exchange would be dominant.

If the electron and the positron are detected in a manner which does not distinguish charge and which is symmetric under the interchange of positron and electron, then the interference term between the one and the two-photon channel will not contribute to the differential cross-section (8). This symmetry between e^+ and e^- can then be used either to eliminate or detect the influence of the two-photon exchange on the nucleon electromagnetic structure.

Figure 3 shows how the total cross-section varies with antiproton energy for the above two assumptions for the form factors.

We emphasize that this graph is not a theoretical prediction but a very

(7) S. Fubini: *Proceedings of the Aix-en-Provence Intern. Conf. on Elementary Particles* (September, 1961).

(8) S. D. Drell: *Ann. Phys.*, **4**, 75 (1958); J. D. Bjorken, S. D. Drell and S. C. Frautschi: *Phys. Rev.*, **112**, 1409 (1958); G. Putzolu: *Nuovo Cimento*, **20**, 542 (1961).

crude guess for the cross-section which in fact could very well be ten times bigger or ten times smaller than the estimate given here.

An experiment on the annihilation at rest would involve the branching ratio for the electromagnetic modes to the total annihilation rate. In order to go from this experimental number to the evaluation of the form factors either the atomic physics of the capture must be eliminated or a separate experiment to determine the complex s-wave phase shifts in $p\bar{p}$ elastic scattering must be performed. Note that 2e (or 2μ) annihilation through the one-photon channel can only occur, in general, from $^3S_1$ and $^3D_1$. In view of these difficulties it appears that the results of the in-flight experiment can be interpreted in a much more unambiguous manner.

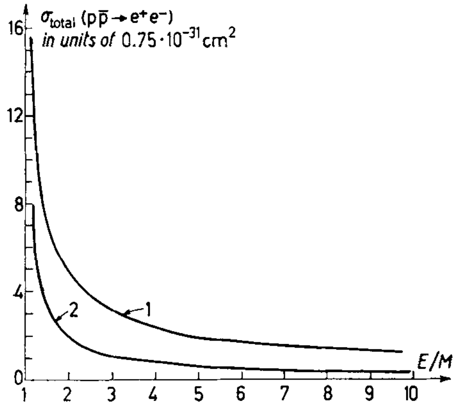

Fig. 3. – In units of $0.75 \cdot 10^{-31}$ cm^2. Upper curve (1) is for pointlike proton with $\mu_p = 1.79$, lower curve (2) is obtained by extrapolating the form factors of reference ([7]).

However, for the determination of the 2μ to 2e ratio, and the consequent exploration of the validity of electrodynamics, formula (8) also applies to annihilation at rest.

2. – The annihilation into intermediate vector bosons.

In this section we consider the possibility of detecting the intermediate vector meson of weak interactions from proton-antiproton annihilation. Vector mesons with semiweak coupling have been suggested as intermediate agents of weak interactions ([9,10]). Production of such mesons from high-energy neutrino beams ([11]), from pion beams ([12]), from photon beams ([13]), and by

([9]) R. P. FEYNMAN and M. GELL-MANN: *Phys. Rev.*, **109**, 193 (1958); also S. GERSTEIN and J. ZELDOVICH: *Žurn. Èksp. Teor. Fiz.*, **29**, 576 (1957).

([10]) T. D. LEE and C. N. YANG: *Phys. Rev.*, **119**, 1410 (1960).

([11]) B. PONTECORVO: *Proceedings of the 9th International Conference of high Energy Physics*, reported by MARSHAK (Moscow, 1960), p. 296; T. D. LEE and C. N. YANG: *Phys. Rev. Lett.*, **4**, 307 (1960).

([12]) J. ZELDOVICH: *Proceedings of the 9th International Conference of High Energy Physics* (Moscow, 1960), p. 296. N. DOMBEY: *Phys. Rev. Lett.*, **5**, 307 (1960).

([13]) M. BASSETTI: *Nuovo Cimento*, **20**, 803 (1961).

electromagnetic pair production [14] has been recently considered. Intermediate vector mesons will decay through their semi-weak coupling in a time $\sim 10^{-17}$ s.

We shall first discuss the annihilation mode of a proton-antiproton system into a pair of such intermediate vector mesons (that we denote by B) via the one photon intermediate state

$$p + \bar{p} \to B + \bar{B}\,. \tag{10}$$

Figure 4 shows the diagram for (10) in the lowest order of electromagnetic coupling.

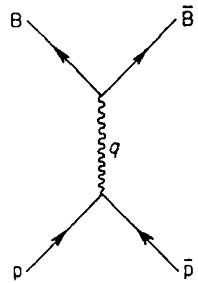

Fig. 4.

The most general form of the electromagnetic vertex, for a spin-one boson is, on invariance grounds,

$$J_\mu = G_1(\varepsilon_1\varepsilon_2)\,p_\mu + (G_1 + \mu G_2 + \varepsilon G_3)\,[(\varepsilon_1 q)\varepsilon_{2\mu} - (\varepsilon_2 q)\varepsilon_{1\mu}] + \\ + \varepsilon G_3 m_B^{-2}\,[(q\varepsilon_1)(q\varepsilon_2) - \tfrac{1}{2}q^2(\varepsilon_1\varepsilon_2)]\,p_\mu\,, \tag{11}$$

where p is the difference of the final four-momenta of B and $\bar{B}$, ε_1 and ε_2 are the polarization vectors of B and $\bar{B}$, m_B is the mass of B, $\mu+\varepsilon$ is a possible anomalous magnetic moment of B and 2ε a possible anomalous electric quadrupole moment. The form factors G_1, G_2 and G_3 depend on the squared momentum transfer q^2.

We also define the bilinear combinations

$$R = \frac{1}{2}|G_1 + \mu G_2 + \varepsilon G_3|^2\left(\frac{E}{m_B}\right)^2,$$

$$S = \frac{1}{2}\left|G_1 + 2\left(\frac{E}{m_B}\right)^2\varepsilon G_3\right|^2 + \frac{1}{4}\left|G_1 + 2\left(\frac{E}{m_B}\right)^2\mu G_2\right|^2.$$

The general expression for the cross-section of (10) is then given in c.m. by

$$\frac{d\sigma(p\bar{p}\to B\bar{B})}{d(\cos\theta)} = \frac{\pi\alpha^2}{2EP}\,\beta_B^3[R(A+B) + SA + (S-R)(B-A)\cos^2\theta]\,, \tag{12}$$

$$\sigma_T(B\bar{B}) = \frac{\pi\alpha^2}{3EP}\,\beta_B^3(2A+B)(2R+S)\,. \tag{13}$$

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[14] R. E. Marshak: *Proceedings of the 9th International Conference of High Energy Physics* (Moscow, 1960), p. 295; S. Bludman and J. A. Young: *Proc. of the 10th Rochester Conference* (1960), in *Interscience Publishers.*

In (12) and (13) β_B is the velocity of B, and

$$A = \frac{1}{2}|F_1 + F_2|^2 \qquad \text{and} \qquad B = \frac{1}{2}\left|\frac{M}{E}F_1 + \frac{E}{M}F_2\right|^2,$$

are exactly the same combinations of the nucleon form factors that determine the angular distribution of

$$p + \bar{p} \to e^+ + e^- .$$

Similarly, 2A+B in (6) also determines the total cross-section for $p+\bar{p} \to e^+ + e^-$. One thus finds for the ratio of $B\bar{B}$ annihilation to e^+e^- annihilation

$$b = \frac{\sigma_T(p\bar{p} \to B\bar{B})}{\sigma_T(p\bar{p} \to e^+e^-)} = \beta_B^3(2R + S) . \tag{14}$$

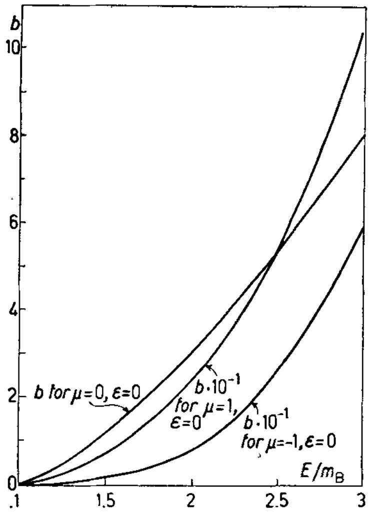

Fig. 5. – Ratio $\dfrac{(p+\bar{p} \to B^+ + B^-)}{(p+\bar{p} \to e^+ + e^-)}$ for different choices of the anomalous magnetic moment of the B mesons, and constant form factors.

Equation (14) holds in the most general case, and is still valid if the antiprotons are at rest.

If B has no anomalous moments and constant form factors, b is simply $b = \beta_B^3[\frac{3}{4} + (E/m_B)^2]$. In Fig. 5 this branching ratio is reported *vs.* E/m_B. Of course E must always be larger than the nucleon mass. One sees that annihilation into a pair of intermediate mesons is favored with respect to annihilation into e^+e^- or $\mu^+\mu^-$ already for c.m. energy larger than $1.5m_B$, provided B has no anomalous electromagnetic properties. In Fig. 5 we have also reported b for $\mu = +1$ and $\mu = -1$, $\varepsilon = 0$ and constant form factors.

