IL NUOVO CIMENTO Vol. 32 C, N. 3-4 Maggio-Agosto 2009
DOI 10.1393/ncc/i2009-10455-y

Colloquia: IFAE 2009

An update on Lattice QCD and Flavour Physics

S. Di Vita(*)

Università Roma Tre and INFN, Sezione di Roma Tre
Via della Vasca Navale 84, I-00146, Rome, Italy

(ricevuto il 19 Settembre 2009; pubblicato online il 13 Novembre 2009)

Summary. — In this paper I give a short update on recent advances in Lattice QCD calculations of quantities relevant for flavour physics. The aim is to give an overview of the state of the art of lattice calculations and to point out the main related issues. I summarize the present status of Lattice QCD machinery and then I focus on a couple of selected results, f_K/f_π and $f_{D_{(s)}}$, using them as examples for discussion.

PACS 12.38.Gc – Lattice QCD calculations.

In this paper I wish to give a brief update on the state of the art of Lattice QCD (LQCD) calculations of quantities relevant for flavour physics, namely weak operators matrix elements. Rather than aiming to give a complete review and to provide the latest numbers, I will focus on a couple of selected results which are new with respect to the IFAE2008 LQCD review talk [1] and use them as examples to point out some typical issues relevant for LQCD calculations.

I will first comment on the current status of precision lattice calculations. I will then discuss the recent results for f_K/f_π, which can be taken as a paradigm for precise, modern lattice calculations. I will then give an update on D-mesons decay constants and comment on the so called f_{D_s} *puzzle*.

1. – Lattice QCD precision era

The precision of the experimental measurements of quantities of interest for flavour physics has now reached the percent level (even better in some cases). Therefore, it is important for the theoretical determinations of the relevant hadronic quantities to match such precision. The precision of LQCD calculations is, in principle, arbitrarily improvable since it essentially depends only on machine power and on the algorithms employed. In the last few years there has been considerable progress in both these directions (see for

(*) E-mail: divita@fis.uniroma3.it

instance ref. [2] for a broad LQCD status report), which allows us to claim we are now entering the era of precision LQCD calculations.

On the one hand, the advent of Tflop machines allowed to go beyond the quenched approximation, in which sea quarks loops are neglected. This is quite relevant, since quenching introduces a systematic error which can only be estimated by comparing the quenched result with the corresponding full QCD (unquenched) value. The state of the art is represented by simulations with $N_f = 2$ (degenerate u/d quarks) and $N_f = 2+1$ (degenerate u/d and s) flavours of dynamical quarks. I should mention that there is also ongoing effort towards $N_f = 2+1+1$ simulations [3].

On the other hand, improvements in simulation algorithms made it possible to easily simulate on the lattice light quarks with masses well below $m_s/2$. This allows to rely on the predictions of Chiral Perturbation Theory (ChPT) in the extrapolations to the physical point.

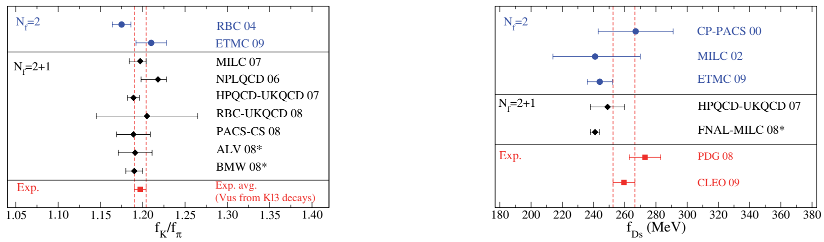
In particular, a recent breakthrough is represented by the pioneering work of the PACS-CS Collaboration [4], which simulated on the lattice quarks almost as light as the physical ones ($m_\pi^{\mathrm{lat}} \simeq 156\,\mathrm{MeV}$). Their results may still suffer from finite-size effects (FSE) since the spatial extent of the simulation is such that $m_\pi^{\mathrm{min}} L \sim 2.3$, while FSE can be under control only with a value of at least 4. Nevertheless, such work is a very important step and opens new perspectives towards *realistic* QCD simulations.