Once B is produced according to (10) it will decay rapidly (in about 10^{-17} s) into its disintegration products (2π, 3π, $\pi+K$, $\mu+\nu$, $e+\nu$, etc.). The annihilation events will exhibit definite angular correlations and in some cases they will be of the kind

$$\begin{aligned} p + \bar{p} \to B^+ + B^- &\to (\mu + \nu) + (\pi + \pi) \\ &\to (\mu + \nu) + (e + \nu) \\ &\to (K^0 + \pi^+) + (\pi^- + \pi^0) \\ &\to \text{etc.}, \end{aligned}$$

which should allow the identification of B. Branching ratios among the various decay modes of B have recently been discussed by BERNSTEIN and FEINBERG [15].

We conclude this section with the observation that vector mesons can also be produced by the reactions

$$\bar{\mathrm{p}}+\mathrm{p}\rightarrow \mathrm{B}^{+}+\pi^{-}\,,\quad \mathrm{B}^{0}+\pi^{0}\,,\quad \mathrm{B}^{-}+\pi^{+}\,,\quad \mathrm{B}^{+}+\mathrm{K}^{-}\,,\quad \text{etc.} \tag{15}$$

$$\bar{\mathrm{p}}+\mathrm{n}\rightarrow \mathrm{B}^{0}+\pi^{-}\,,\quad \mathrm{B}^{-}+\pi^{0}\,,\quad \mathrm{B}^{0}+\mathrm{K}^{-}\,,\quad \text{etc.} \tag{16}$$

These reactions occur through the semi-weak coupling of the vector meson and on dimensional grounds should have a cross-section

$$\sigma \sim G \sim 0.4\cdot 10^{-32}\ \mathrm{cm}^{2}\,, \tag{17}$$

where G is the weak-coupling constant. A more refined estimate than (17) would involve the complications of strong interactions at rather high energies. If the vector weak current is conserved [9] the vector part of the amplitudes for $\bar{\mathrm{p}}\mathrm{p}\rightarrow \mathrm{B}\pi$ and $\bar{\mathrm{p}}\mathrm{n}\rightarrow \mathrm{B}\pi$ are related to the isovector amplitudes for

$$\bar{\mathrm{p}}+\mathrm{p}\rightarrow \pi^{0}+\gamma\,,$$

$$\bar{\mathrm{p}}+\mathrm{n}\rightarrow \pi^{-}+\gamma\,,$$

with the γ off-mass-shell in the form

$$\frac{\sigma(\mathrm{p}\bar{\mathrm{p}}\rightarrow \mathrm{B}\pi)}{\sigma(\mathrm{p}\bar{\mathrm{p}}\rightarrow \gamma\pi)} \geqslant \frac{\sigma_{\mathrm{V}}(\mathrm{p}\bar{\mathrm{p}}\rightarrow \mathrm{B}\pi)}{\sigma(\mathrm{p}\bar{\mathrm{p}}\rightarrow \gamma\pi)} = \frac{Gm_{\mathrm{B}}^{2}}{4\pi\sqrt{2}\alpha}\,\frac{P_{\mathrm{B}}}{P_{\gamma}}\,x = 0.77\cdot 10^{-4}\,\frac{P_{\mathrm{B}}}{P_{\gamma}}\left(\frac{m_{\mathrm{B}}}{m_{\mathrm{D}}}\right)^{2} x\,, \tag{18}$$

where σ_{V} is the contribution from the weak vector current (this does not interfere with the axial contribution in the rate) and x is a number that differs from unity for two reasons: because the correspondence holds only with the γ off-shell, and also it holds only for the iso-vector electromagnetic amplitude. From angular momentum, parity, and charge conjugation one can show that $\mathrm{p}+\bar{\mathrm{p}}\rightarrow \mathrm{B}^{0}+\pi^{0}$ from S-states goes only through vector coupling, so that the $\geqslant$ in (18) becomes an equality sign in this case. Furthermore in the schizon's theory of Lee and Yang [10] $\sigma(\mathrm{p}\bar{\mathrm{p}}\rightarrow \mathrm{B}^{+}\pi^{-})\geqslant\sigma(\bar{\mathrm{p}}\mathrm{p}\rightarrow \mathrm{B}^{0}\pi^{0})$.

In conclusion we would like to stress the fact that even though the study of these rare annihilation modes are very difficult experiments, definitive re-

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[15] J. BERNSTEIN and G. FEINBERG: *Report at the Conference on Elementary Particles* (Aix-en-Provence, 1961).

sults would be of great importance in the understanding of strong, electromagnetic and weak interactions.

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We would like to thank Prof. S. D. DRELL for informative discussions.

RIASSUNTO (*)

La possibilità di ottenere fasci di antiprotoni di intensità relativamente alta ha suggerito alcune considerazioni sui canali di annichilazione del sistema protone antiprotone, che sono alquanto rari. Ci proponiamo: i) di studiare il modo a due elettroni come mezzo per investigare la struttura elettromagnetica del protone per trasferimenti di impulso di tipo temporale; ii) di studiare il modo a due muoni e confrontarlo con il modo a due elettroni per vedere se il muone si comporta come un elettrone pesante per grandi trasferimenti di impulsi di tipo temporale; iii) ricercare l'esistenza di bosoni vettoriali deboli con i modi $p+\bar{p}\to B+\bar{B}$ e $p+\bar{p}\to B+\pi$. Sebbene non si possano fare precise predizioni teoriche, stime grossolane indicano che la sezione d'urto per questi quattro canali dovrebbe essere approssimativamente dello stesso ordine di grandezza.

(*) *Traduzione a cura della Redazione.*

IL NUOVO CIMENTO VOL. XXVII, N. 1 1° Gennaio 1963

Proton Antiproton Annihilation into Electrons, Muons and Vector Bosons.

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(*Nuovo Cimento*, **24**, 170 (1962))

Formulas (4) and (5) contain some errors and should be substituted by:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}=\frac{\mathrm{d}(\cos\theta_0)}{\mathrm{d}(\cos\theta)}\,\frac{\alpha^2\mathscr{E}_e^2}{4P(\mathscr{E}+M)}\left[2|F_1+F_2|^2+\operatorname{ctg}^2\frac{\theta}{2}\left\{|F_2|^2-\frac{2M}{\mathscr{E}+M}|F_1|^2\right\}\right], \tag{4}$$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega_0}=\frac{\alpha^2\mathscr{E}_e^2}{4P(\mathscr{E}+M)}\left[2|F_1+F_2|^2+\frac{\mathscr{E}-M}{2M}g(\theta_0)\left\{|F_2|^2-\frac{2M}{\mathscr{E}+M}|F_1|^2\right\}\right]. \tag{5}$$

* * *

We thank Dr. K. J. BARNES for pointing out these errors to us.

Dynamical Equations and Angular Momentum.

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(*Nuovo Cimento*, **26**, 1029 (1962))

The authors of the paper *Dynamical equations and angular momentum* would like to point out that, since the note added in proof at the bottom of p. 1045 refers to Appendix A only and not to the whole paper, it should be placed at the end of that appendix.

The location of the curve on the θ axis can be shifted to larger angles by increasing V_2 and R (thus maintaining the well-known VR ambiguity in the optical model) and to smaller angles by increasing V_1 and $|\eta|$, the energy difference between entrance and exit channels, which is determined experimentally and not treated as a parameter. The effect of varying V_2 is much larger than that of varying V_1, since V_2 determines two optical-model wave functions, V_1 only determines one. It was found that a large difference between V_2 and V_1 was necessary to locate the curves properly. The values quoted are not unique.

The over-all width is determined almost exclusively by R_b. Increasing R_b decreases the over-all width and increases the magnitude of the cross section at the center of the curve. It is found that when the best value of R_b is used in each state, the relative magnitudes are automatically fitted well.

The effects of increasing W_1, W_2, and a are small. Increasing W_1 and W_2 decreases the magnitude of both curves slightly. In fitting the p-state curve, V_2 and V_1 have opposite effects on the ratio of peak heights. Increasing V_2 increases the ratio. Increasing both V_1 and V_2 reduces the depth of the minimum by a very small amount.

The physical conclusions which we tentatively draw from this calculation are rather significant. For finite potentials there cannot be significant differences between single-particle wave functions whose principal quantum number, angular momentum, binding energy, and rms radius are given. Hence it seems that a distorted-wave analysis of $(p, 2p)$ experiments determines the single-particle wave functions very well.

The rms radius of the charge distribution in C^{12} given by our empirical values of R_b is 2.5 F. The experimental value obtained from electron scattering is 2.4 F. The rms radius for s-state protons is 1.7 F, which is the experimental value for the α particle. Whether this is true for s states in other light nuclei is, at present, being investigated by a systematic study of the available data. Finer points concerning curve fitting are also being investigated.

We would like to thank Dr. M. A. Melkanoff, Dr. J. S. Nodvik, Dr. D. S. Saxon, and Dr. D. G. Cantor for the use of their optical-model code SCAT 4 which was used to calculate our optical-model wave functions, and Dr. C. A. Hurst and Mr. K. A. Amos for valuable discussions.

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[1]A. J. Kromminga and I. E. McCarthy, Phys. Rev. Letters 4, 288 (1960).

[2]K. F. Riley, H. G. Pugh, and T. J. Gooding, Nucl. Phys. 18, 65 (1960); T. Berggren and G. Jacob, Phys. Letters 1, 258 (1962); K. L. Lim and I. E. McCarthy, Proceedings of the International Symposium on Direct Interactions and Nuclear Reaction Mechanisms, Padua, 1962 (Gordon and Breach, New York, 1963). Further references are given in these papers.

[3]J. P. Garron, J. C. Jacmart, M. Riou, C. Ruhla, J. Teillac, and K. Strauch, Nucl. Phys. 37, 126 (1962).

UNITARY SYMMETRY AND LEPTONIC DECAYS

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(Received 29 April 1963)

We present here an analysis of leptonic decays based on the unitary symmetry for strong interactions, in the version known as "eightfold way,"[1] and the V-A theory for weak interactions.[2,3] Our basic assumptions on J_μ, the weak current of strong interacting particles, are as follows:

(1) J_μ transforms according to the eightfold representation of SU_3. This means that we neglect currents with $\Delta S = -\Delta Q$, or $\Delta I = 3/2$, which should belong to other representations. This limits the scope of the analysis, and we are not able to treat the complex of K^0 leptonic decays, or $\Sigma^+ \to n + e^+ + \nu$ in which $\Delta S = -\Delta Q$ currents play a role. For the other processes we make the hypothesis that the main contributions come from that part of J_μ which is in the eightfold representation.

(2) The vector part of J_μ is in the same octet as the electromagnetic current. The vector contribution can then be deduced from the electromagnetic properties of strong interacting particles. For $\Delta S = 0$, this assumption is equivalent to vector-

current conservation.[2]

Together with the octet of vector currents, j_μ, we assume an octet of axial currents, g_μ. In each of these octets we have a current with $\Delta S=0$, $\Delta Q=1$, $j_\mu^{(0)}$, and $g_\mu^{(0)}$, and a current with $\Delta S=\Delta Q=1$, $j_\mu^{(1)}$, and $g_\mu^{(1)}$. Their isospin selection rules are, respectively, $\Delta I=1$ and $\Delta I=1/2$.

From our first assumption we then get

$$J_\mu=a(j_\mu^{(0)}+g_\mu^{(0)})+b(j_\mu^{(1)}+g_\mu^{(1)}). \quad (1)$$

A restriction $a=b=1$ would not ensure universality in the usual sense (equal coupling for all currents), because if J_μ [as given in Eq. (1)] is coupled, we can build a current, $b(j_\mu^{(0)}+g_\mu^{(0)})-a(j_\mu^{(1)}+g_\mu^{(1)})$, which is not coupled. We want, however, to keep a weaker form of universality, by requiring the following:

(3) J_μ has "unit length," i.e., $a^2+b^2=1$.

We then rewrite J_μ as[4]

$$J_\mu=\cos\theta(j_\mu^{(0)}+g_\mu^{(0)})+\sin\theta(j_\mu^{(1)}+g_\mu^{(1)}), \quad (2)$$

where $\tan\theta=b/a$. Since J_μ, as well as the baryons and the pseudoscalar mesons, belongs to the octet representation of SU_3, we have relations (in which θ enters as a parameter) between processes with $\Delta S=0$ and processes with $\Delta S=1$.

To determine θ, let us compare the rates for $K^+\to\mu^++\nu$ and $\pi^+\to\mu^++\nu$; we find

$$\Gamma(K^+\to\mu\nu)/\Gamma(\pi^+\to\mu\nu)$$
$$=\tan^2\theta M_K(1-M_\mu^2/M_K^2)^2/M_\pi(1-M_\mu^2/M_\pi^2)^2. \quad (3)$$

From the experimental data, we then get[5,6]

$$\theta=0.257. \quad (4)$$

For an independent determination of θ, let us consider $K^+\to\pi^0+e^++\nu$. The matrix element for this process can be connected to that for $\pi^+\to\pi^0+e^++\nu$, known from the conserved vector-current hypothesis (2nd assumption). From the rate[6] for $K^+\to\pi^0+e^++\nu$, we get

$$\theta=0.26. \quad (5)$$

The two determinations coincide within experimental errors; in the following we use $\theta=0.26$.

We go now to the leptonic decays of the baryons, of the type $A\to B+e+\nu$. The matrix element of any member of an octet of currents among two baryon states (also members of octets) can be expressed in terms of two reduced matrix elements[7]

$$\langle A|j_\mu^{(i)}+g_\mu^{(i)}|B\rangle=if_{ABi}O_\mu+d_{ABi}E_\mu; \quad (6)$$

the f's and d's are coefficients defined in Gell-Mann's paper.[1,7] It is sufficient to consider only allowed contributions and write

$$O_\mu, E_\mu=F^{O,E}\gamma_\mu+H^{O,E}\gamma_\mu\gamma_5. \quad (7)$$

From the connection with the electromagnetic current we get the vector coefficients: $F^O=1$, $F^E=0$; from neutron decay we get

$$H^O+H^E=1.25. \quad (8)$$

We remain with one parameter which can be determined from the rate for $\Sigma^-\to\Lambda+e^-+\bar\nu$. The relevant matrix element for this is

$$\cos\theta\langle\Sigma^-|j_\mu^{(0)}+g_\mu^{(0)}|\Lambda\rangle$$
$$=\cos\theta(\tfrac{2}{3})^{1/2}E_\mu=(\tfrac{2}{3})^{1/2}\cos\theta H^E\gamma_\mu\gamma_5. \quad (9)$$

Taking the branching ratio for this mode to be 0.9×10^{-4},[8] we get

$$H^E=\pm0.95. \quad (10)$$

The negative solution can be discarded because it produces a large branching ratio for $\Sigma^-\to n+e^-+\bar\nu$, of the order of 1%. The positive solution ($H^E=0.95$, $H^O=0.30$) is good, because it produces a cancellation of the axial contribution to this process. This explains the experimental result that this mode is more depressed than the $\Lambda\to p+e^-+\bar\nu$ in respect to the predictions of Feynman and Gell-Mann.[2] In Table I we give a summary of our predictions for the electron modes with $\Delta S=1$. The branching ratios for $\Lambda\to p+e^-+\bar\nu$ and $\Sigma^-\to n+e^-+\bar\nu$ are in good agreement with experimental data.[9]

As a final remark, the vector-coupling constant for β decay is not G, but $G\cos\theta$. This gives a correction of 6.6% to the ft value of Fermi transitions, in the right direction to eliminate the discrepancy between O^{14} and muon lifetimes.

Table I. Predictions for the leptonic decays of hyperons.

Decay	Branching ratio From reference 2	Branching ratio Present work	Type of interaction
$\Lambda\to p+e^-+\bar\nu$	1.4 %	0.75×10^{-3}	$V-0.72A$
$\Sigma^-\to n+e^-+\bar\nu$	5.1 %	1.9×10^{-3}	$V+0.65A$
$\Xi^-\to\Lambda+e^-+\bar\nu$	1.4 %	0.35×10^{-3}	$V+0.02A$
$\Xi^-\to\Sigma^0+e^-+\bar\nu$	0.14 %	0.07×10^{-3}	$V-1.25A$
$\Xi^0\to\Sigma^++e^-+\bar\nu$	0.28 %	0.26×10^{-3}	$V-1.25A$

The correction is, however, too large, leaving about 2% to be explained.[10]

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[1]M. Gell-Mann, California Institute of Technology Report CTSL-20, 1961 (unpublished); Y. Ne'eman, Nucl. Phys. 26, 222 (1961).

[2]R. P. Feynman and M. Gell-Mann, Phys. Rev. 109, 193 (1958).

[3]R. E. Marshak and E. C. G. Sudarshan, Proceedings of the Padua-Venice Conference on Mesons and Recently Discovered Particles, September, 1957 (Società Italiana di Fisica, Padua-Venice, 1958); Phys. Rev. 109, 1860 (1958).

[4]Similar considerations are forwarded in M. Gell-Mann and M. Lévy, Nuovo Cimento 16, 705 (1958).

[5]The lifetimes from W. H. Barkas and A. H. Rosenfeld, Proceedings of the Tenth Annual International Rochester Conference on High-Energy Physics, 1960 (Interscience Publishers, Inc., New York, 1960), p. 878. The branching ratio for $K^+ \to \mu^+ + \nu$ is taken as 57.4%. W. Becker, M. Goldberg, E. Hart, J. Leitner, and S. Lichtman (to be published).

[6]B. P. Roe, D. Sinclair, J. L. Brown, D. A. Glaser, J. A. Kadyk, and G. H. Trilling, Phys. Rev. Letters 7, 346 (1961). These authors give the branching ratio for $K^+ \to \mu^+ + \nu$ as 64%, from which $\theta = 0.269$. Also this value agrees with that from $K^+ \to \pi^0 + e^+ + \nu$ within experimental errors.

[7]N. Cabibbo and R. Gatto, Nuovo Cimento 21, 872 (1961). Our notation for the currents is different from the one used in this reference and by Gell-Mann; the connection is $j_\mu^{(0)} = j_\mu^1 + ij_\mu^2$, $j_\mu^{(1)} = j_\mu^4 + ij_\mu^5$.

[8]W. Willis et al. reported at the Washington meeting of the American Physical Society, 1963 [W. Willis et al., Bull. Am. Phys. Soc. 8, 349 (1963)] this branching ratio as $(0.9^{+0.5}_{-0.4}) \times 10^{-4}$. If it is allowed to vary between these limits, our predictions for the $\Sigma^- \to ne^-\bar{\nu}$ varies between 0.8×10^{-3} and 4×10^{-3}, and that for $\Lambda^0 \to pe^-\bar{\nu}$ between 1.05×10^{-3} and 0.56×10^{-3}. I am grateful to the members of this group for prepublication communication of their results.