Despite these remarkable developments, it has to be admitted that systematic errors in the unquenched measurements available nowadays are not always well under control, in contrast with recent (less expensive) quenched simulations. In this respect, much work still has to be done in the context of unquenched simulations. It is important to note that calculations done with different lattice formulations (fermionic and gauge actions) are affected by different systematic errors. Therefore it is a crucial check of LQCD methods to verify that they give consistent results in the continuum limit. For instance, this could help in clarifying the issues concerning the so-called *fourth root trick* in the evaluation of the fermionic determinant in the staggered formulation (see ref. [2] for some comments and references). Indeed, despite the hints which come from comparison of lattice results with experimental observables, a proof that QCD is the continuum limit of staggered lattice QCD is still missing. Still, quite often only a few (sometimes just one or none) independent unquenched determinations of a given physical quantity are available. Moreover, finite volume and discretization effects are not always well accounted for and non-perturbative renormalization is not always implemented.

2. – A paradigm: f_K/f_π

In order to clarify the above comments, I want to dwell on a specific example which I would take as a paradigm for precise lattice calculations. The ratio of the K-meson and π-meson decay constants (f_K/f_π) represents indeed one of the few unquenched LQCD results in which systematics are well under control. A collection of unquenched calculations, obtained from $N_f = 2$ and $N_f = 2+1$ lattice simulations, is shown in fig. 1(a).

I would like to point out that most of these lattice measurements employ different lattice formulations for fermions (staggered quarks, twisted mass fermions, domain wall fermions, Wilson fermions). The NPLQCD, MILC, HPQCD and ALV collaborations use the publicly available MILC gauge field ensembles, while the others have independetly generated their own gauge configurations.



(a) (b)

Fig. 1. – (a) Shows unquenched results for f_K/f_π [4-13]. The experimental value is inferred from [14]. (b) Shows unquenched results for f_{D_s} [6, 10, 15-17]. A star denotes preliminary results.

One can appreciate the overall excellent agreement between the many lattice measurements and the experimental determination [14] (which takes $|V_{us}|$ from $K\ell 3$ decays).

It is worth noting that, in principle, quenching the strange quark is expected to have some effect. Nevertheless it can be seen, by comparing $N_f = 2$ and $N_f = 2 + 1$ results, that this source of error is still less important than the other ones. This observation holds for any other quantity (including the mass of the Ω baryon, which consists of three strange quarks).

The above lattice simulations are performed with $m_{ud} \lesssim m_s/2$ and the data show evidence of the chiral logs predicted by ChPT. This gives us confidence that it is sensible to perform ChPT fits, thus not having to rely on arbitrary ansatzs for the dependence on the pion mass. $SU(2)$ performs quite well, at variance with $SU(3)$ which typically fails because in $N_f = 2 + 1$ simulations the strange quark is not degenerate with the others.

As regards the continuum limit, excluding the calculations of NPLQCD, RBC-UKQCD and PACS-CS, every other lattice result has been extrapolated to zero lattice spacing by including in the analysis simulations performed at least at three different lattice spacings.

Finally, finite-size effects are safely taken into account in all of the above measurements (except for the aforementioned PACS-CS one).

3. – The D-mesons decay constants and the f_{D_s} *puzzle*

The CKM matrix elements $|V_{cs}|$ and $|V_{cd}|$, which control the leptonic decay rates for the processes $D_s^+ \to \ell^+\nu_\ell$ and $D^+ \to \ell^+\nu_\ell$, are well constrained by the unitarity of the CKM matrix in the Standard Model. The above decay rates have been recently measured with a great accuracy at BaBar [18], Belle [19], BES [20] and CLEO-c [21-23] and the decay constants f_{D_s} and f_D have been extracted by making use of the predicted values for the relevant CKM matrix elements. A comparison with the theoretical LQCD predictions can serve as a valuable crosscheck of lattice methods, in order to give us confindence in applying the same methods in the B sector.

There has always been excellent agreement between LQCD predictions for f_D and its experimental determinations. Instead, in the past few years there has been some tension between theory and data in the D_s sector, usually reffered to as the f_{D_s} *puzzle* (see fig. 1(b)). The extremely accurate (1%) determination published by HPQCD [10] is almost $3\,\sigma$ away from the PDG2008 [24] average and possible new physics explanations were invoked in ref. [25].