[9]R. P. Ely, G. Gidal, L. Oswald, W. Singleton, W. M. Powell, F. W. Bullock, G. E. Kalmus, C. Henderson, and R. F. Stannard [Proceedings of the International Conference on High-Energy Nuclear Physics, Geneva, 1962 (CERN Scientific Information Service, Geneva, Switzerland, 1962), p. 445] give the branching ratio for $\Lambda \to p + e^- + \bar{\nu}$ as $(0.85 \pm 0.3) \times 10^{-3}$, while that for $\Sigma^- \to n + e^- + \bar{\nu}$ is given (see preceding reference) as $(1.9 \pm 0.9) \times 10^{-3}$.

[10]R. P. Feynman, Proceedings of the Tenth Annual International Rochester Conference on High-Energy Physics, 1960 (Interscience Publishers, Inc., New York, 1960), p. 501. Recent measurements of the muon lifetime have slightly increased the discrepancy. We think that more information will be needed to decide whether our 3rd assumption can be maintained.

EXPERIMENTAL EVIDENCE ON π - π SCATTERING NEAR THE ρ AND f^0 RESONANCES, FROM $\pi^- + p \to \pi + \pi + \text{NUCLEON}$, AT 3 BeV/$c$†

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This note reports some preliminary results on π - π scattering, near the 770-MeV ρ and 1250-MeV f^0 resonances. The experiment is the one reported earlier[1]; with more data measured (now about 75% of the two-prong events), we have examined the data to see to what extent they seem analyzable in terms of π - π scattering. We give a brief summary of the results, and then a few details. A more detailed report will be available later.

(1) There is evidence of a major contribution from the one-pion-exchange mechanism ("peripheral collision"), for low nucleon recoil momentum. We take the region of $\Delta^2 < \Delta_{\min}^2 + 10$ to be interpretable in terms of π - π scattering. (Δ^2 is the square of the four-momentum transfer to the nucleon, in units of the pion mass squared; $\Delta_{\min}^2$ is the lower kinematic limit, which is a function of the π - π "mass" and the incident energy)

(2) We then consider these "peripheral" (i.e., peripheral-collision) events to be representative of the angular distribution of π - π scattering. Two obvious points of caution must be mentioned here: (a) Interference effects arise from nucleon isobar production, and (b) the effective π - π scattering is off the energy shell. From detailed examination of the data, we believe neither of these effects is so severe as to grossly affect the further conclusions below. A third possible complicating effect is interference from two-pion decay of the ω, into $\pi^+\pi^-$; the possible magnitude of this effect is at present difficult for us to estimate.

(3) The spin of the f^0 is greater than zero, as reported earlier by Veillet et al.[2] We believe it is difficult to draw any conclusion from these data as to whether the spin is 2 or greater than 2. (Isospin arguments, and the data directly, exclude spin 1.)

(4) The π^- - π^0 scattering in the ρ region is con-

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Possible Vanishing of Strong Interaction Cross-Section at Infinite Energies.

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It has been proposed (¹) that high-energy scattering can be described in terms of two nonets of algebraic operators coupled to Regge trajectories. In this way, one obtained very satisfactory relations among total cross-sections, some of which had been previously obtained in the composite model (***), in particular the asymptotic relation

$$\frac{\sigma_{\pi p}}{\sigma_{pp}} = \frac{2}{3}. \tag{1}$$

A careful investigation, however, of the energy-dependence of cross-sections within the model of I shows that a good fit to the entire experimental data cannot be obtained —without altering the algebraic structure— with the assumption that all total cross-sections approach nonzero asymptotic limits. Moreover, an excellent fit is obtained if one takes the value of 0.925 ± 0.008 for the intercept of the

(*) Supported in part by the Swiss National Fund.
(**) Sponsored in part by the Air Force Office of Scientific Research under Grant AF EOAR 66-39 through the European Office of Aerospace Research (OAR) United States Air Forces.
(¹) N. CABIBBO, L. HORWITZ and Y. NE'EMAN: CERN preprint TH. 680, to be published in *Phys. Lett.* Hereafter referred to as I.
(***) See I for references.

Pomeranchuk trajectory. This implies the vanishing of all total cross-sections (*) as

$$\sigma \sim s^{-0.075\pm0.008}\,, \tag{2}$$

where s is the square of the c.m. energy.

Although this conclusion runs against the presently held theoretical beliefs, the existing data, including cosmic ray results (**), are not sufficient to distinguish between a constant cross-section and one which vanishes as slowly as suggested by (2).

This vanishing of the cross-sections has of course far reaching implications of a general nature, in particular, as far as dispersion relations and sum rules are concerned.

Furthermore, the fact that the Pomeranchuk trajectory has an intercept different from unity also implies that for all elastic amplitudes T,

$$\left(\frac{\mathrm{Re}\,T}{\mathrm{Im}\,T}\right)_{t=0} \xrightarrow[s\to\infty]{} -\mathrm{tg}\left[\frac{1-\alpha_0^s(0)}{2}\,\pi\right], \tag{3}$$

where $\alpha_0^s(0)$ is the intercept of the Pomeranchuk trajectory. For $\alpha_0^s(0)=0.925$, this ratio becomes -0.118.

We begin by writing the results of I for the averaged total cross-sections. From the Table given in I, we have

$$S_{\mathcal{N}} = 6t_0^s + 3t_8^s\,, \tag{4a}$$

$$S_\pi = 4t_0^s + 2t_8^s\,, \tag{4b}$$

$$S_{\mathrm{K}} = 4t_0^s - t_8^s\,, \tag{4c}$$

where

$$S_{\mathcal{N}} = \tfrac{1}{4}(\mathrm{pp}+\mathrm{p}\mathcal{N}+\bar{\mathrm{p}}\mathrm{p}+\bar{\mathrm{p}}\mathcal{N})\,, \tag{5a}$$

$$S_\pi = \tfrac{1}{2}(\pi^+\mathrm{p}+\pi^-\mathrm{p})\,, \tag{5b}$$

$$S_{\mathrm{K}} = \tfrac{1}{4}(\mathrm{K}^+\mathrm{p}+\mathrm{K}^-\mathrm{p}+\mathrm{K}^+\mathcal{N}+\mathrm{K}^-\mathcal{N})\,, \tag{5c}$$

and t_0^s and t_8^s are the reduced absorptive parts for the unitary singlet and octet eighth-component even signature trajectories. AB represents the total cross-section for scattering of hadrons A and B. If we assume t_0^s to be a constant (which corresponds to $\alpha_8^s(0)=1$), eqs. (4) would imply that S_{K} approaches its asymptotic

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(*) Note that the value of the exponent approximates the Bond factor 0.07, a fact which may have important though obscure implications (2).

(2) I. FLEMING: for instance, *In Her Majesty's Secret Service* (London, 1963). The relevance of the Bond factor in quantum electrodynamics has been previously noted by B. TOUSCHEK (private communication).

(**) Cosmic ray data on proton-nucleus total cross-sections exist up to very high energy (3) indicating constant « geometric » cross-sections. G. COCCONI pointed out that this result does not imply a constancy of the p-nucleus cross-section, which can well decrease by a factor ~ 2 in 10^6 GeV, as suggested by eq. (2). The reason is that the nucleus would still behave as an essentially opaque sphere and the cross-section would remain $\approx \pi R^2$. In fact a slowly decreasing p-nucleon cross-section could even correspond, in a large energy range, to an increasing p-nucleus cross-sections due to the shrinking of the diffraction peak.

(3) D. PERKINS: Berkeley High-Energy Physics Study, UCRL 10022 (1962).

limit from below, while $S_{\mathcal{N}}$ and S_π approach their limits from above. Experimentally, all cross-sections are known to be decreasing functions of the energy, so that eqs. (4) would not fit the data.

In order to obtain a better result, one may introduce the following modifications:

a) it was pointed out in I that an admixture of D-type octet coupling in the baryon vertex strengths should be expected to occur, and is in fact necessary to achieve a fit to the data at any given energy. This in itself, however, does not supply a good fit at *all* energies if we assume constant t_0^s, since it multiplies t_8^s in both S_π and S_K by the same factor. We cannot therefore get both S_π and S_K decreasing properly with a D/F correction alone.

b) a mixing between t_0^s and t_8^s analogous to φ/ω mixing. This represents SU_3 breaking and introduces differences between the asymptotic σ_{Kp} and $\sigma_{\pi p}$.

Using both *a*) and *b*), formulae (4*a*)-(4*c*) become:

$$S_{\mathcal{N}} = (\alpha\sqrt{6} + \beta\lambda\sqrt{3})^2 t_0^s + (\beta\sqrt{6} - \alpha\lambda\sqrt{3})^2 t_8^s\,, \tag{6a}$$

$$S_\pi = 2(\alpha\sqrt{2} + \beta)(\alpha\sqrt{2} + \beta\lambda)t_0^s + 2(\beta\sqrt{2} - \alpha)(\beta\sqrt{2} - \alpha\lambda)t_8^s\,, \tag{6b}$$

$$S_K = (2\alpha\sqrt{2} - \beta)(\alpha\sqrt{2} + \beta\lambda)t_0^s + 2(\beta\sqrt{2} + \alpha)(\beta\sqrt{2} - \alpha\lambda)t_8^s\,, \tag{6c}$$

where λ describes the F/D mixing,

$$\lambda = F - \frac{1}{3}D\,, \tag{7}$$

and α and β represent the t_0^s, t_8^s mixture, with mixing angle φ:

$$\alpha = \cos\varphi\,, \qquad \beta = \sin\varphi\,. \tag{8}$$

We tried to fit eqs. (6) to the experimental data of GALBRAITH *et al.* (4). No acceptable fit was achieved for constant t_0^s; we observe that trying to obtain the correct decreasing behaviour for $S_{\mathcal{N}}$, S_π and S_K destroys the fit as far as the ratio of S_π/S_K at all energies is concerned.

The only available alternative appears to consist in abandoning the idea that t_0^s represents the contribution of a single pole with intercept $\alpha_0^s(0) = 1$. One such possibility would be to adjoin to the Pomeranchuk pole an additional SU_3 singlet scalar trajectory. This has been known to give a good fit to the data (5), but it would imply some extension of our algebraic approach. In fact, at least one of the two singlets, or a linear combination thereof would be outside the algebra of $[U_6 \otimes U_6]_\beta$. Considering that there is no natural prescription for the matrix structure of the additional operator, we would lose the asymptotic prediction of eq. (1) which appears experimentally validated (*).

(4) W. GALBRAITH, *et al.*: *Phys. Rev.*, **138**, B 913 (1965).

(5) V. BARGER and M. OLSSON: University of Madison preprint.

(*) If a second even signature singlet were necessary, it would seem more natural to propose a complete doubling of the whole family of trajectories, the new family being also coupled to members of a $U_6 \otimes U_6$ algebra. In this way one could preserve the prediction of eq. (1). It is simpler—and seems to us more natural—to assume a single family of trajectories.

We wish, however, to remain in the framework of the model of I, and we therefore consider the possibility that t_0^s corresponds to a single pole with intercept $\alpha_0^s(0) < 1$; for momentum transfer $t = 0$, we write

$$t_0^s(\nu) = t_0^s(1)\,\nu^{\alpha_0^s(0)-1}\,, \tag{9}$$

$$t_8^s(\nu) = t_8^s(1)\,\nu^{\alpha_8^s(0)-1}\,, \tag{10}$$

with $\nu = s - m_A^2 - m_B^2$.

We substitute eqs. (9) and (10) into eqs. (6*a*)-(6*c*) and get an excellent fit to the GALBRAITH *et al.* data, with the following parameters

$$\left\{\begin{array}{llll} t_0^s(1) = & 7.43 \pm 0.56\,, & t_8^s(1) & = 2.08 \pm 0.26\,, \\ \alpha_0^s(0) = & 0.925 \pm 0.008\,, & \alpha_8^s(0) & = 0.76 \pm 0.04\,, \\ \operatorname{tg}\varphi = & -0.066 \pm 0.048\,, & \lambda & = 2.44 \pm 0.09\,. \end{array}\right. \tag{11}$$

The χ^2 for the fit is 5.8, to be compared with 14 (the number of degrees of freedom). The above results remain essentially unchanged when we take $\varphi = 0$.

From eqs. (6) and the above values of the parameters, we find for $s \to \infty$

$$\frac{S_{\mathcal{N}}}{S_\pi} = 1.39\,, \qquad \frac{S_\pi}{S_K} = 0.93. \tag{12}$$

If there were no mixing, one would obtain for $S_{\mathcal{N}}/S_\pi$ the value given by eq. (1), and S_π/S_K would reduce to unity. Note, however, that substituting (11) into eqs. (6), one finds $S_\pi > S_K$ up to momenta of 10^5 GeV/c, after which they cross, and the asymptotic limit of eq. (12) is approached at extremely high energies.

The Figure displays the comparison between the GALBRAITH *et al.* ([4]) data for $S_{\mathcal{N}}$, S_π, S_K and our fit.

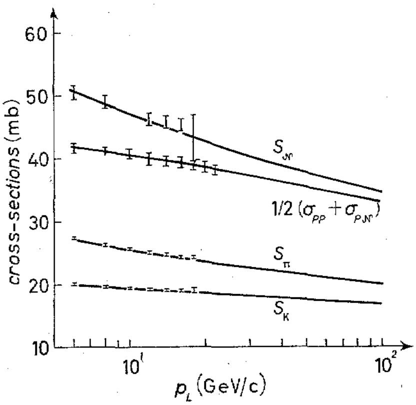

Fig. 1. – Comparison between the data of GALBRAITH *et al.* ([4]) for $S_{\mathcal{N}}, S_\pi, S_K$ and $\frac{1}{2}[\mathrm{pp} + \mathrm{p}\mathcal{N}]$ and our fit.

We note that at high energy, the t_0^s contribution still dominates each cross-section, so that in a ($\log\sigma$, $\log\nu$) plot, they approach a straight line asymptotically. A cross-section which approaches this asymptote from below may appear to be constant throughout a large energy region. This explains the behaviour of K^+-nucleon and p-nucleon cross-sections above 5 GeV/c. Moreover, all elastic amplitudes in the forward direction go asymptotically to the form

$$T(\nu, 0) \propto -\frac{1 + \exp[-i\pi\alpha_0^s(0)]}{\sin \pi\alpha_0^s(0)}\,\nu^{\alpha_0^s(0)}\,, \tag{13}$$

so that the ratio of real to imaginary part approaches the negative limit of eq. (3).

We have extended our fit to include individual cross-sections at all energies. This is done by using the total cross-sections Table of I, with the above-mentioned modifications for the t_0^s and t_8^s. The four contributions t_3^s, t_3^v, t_0^v, t_8^v are proportional to differences of cross-sections, so that they cannot be determined accurately. For the $I=0$ poles, we find

$$t_0^v(\nu) = 0.65\left(\frac{\nu}{24}\right)^{-0.65},$$

$$t_8^v(\nu) = 0.65\left(\frac{\nu}{24}\right)^{-0.56}.$$

No φ-ω mixing was required by the data, and we also assume no D/F admixture in the vector contributions. The combination $\frac{1}{2}(\mathrm{pp}+\mathrm{p}\mathcal{N})$, *e.g.*, is expressed as:

$$\tfrac{1}{2}(\mathrm{pp}+\mathrm{p}\mathcal{N}) = S_{\mathcal{N}} - 6t_0^v - 3t_8^v.$$

This is also compared to experimental points in the Figure, showing excellent agreement. In going to individual cross-sections, one needs also t_3^s and t_3^v. These can be determined again from meson-nucleon cross-sections. Their intercepts are found to be consistent with those determined from the analysis of $\pi^-\mathrm{p}\to\eta\mathcal{N}$ (6) and $\pi^-\mathrm{p}\to\pi^0\mathcal{N}$ (7). Using the results to compute individual nucleon-nucleon cross-sections gives again a fair agreement—albeit a very unenlightening one, given the large experimental errors in the differences pp-p$\mathcal{N}$ and $\bar{\mathrm{p}}$p-$\bar{\mathrm{p}}\mathcal{N}$.

Finally, we used all of our numbers to compute Re T/Im T for π^+p, π^-p, pp and $\bar{\mathrm{p}}$p; we list a few values in the following Table:

TABLE I. – *Ratio of real to imaginary parts of elastic amplitudes in the forward direction.*

P_L (GeV/c)	pp	p$\bar{\mathrm{p}}$	π^+p	π^-p
1	−0.434	−0.119	−0.244	−0.125
12	−0.385	−0.133	−0.231	−0.129
14	−0.372	−0.135	−0.227	−0.129
16	−0.362	−0.140	−0.226	−0.132
18	−0.349	−0.140	−0.219	−0.130

The vanishing of all total cross-sections as $\nu\to\infty$ is seen to be strongly suggested by our algebraic interpretation of the Regge pole model; it is interesting that existing experimental data do not contradict the relinquishing of the constant asymptotic cross-section hypothesis. We thus feel that it would be extremely impor-

(6) R. J. N. PHILLIPS and W. RARITA: *Phys. Rev. Lett.*, **15**, 807, 942 (1966).
(7) G. HÖHLER *et al.*: *Phys. Lett.*, **20**, 79 (1966).

tant to obtain accurate measurements of cross-sections at higher energies than now available; such as could be obtained by the use of an intersecting storage ring (ISR), or cosmic ray experiments on hydrogen. An accurate measurement of the pp cross-sections in the 30 to 70 GeV/c range—available at the new Serpukhov accelerator—would also be relevant since our fit predicts a drop of ≈ 3 mb in this range (see the Figure).

Although the conclusion that the Pomeranchuk trajectory has an intercept $\alpha_0^s < 1$ may seem at first sight shocking, we find it rather pleasing: $\alpha = 1$ is known to be an upper limit to acceptable intercepts, and therefore, in a certain sense, a point of high singularity, such that a pole with $\alpha(0) = 1$ would be entirely set apart from other poles with $\alpha(0) < 1$. Since we assume the s_0 trajectory to be a member of a nonet, we find it natural that its properties should be quantitatively—not qualitatively—different from those of other poles of even signature (*).

* * *

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(*) After this work had been completed, a paper by GOLDBERGER and JONES (*Phys. Rev. Lett.*, **17**, 105 (1966)) came to our attention in which it was shown that the existence of a pole with $\alpha(0) = 1$ may lead to an inconsistency between the requirements of the Mandelstam representation and analyticity in the l-plane.

Hadron Production in e^+e^- Collisions (*).

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(ricevuto il 30 Maggio 1970)

1. – The simple properties of deep inelastic electron-proton scattering has suggested models where these processes arise as interactions of virtual photons with an « elementary » component of the proton. These as yet unspecified elementary components of the proton have been given the name of « partons » by FEYNMAN (^{1}). The model has been studied by BJORKEN and PASCHOS (^{2}) and successively by DRELL, LEVY and TUNG MOW YAN (^{3}) who gave a field-theoretical treatment of the parton model, and were able to recover some of the experimentally observed properties of this process. In this letter we wish to extend the method of ref. (^{3}) to the study of the total cross-section of electron-positron annihilation into hadrons.