With respect to the last year, there have been some news both from the experimental and the theoretical sides. On the one hand, the recent higher statistics, improved experimental determination provided by CLEO-c [22, 23] lowered this tension to the $2.3\,\sigma$ level and weakened its interpretation as a new physics effect. On the other hand, ETMC [6] produced new results for f_{D_s} and f_D (with $N_f = 2$ dynamical twisted mass quarks at three lattice spacings and two different volumes) with 3% and 4% accurcacy, respectively. Both their determinations are in very good agreement with the other lattice data, thus providing an independent check of lattice methods. For what concerns f_D, this strenghtens the full compatibility between lattice and experimental determinations. As far as f_{D_s} is concerned, even if well aligned to the other lattice data, the result from ETMC is in better agreement with the new CLEO-c measurement than the HPQCD result. Still, this might be due to the larger errors quoted. Further details about the analysis performed by HPQCD to get their impressive precision would be very welcome in order to better understand the discrepancy.

* * *

I wish to thank the organizers, and in particular the Flavour Physics session conveners M. Pierini and S. Tosi, for the invitation to this enjoyable conference. Also, I would like to thank V. Lubicz for the valuable discussions and for the precious advices given to me during the preparation of this brief review talk. Last, but not least, I wish to thank all the people at IFAE2009 who made my stay in Bari a great time.

REFERENCES

[1] Lubicz V. and Tarantino C., *Nuovo Cimento B*, **123** (2008) 674 [arXiv:0807.4605].
[2] Jansen K., *PoS*, LATTICE2008 (2008) 010 [arXiv:0810.5634].
[3] Baron R. *et al.* (ETMC), *PoS*, LATTICE2008 (2008) 094 [arXiv:0810.3807].
[4] Aoki S. *et al.* (PACS-CS), arXiv:0807.1661.
[5] Aoki Y. *et al.*, *Phys. Rev. D*, **72** (2005) 114505 [hep-lat/0411006].
[6] Blossier B. *et al.*, arXiv:0904.0954.
[7] Aubin C. *et al.* (MILC), *Phys. Rev. D*, **70** (2004) 114501 [hep-lat/0407028].
[8] Bernard C. *et al.* (MILC), *PoS*, LAT2007 (2007) 090 [arXiv:0710.1118].
[9] Beane S. R. *et al.*, *Phys. Rev. D*, **75** (2007) 094501 [hep-lat/0606023].
[10] Follana E. *et al.* (HPQCD), *Phys. Rev. Lett.*, **100** (2008) 062002 [arXiv:0706.1726].
[11] Allton C. *et al.* (RBC-UKQCD), *Phys. Rev. D*, **78** (2008) 114509 [arXiv:0804.0473].
[12] Lellouch L., *PoS*, LATTICE2008 (2008) 015 [arXiv:0902.4545].
[13] Aubin C., Laiho J. and Van de Water R. S., *PoS*, LATTICE2008 (2008) 105 [arXiv:0810.4328].
[14] Antonelli M. *et al.*(FlaviaNet Working Group on Kaon Decays), *Nucl. Phys. Proc. Suppl.,*, **181-182** (2008) 83 [arXiv:0801.1817].
[15] Ali Khan A. *et al.* (CP-PACS), *Phys. Rev. D*, **64** (2001) 034505 [hep-lat/0010009].
[16] Bernard C. *et al.* (MILC), *Phys. Rev. D*, **66** (2002) 094501 [hep-lat/0206016].
[17] Bernard C. *et al.*, *PoS*, LATTICE2008 (2008) 278 [0904.1895].
[18] Aubert B. *et al.* (BaBar), *Phys. Rev. Lett.*, **98** (2007) 141801 [hep-ex/0607094].
[19] Abe K. *et al.* (Belle), *Phys. Rev. Lett.*, **100** (2008) 241801 [arXiv:0709.1340].
[20] Ablikim M. *et al.* (BES), *Phys. Lett. B*, **610** (2005) 183 [hep-ex/0410050].
[21] Eisenstein B. I. *et al.* (CLEO), *Phys. Rev. D*, **78** (2008) 052003 [arXiv:0806.2112].
[22] Onyisi P. U. E. *et al.* (CLEO), *Phys. Rev. D*, **79** (2009) 052002 [arXiv:0901.1147].
[23] Alexander J. P. *et al.* (CLEO), *Phys. Rev. D*, **79** (2009) 052001 [arXiv:0901.1216].
[24] Amsler C. *et al.* (Particle Data Group), *Phys. Lett. B*, **667** (2008) 1.
[25] Dobrescu B. A. and Kronfeld A. S., *Phys. Rev. Lett.*, **100** (2008) 241802 [arXiv: 0803.0512].