This treatment leads to an asymptotic (very high cross-section c.m. energy, $2E$) of the form

$$\sigma \to \frac{\pi\alpha^2}{12E^2}\left[\sum_{\text{spin }0}(Q_i)^2 + 4\sum_{\text{spin }\frac{1}{2}}(Q_i)^2\right], \tag{1}$$

where Q_i is the charge of the i-th parton in units of e. This is simply the sum of the contributions of the single partons considered as pointlike (^{4}). Each parton contributes a different kind of events to the total cross-section. The typical high-energy event should consist in the production of a pair of virtual partons, each of which develops into a jet of physical hadrons.

(*) This research has been sponsored in part by the Air Force Office of Scientific Research, through the European Office of Aerospace Research OAR, United States Air Force, under contract f 61 05267 C 0084.

(^{1}) R. P. FEYNMAN: unpublished.
(^{2}) J. D. BJORKEN and E. A. PASCHOS: *Phys. Rev.*, **185**, 1976 (1969).
(^{3}) S. D. DRELL, D. J. LEVY and TUNG MOW YAN: *Phys. Rev. Lett.*, **22**, 744 (1969); *Phys. Rev.*, **187**, 2159 (1969).
(^{4}) Equation (1) extends the well-known result obtained by J. D. BJORKEN: *Phys. Rev.*, **148**, 1467 (1966) in the case of spin-$\frac{1}{2}$ partons.

The quantum numbers of each jet (total charge, total isospin, etc.) coincide with that of the parton from which it originates. This suggests the possibility of using high-energy e^--e^- collisions to identify the quantum numbers of the « elementary » particles. At lower energies (presumably the critical energy should be connected with the parton mass) the two jets can interact with each other, and exchange energy. In this region one would expect either a resonant behaviour, or a « statistical » behaviour ([5]), or a mixture of the two.

Preliminary results of the initial operation of the Frascati electron-positron ring ADONE indicate the possibility of an abundant production of hadrons in the range of $(1.6 \div 2)$ GeV e^-e^- c.m. energy ([6]). In particular the production of meson pairs seems to be higher than what is predicted on the basis of the dominance of ρ, ω and φ ([7]).

We discuss the possibility that in this energy range the description in terms of jets is relevant. This would require the pion and possibly the kaon to be partons, *i.e.* to be treated as elementary particles. $SU_3 \times SU_3$-type arguments would then suggest that all 18 the members of the $(3, \bar{3}) \ominus (\bar{3}, 3)$ representation of scalar and pseudoscalar mesons are to be treated as elementary. If a description in terms of « π-jets » and « K-jets » is relevant for part of the observed events at $(1.6 \div 2)$ GeV, the production of « jets » with the quantum numbers of the $\delta^+(962)$ and $\varkappa(1100)$ should be an important feature of the processes at higher energies (possibly in the 3 GeV range). The production a pair of « δ-jets » should give rise among others to an high-multiplicity final state $2\eta\pi^+\pi^-$.

Another consequence of this picture would be the absence of $\pi^+\pi^-\pi^0$ and $K^+K^-\pi^0$ final states (and in general of states with an odd number of pions) which cannot arise from a pair of charge-conjugate « jets ». This would allow a critical test of the jet hypothesis, since a suppression of three-particle final states would not be expected if other mechanisms, such as resonant production, are operative.

2. – The total cross-section for hadron production is proportional ([8]) to the absorptive part of the two-point correlation function of the e.m. current

$$\sigma = \frac{\alpha^2 4\pi^3}{E^2} \Pi(4E^2), \tag{2}$$

where

$$\Pi(q^2)(q_\mu q_\nu - q^2 \delta_{\mu\nu}) = (2\pi)^3 \sum_n \delta^4(p_n - q) \langle 0|I_\mu(0)|n\rangle \langle n|I_\nu(0)|0\rangle . \tag{3}$$

Following the method of ref. ([3]) we employ the noncovariant perturbation expansion in the $P \to \infty$ frame. This limit is obtained by sending $|\boldsymbol{q}| \to \infty$ at fixed q^2. (The features of this limit, originated by FUBINI and FURLAN, have been clearly spelled out by WEINBERG ([9]).)

([5]) J. D. BJORKEN and S. J. BRODSKY: SLAC preprint, have considered the alternative between a jet model and a statistical model, but have limited their discussion to the latter.

([6]) We are grateful to the members of the experimental groups working at ADONE for private communications of the preliminary results of the work now in progress.

([7]) We thank the members of the Bologna-Frascati Group, and in particular A. ZICHICHI, for a private communication on this point.

([8]) N. CABIBBO and R. GATTO: *Phys. Rev.*, **124**, 1577 (1961).

([9]) S. FUBINI and G. FURLAN: *Physics*, 1, 229 (1966); S. WEINBERG: *Phys. Rev.*, **150**, 1313 (1966).

The first step is to re-express $\Pi(q^2)$ in terms of free currents (using only the « good » component J_3):

$$(4)\qquad \sum_n \langle 0|J_3|n\rangle\langle n|J_3|0\rangle\,\delta^4(p_n-q)=$$
$$=\sum_n \langle U0|j_3|Un\rangle\langle Un|j_3|U0\rangle\,\delta^4(p_n-q)\xrightarrow[q_3\to\infty]{}\sum_n \langle 0|j_3|Un\rangle\langle Un|j_3|0\rangle\,\delta^4(p_n-q)\,.$$

The latter equality follows from the fact that in the limit $|\boldsymbol{q}|\to\infty$ states in $\langle 0|U^{-1}$, different from the vacuum itself, which could contribute to the sum, are killed by factors of order $|\boldsymbol{q}|^{-1}$.

As in ref. (3) the basic assumption needed to derive our results is that there is a definite cut-off in the transverse momenta of the particles which appear in intermediate states (10). This assumption is of course artificial in a field-theoretic model, but it is suggested by existing information on high-energy processes, and it led to good theoretical predictions in the case of deep inelastic scattering. Under this hypothesis it follows that, in the region where q^2 is very large compared with the mass of produced particles and the transverse cut-off, we need only consider, for the amplitude $\langle 0|j_\mu U|n\rangle$, graphs of the kind of Fig. 1, where the final state is divided into two jets. The relative momentum transfer between these jets is of the order of q^2, while the transverse momenta within each jet are assumed to be $\ll q^2$. The two jets are then well separated in phase space and there is no overlap between them. In these conditions one can see, by the argument of ref. (3), that the energy of the state $U|n\rangle$ is equal to that of the state $|n\rangle$ up to terms of order $M^2/|\boldsymbol{q}|$ and $(k_\perp)^2/|\boldsymbol{q}|$, which can be neglected with respect to the main terms which are of order $|\boldsymbol{q}|$ and $q^2/|\boldsymbol{q}|$. This allows us to replace in the δ-function p_n by p_{Un} and by use of translational invariance and completeness of physical states to rewrite this equation as

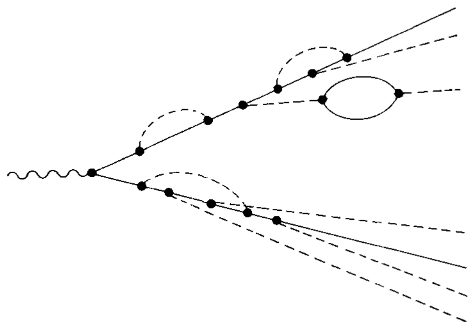

Fig. 1.

$$(5)\qquad \frac{1}{(2\pi)^4}\int d^4x \exp[iqx]\langle 0|j_3(x)\,j_3(0)|0\rangle\,.$$

Since j_μ is now the free current of the partons, this leads directly to eq. (1).

3. – We have shown that under a restrictive hypothesis on transverse momenta, for very high q^2, the final state is expected to be composed of two jets which, in the center-of-mass frame, move in opposite directions. We note that:

i) There is one such contribution to the total cross-section for each individual charged parton. The study of these jets at very high c.m. energy would allow the identification of these « elementary » components of the hadrons.

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(10) See R. P. FEYNMAN: *Phys. Rev. Lett.*, 23, 1415 (1969).

ii) The differential cross-section for the production of two jets of mass m_1 and m_2 is asymptotically given by:

a) spinless parton of charge Q

$$\frac{\mathrm{d}\sigma}{\mathrm{d}m_1^2\,\mathrm{d}m_2^2\,\mathrm{d}\cos\theta}=\frac{\pi}{16}\,\frac{\alpha^2Q^2}{E^2}\,\varrho(m_1^2)\,\varrho(m_2^2)\sin^2\theta\,; \tag{6}$$

b) spin-$\frac{1}{2}$ parton

$$\frac{\mathrm{d}\sigma}{\mathrm{d}m_1^2\,\mathrm{d}m_2^2\,\mathrm{d}\cos\theta}=\frac{\pi}{8}\,\frac{\alpha^2Q^2}{E^2}\,\varrho(m_1^2)\,\varrho(m_2^2)(1+\cos^2\theta)\,. \tag{7}$$

We have assumed spin-$\frac{1}{2}$ partons to have no bare anomalous magnetic moment, and we have not considered the possibility of charged partons of spin greater than $\frac{1}{2}$; either of these would lead to cross-sections which do not even decrease as E^{-2} and to unrenormalizability of quantum electrodynamics. In eqs. (6) and (7), ϱ is the spectral function of the parton propagator (11). If the parton field corresponds to a single-particle state of mass M, we have

$$\varrho(m^2)=Z\delta(m^2-M^2)+\sigma(m^2)\,. \tag{8}$$

By integrating eqs. (6), (7) and using the sum rule $\int\varrho(\mu^2)\,\mathrm{d}\mu^2=1$ one recovers eq. (1). In particular for the production of a parton-antiparton pair eqs. (6), (7) coincide with the usual perturbative expression (8) with the form factor equal to the renormalization constant of the parton. This is a well-known result (12).

4. – We wish now to speculate about the possibility that the mechanism discussed above is already operative in the range of $(1.6\div2)$ GeV in the c.m. energy explored by ADONE. This is possible if the pion itself is one of the partons and its spectral function is rapidly converging.

We note some consequences of this assumption:

a) A rapid convergence of the pion spectral function would imply a relatively large value for the renormalization constant and therefore a substantial cross-section for the production of $\pi^-\pi^-$ pairs which may be in agreement with experimental indications (7).

b) From SU_3 symmetry we expect all the pseudoscalar octet to be elementary if the pions is. In this way one would expect the production of K^+K^- pairs and related jets to be comparable with that of $\pi^+\pi^-$ and « pion jets ».

c) The assignment of the pion to a $(3,\bar{3})\oplus(\bar{3},3)$ representation of $SU_3\odot SU_3$ would suggest the elementarity of two charged scalar bosons. These are usually identified with the $\delta^+(962)$ and the $\varkappa^+(1100)$. Theoretical models based on this assignment (13) are in good agreement with the hypothesis of equal renormalization constants

(11) In the case of spin-$\frac{1}{2}$ parton the propagator contains two spectral functions: $\varrho(\mu^2)$ and $\tilde{\varrho}(\mu^2)$, which multiply respectively γp and μ. Assuming a suitable convergence property, only $\varrho(\mu^2)$ is relevant at very high q^2.

(12) M. GELL-MANN and F. E. LOW: *Phys. Rev.*, **95**, 1300 (1954).

(13) S. L. GLASHOW and S. WEINBERG: *Phys. Rev. Lett.*, **20**, 224 (1968); M. GELL-MANN, R. J. OAKES and B. RENNER: *Phys. Rev.*, **175**, 2195 (1969); G. PARISI and M. TESTA: *Nuovo Cimento*, **67** A, 13 (1970).

for all these particles. A test of this prediction would require an higher c.m. energy because of their higher masses.

A critical test of the hypothesis that the main mechanism of hadron production above 1.5 GeV is the one just discussed would be given by the absence of the processes $e^+e^- \to \pi^+\pi^-\pi^0$ and $e^+e^- \to K^+K^-\pi^0$. These processes cannot arise from two «jets» whose quantum numbers are charge conjugate of each other. In Table I we list the

TABLE I.

Parton	Particles in jet	Branching ratio
π^+	π^+	—
π^+	$2\pi^+ + \pi^-$, $2\pi^0 + \pi^+$	(4:1) (*)
K^+	K^+	—
K^+	$K^+\pi^+\pi^-$, $K^+\pi^0\pi^0$, $K^0\pi^+\pi^0$	(2:1:0) (*)
δ^+	$\eta\pi^+$, $K^0\bar{K}^0$	—
$\varkappa^+$	$K^+\pi^0$, $K^0\pi^+$	(1:2)
$\varkappa^+$	$K^+\eta$	—

(*) Indicative only: computed assuming S-wave emission in the center of mass of the jet, with no correction for phase space.

simplest set of particles associated with each of the four kinds of partons we have discussed. From this Table it is easy to find the various processes allowed by our picture. All the possibilities listed in the Table give rise to final states containing an even number of pseudoscalar mesons.

Final states with an odd number of pseudoscalar mesons could arise from more complicated fragmentations, or decays of the η or of the K^0 mesons. The prohibition of 3-meson final states cannot be escaped.

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Deep Inelastic Scattering and the Nature of Partons (*).

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1. – The results of inelastic-electron-scattering experiments have suggested the existence of pointlike constituents of the hadrons which have been named « partons » by Feynman (1). Two alternatives are possible:

1) Partons are identified with quarks or other mythical components of the hadron.

2) Partons are associated with some of the usual hadrons.

The second possibility has been adopted in the field-theoretical model of Drell, Levy and Yan (2). We have recently analised the consequences of this hypothesis for the electron-positron annihilation (3). In particular we have discussed the possibility that pseudoscalar (and possibly scalar) mesons are partons.

In this paper we show that in the second alternative one can obtain a simple description of the behaviour of the two structure functions W_1 and νW_2 in the limit of very large $\omega = -2M\nu/q^2$, where ν is the energy loss of the electron and q^2 is the invariant momentum transfer. In particular νW_2 reaches a constant for large values of ω in agreement with the experimental results. Also the ratio of longitudinal to the transverse cross-sections approaches a constant value. The simple hypothesis of considering the

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(*) This research has been sponsored in part by the Air Force Office of Scientific Research, through the European Office of Aerospace Research, OAR, United States Air Force, under contract F 61 05267 C 0084.

(1) R. P. Feynman: unpublished.

(2) S. D. Drell, D. J. Levy and Tung Mow Yan: *Phys. Rev. Lett.*, **22**, 744 (1969); *Phys. Rev.*, **187**, 2159 (1969).

(3) N. Cabibbo, G. Parisi and M. Testa: *Lett. Nuovo Cimento*, **4**, 35 (1970).

octet of $\frac{1}{2}^{+}$ baryons and the pseudoscalar octet as partons leads to $\sigma_{\text{long}}/\sigma_{\text{tran}} \approx 0.15 \div 0.2$. If also the scalar octet is included the ratio is increased to $0.3 \div 0.4$. Either of these possibilities seems in agreement with the scanty experimental information [4].

2. – Under the hypothesis that partons are associated with normal hadrons, the one-parton exchange diagrams of Fig. 1 are expected to give the dominant mechanism of deep inelastic electron scattering. Arguments to this effect have been clearly stated in ref. [2]. The virtual photon of momentum q creates a pair of bare partons: the first one of timelike momentum P_{N_1} dresses up into a jet of physical hadrons, the other one of spacelike momentum r interacts with the target of momentum p and mass M and produces a group of particles N_2. The kinematics of this composite process is such that we cannot consider the second parton as ejected from the proton, but rather as a bare particle produced by the photon, and interacting with the target.

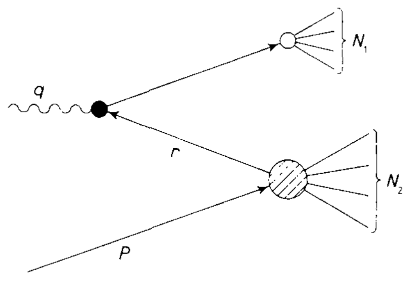

Fig. 1.

If this model were correct, deep inelastic scattering would not really investigate the electromagnetic structure of the proton, but that of the vacuum.

We will consider both the cases where the parton is a spinless boson or a spin-$\frac{1}{2}$ fermion; for simplicity we will give details of the boson case only. The fermion case is carried on along similar lines and only the results will be given.

The contribution of Fig. 1 to the correlation function $W_{\mu\nu}$ [5] is

$$W_{\mu\nu} = \frac{1}{2\pi}\int \mathrm{d}^4 r (2q + r)_\mu (2q + r)_\nu \theta(q + r)\, \varrho[(q + r)^2]\, f(r^2, r\cdot p)\,, \tag{1}$$

where ϱ is the spectral function of the unrenormalized parton propagator, and

$$f(r^2, r\cdot p) \equiv \int \exp[-irx]\langle p|[\pi(x), \pi(0)]|p\rangle\, \mathrm{d}^4 x\,, \tag{2}$$

$\pi(x)$ being the parton field.

Equation (1) can be rewritten as

$$W_{\mu\nu} = \int \mathrm{d}\sigma^2\, \mathrm{d}\mu^2\, \mathrm{d}\tau\, f(\mu^2, \tau)\, \varrho(\sigma^2)\, H_{\mu\nu}\,, \tag{3}$$

where

$$H_{\mu\nu} = \frac{1}{2\pi}\int \mathrm{d}^4 r (2r + q)_\mu (2r + q)_\nu \theta(q + r)\, \delta[(q + r)^2 - \sigma^2]\, \delta(r^2 - \mu^2)\, \delta(r\cdot p - \tau)\,. \tag{4}$$

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(4) E. D. BLOOM, D. H. COWARD, H. DE STAEBLER, J. DREES, G. MILLER, L. W. MO, R. E. TAY, M. BREIDENBACH, J. I. FRIEDMAN, G. C. HARTMANN and H. W. KENDALL: *Phys. Rev. Lett.*, **23**, 930 (1969); R. TAYLOR: SLAC Report No. SLAC-PUB-677 (1969) (to be published).

(5) For convenience we adopt the notation of J. D. BJORKEN: *Phys. Rev.*, **179**, 1547 (1969).

We are interested in computing eq. (3) in the deep inelastic region where $|q^2| \to \infty$ at fixed $\omega = -2M\nu/q^2$. In this region we can neglect σ^2 and M^2 with respect to q^2. In this limit $H_{\mu\nu}$ is independent of σ^2 and the integral over σ^2 is simply performed by use of the Lehmann sum rule

$$\int_0^\infty \varrho(\sigma^2)\, d\sigma^2 = 1 . \tag{5}$$

Integrating eq. (4) one finds that the integral over μ^2 and τ is to be performed over the shaded region in Fig. 2. This region is enclosed by the straight line a of equation (6)

$$\tau = \tfrac{1}{2}\mu^2 \tag{6}$$

and the hyperbola b of equation

$$\mu^4 + \frac{4}{M^2} q^2\tau^2 + \frac{4}{M} q_0\mu^2\tau + 2(q^2 + 2|\boldsymbol{q}|^2)\mu^2 + \frac{4}{M} q^2 q_0 \tau + q^4 = 0 . \tag{7}$$

In order to obtain a scaling behaviour we must assume that $f(\mu^2, \tau)$ is a rapidly decreasing function of μ^2 in the integration region so that the relevant values of μ^2 are much smaller than q^2 in the Bjorken limit (5). With this hypothesis we can simplify the lower bound of the integration region substituting it by the straight line b' (see Fig. 2) of equation

$$\tau = \frac{\mu^2 + M^2/\omega^2}{2}\,\omega . \tag{8}$$

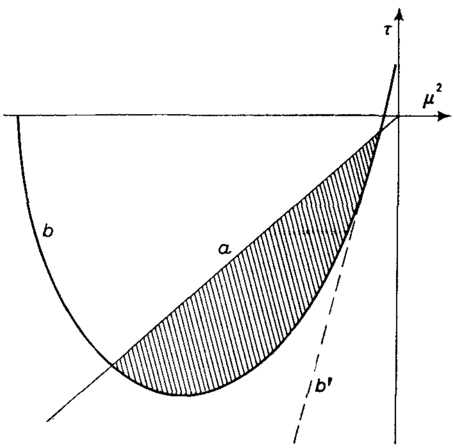

Fig. 2.

We note that the integration region over τ (the negative of laboratory energy of the exchanged parton) increases linearly with ω. Since f is related to the absorptive part of proton-parton scattering the large-ω limit of eq. (3) can then be evaluated by keeping only the Pomeron contribution in f.

For simplicity let us assume that the parton field $\pi(x)$ corresponds to a set of discrete states $|\pi_N\rangle$ of mass m_N. We can define renormalization constants for each of these states by

$$\langle 0|\pi(0)|\pi_N\rangle = \frac{1}{\sqrt{(2\pi)^3 2P_0^{(N)}}}\sqrt{Z_N} . \tag{9}$$

(6) Equation (6) is valid in the case where the lowest state of the group N_2 is a single nucleon. The corrections for higher or lower thresholds are easily done, but are irrelevant in the limit of large ω in which we are interested.

In this approximation eq. (5) becomes

$$\sum_N Z_N = 1 . \tag{10}$$

On the other hand one can write f in the following form:

$$f = \sum_{ik} \frac{\sqrt{Z_i Z_k}\, \mathrm{Im}\, A_{ik}}{(2\pi)^3 (\mu^2 - m_i^2)(\mu^2 - m_k^2)} F(\mu^2) . \tag{11}$$

where A_{ik} is the amplitude of the forward scattering $\pi_i p \to \pi_k p$ and we have introduced a factor $F(\mu^2)$ which should control the behaviour of f as a function of μ^2 far from the poles (7). We will assume that:

1) A_{ik} can be approximated by the exchange of a universally coupled Pomeron;

2) the coupling of the Pomeron is essentially diagonal (8).

Under these hypotheses, keeping in mind eq. (10), we will approximate f as

$$f = \frac{1}{(2\pi)^3} \frac{F(\mu^2)}{M} \frac{\sigma\tau}{(\mu^2 - m_{av}^2)^2} . \tag{12}$$

where σ is the asymptotic value of the total proton-parton cross-section and m_{av}^2 is some average mass of the π_N states. We would obviously arrive at the same expression even if the states associated with the field $\pi(x)$ belong to a continuum.

We can now perform the integration in eq. (3) obtaining

$$\left\{ \begin{aligned} &(W_1)_{\text{boson}} \underset{\omega\to\infty}{\longrightarrow} 0 , \\ &(\nu W_2)_{\text{boson}} \underset{\omega\to\infty}{\longrightarrow} \frac{\sigma A_b^2}{8(2\pi)^3} , \end{aligned} \right. \tag{13}$$

where

$$A_b^2 = \int \frac{\mu^4 F(\mu^2)\, \mathrm{d}\mu^2}{(\mu^2 - m_{av}^2)^2} . \tag{14}$$

The same argument can be carried on for a spin-$\frac{1}{2}$ parton and leads to

$$(\nu W_2)_{\text{fermion}} \underset{\omega\to\infty}{\longrightarrow} \frac{2M}{\omega} (W_1)_{\text{fermion}} = \frac{\sigma A_f^2}{4(2\pi)^3} . \tag{15}$$

where A_f^2 is defined by an equation similar to (14).

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(7) The need for « cut-off » on the exchanged momenta is a general feature of peripheral models (E. FERRARI and F. SELLERI: *Suppl. Nuovo Cimento*, **24**, 453 (1962); S. D. DRELL and A. C. HEARN: *High-Energy Physics*, vol. **2**, edited by BURHOP (London, 1967)).

(8) This is born out, *e.g.*, by the small cross-section of diffractive production in pp interactions: E. W. ANDERSON, E. J. BLESER, G. B. COLLINS, T. FUJII, J. MENES, F. TURKOT, R. A. CARRIGAN jr., R. M. EDELSTEIN, N. C. HIEN, T. J. MCMAHON and I. NADELHAFT: *Phys. Rev. Lett.*, **16**, 855 (1966).

3. – Given our ignorance about the cut-off function $F(\mu^2)$ and about the exact value of m^2_{av}, we can only draw semi-quantitative *conclusions:*

1) We obtain a scaling behaviour for the structure functions W_1 and νW_2. Although we have given formulae valid only in the limit of large ω, this is true quite in general for the present model.

2) The contribution of a single bosonic or fermionic part to νW_2 tends to a constant as ω tends to infinity in good agreement with the observed behaviour [4].

3) The ratio of longitudinal to transversal cross-section would depend (for large ω) upon the nature and number of bosonic and fermionic parton contributions:

a) One could assume that the pseudoscalar octet and the $\frac{1}{2}^+$ barion octet behave as partons. In this case one would have 8 fermion contributions (p, Σ^+, Σ^-, Ξ^- and their antiparticles) of the form of eq. (15) where we would identify σ as the asymptotic pp cross-section. There would also be 4 boson contributions (π^+, π^-, K^+, K^-) of the form of eq. (13) where $\sigma = \sigma_{\pi p}$. Assuming $\Lambda_b^2 \approx \Lambda_f^2$ one would find

$$\frac{\sigma_{\text{long}}}{\sigma_{\text{tran}}} = \frac{1}{2}\,\frac{4}{8}\,\frac{\sigma_{\pi p}}{\sigma_{pp}} \approx \frac{1}{6}\,.$$

b) If also a 0^+ octet is included, which may be suggested by $SU_3 \otimes SU_3$ arguments [3], the simplest guess on total cross-sections would lead to

$$\frac{\sigma_{\text{long}}}{\sigma_{\text{tran}}} \approx \frac{1}{3}\,.$$

Either of these numbers can be in agreement with the experimental data [4].

The possibilities *a*) and *b*) are purely indicative and other schemes can be simply analised on the basis of eqs. (13) and (15).

4) The experimental data on νW_2 suggest a Λ^2 (counting 8 fermion contributions) of the order $20 m_\pi^2$, which is quite reasonable.

We note that apart from these results which are in good agreement with the experimental facts, the low value of Λ^2, as well as the fact that normal masses are involved, is consistent with the observed onset of the scaling behaviour at relatively low values of q^2 and ν.

Although in the present paper we have discussed the large-ω region, it would be relatively simple to perform a complete calculation to study the behaviour of the contribution of Fig. 1 for smaller values of ω, always in the Bjorken limit. Such a computation would become sensitive to the behaviour of f in the finite-τ region and, although indicative, would not be as reliable. We note that our computation does not lead to a « quasi-elastic peak » [9]. The reason for this is that our « partons » cannot be thought of as components of the proton, but rather as the elementary hadronic constituents of the vacuum.

Another consequence of this model is in the predicted structure of the final states: these states should include a jet of particles highly collimated in the direction of q

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(9) J. D. BJORKEN: in *Proceedings S.I.F.*, Course XLI, edited by J. STEINBERGER (New York, 1969).

having the quantum numbers of one of the contributing partons. If these belong to SU_3 octets, a large fraction of events should be associated with jets of nonzero strangeness, a point which could be tested in coincidence experiments. Since this model is derived from that of ref. (2), the correlations among produced hadrons should have the simple properties discussed by DRELL and YAN (10).

Alternative parton models are based on quarks (or other heavy baryons) as partons. In such models it seems difficult to obtain the results we have discussed, although it is not excluded that such a quark model could lead to the experimentally observed features. In a quark model, however, we would not expect a jet structure and the final states should appear as the decay products of an excited proton. Contrary to the model discusses here, we would not expect, in a quark model, an abundant production of strange particles.

The idea of treating deep inelastic scattering as a peripheral process is also discussed in a recent paper of WEST (11), which we noticed after completing this work. Our results seem however different from his, especially because of our discussion of the large-ω limit and of the ratio $\sigma_{\text{long}}/\sigma_{\text{tran}}$.

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One of us (A.V.) is grateful to Prof. R. QUERZOLI for the hospitality extended to him at the Frascati National Laboratories during the completion of this work.

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(10) S. D. DRELL and TUNG MOW YAN: *Phys. Rev. Lett.*, **24**, 855 (1970).
(11) G. B. WEST: *Phys. Rev. Lett.*, **24**, 1206 (1970).