IL NUOVO CIMENTO Vol. 32 C, N. 3-4 Maggio-Agosto 2009
DOI 10.1393/ncc/i2009-10495-3

Colloquia: IFAE 2009

Tests of Chiral Perturbation Theory at NA48/II

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(ricevuto il 19 Settembre 2009; pubblicato online il 3 Dicembre 2009)

Summary. — During the 2003 and 2004 data taking the NA48/II experiment has collected the world largest samples of the decays $K^\pm \to \pi^\pm\pi^\mp e^\pm \nu_e$ (K_{e4}) and $K^\pm \to \pi^\pm\pi^0\gamma$. The measurement performed on these samples provided excellent tests of Chiral Perturbation Theory.

PACS 11.30.Rd – Chiral symmetries.
PACS 13.20.Eb – Decays of K mesons.
PACS 13.75.Lb – Meson-meson interactions.

1. – Introduction

The main goal of NA48/II was the search for direct CP violation in $K^\pm$ decays into three pions [1]. However, given the high statistics collected on many rare kaon decays, tests of Chiral Perturbation Theory (χPT) were also possible. The beam and detector are described in details elsewhere [2].

The NA48/II experiment has performed high-precision measurement of the form factors of K_{e4} decays and s-wave $\pi\pi$ scattering lengths a_0 and a_2 for isospin 0 and 2, respectively. The preliminary result on the full available statistics of more than 1 million K_{e4} decays achieves a precision similar to the theoretical one and it is described in the next section. Concerning $K^\pm \to \pi^\pm\pi^0\gamma$, Direct Emission (DE) and Interference (INT) fractions with respect to the internal bremsstrahlung (IB) have been measured.

2. – The $K^\pm \to \pi^\pm\pi^\mp e^\pm\nu_e$ decay

The selection of the K_{e4} decays requires three charged tracks forming a common vertex. Only one of them should be consistent with the electron hypothesis, *i.e.* an associated energy deposit in the calorimeter consistent with the measured track momentum. The other two tracks should have opposite signs and are considered pions (unless they give signal in muon detector). Specific cuts are adopted to enhance the efficiency of electron-pion separation and to keep the background at low level. The main source of background is $K^\pm \to \pi^\pm\pi^+\pi^-$ with $\pi \to e\nu_e$ decays or pion misidentified as an electron. The suppression of these decays is achieved by requiring the event to be outside an ellipse in the $(p_t, M_{3\pi})$-plane (where p_t and $M_{3\pi}$ are the transverse momentum and the invariant mass of the three charged particles under the 3π hypothesis) centred at $(0, M_K)$, with semi-axes $\pm 35\,\mathrm{MeV}/c$ and $\pm 20\,\mathrm{MeV}/c^2$. $K^\pm \to \pi^\pm\pi^0\pi^0$ also can fake the signal in case of subsequent Dalitz decay of a π^0, a lost photon and one electron misidentified as pion. The decays $K^\pm \to \pi^\pm\pi^\pm e^\mp\nu_e$ (called "wrong sign" events—WS) are highly suppressed by the $\Delta S = \Delta Q$ rule. They can be used to evaluate the background contribution. The relative level of background obtained with WS events to the signal is $\sim 0.5\%$, which has been crosschecked with Monte Carlo simulation of the contributing decays. The total amount of selected K_{e4} decays is 1.15 million.

The form factors of the K_{e4} decay can be parameterized as a function of five kinematic variables: the invariant masses $M_{\pi\pi}$ and $M_{e\nu}$ between the two pions and the two leptons, respectively, and three angles in the rest frame of the charged kaon θ_π (angle between the direction of the dipion and the direction of the π^+), θ_e (angle between the direction of the dilepton and the direction of the electron) and ϕ (angle between the dipion and the dilepton planes). The hadronic part of the matrix element can be described in terms of two axial (F and G) and one vector (H) complex form factors [3]. Their expansions into partial s and p waves (neglecting d waves and assuming isospin symmetry) are further developed in Taylor series in $q^2 = (M^2_{\pi\pi}/4m^2_{\pi^\pm} - 1)$ and $M^2_{e\nu}/4m^2_\pi$. This allows to determine the form factor parameters from the experimental data [4]:

$$F = F_s e^{i\delta_s} + F_p \cos\Theta_\pi e^{i\delta_p}, \qquad G = G_p e^{i\delta_g}, \qquad H = H_p e^{i\delta_h},$$

where

$$\begin{aligned} &F_s = f_s + f'_s q^2 + f''_s q^4 + f'_e M^2_{e\nu}/4m^2_\pi + \ldots, \qquad F_p = f_p + f'_p q^2 + \ldots, \\ &G_p = g_p + g'_p q^2 + \ldots, \qquad H_p = h_p + h'_p q^2 + \ldots . \end{aligned}$$

Since in this analysis the branching fraction of K_{e4} is not measured, only relative form factors with respect to f_s are accessible.

The following method is used to extract the form factor parameters: In a first step, $10\times5\times5\times5\times12$ iso-populated bins are defined in $(M_{\pi\pi}, M_{e\nu}, \theta_\pi, \theta_e, \phi)$-space. Each box is filled with 49 K^+_{e4} events (739000 total) and 27 K^-_{e4} events (411000 total). For each bin in $M_{\pi\pi}$, comparing data and Monte Carlo simulation, ten independent five-parameter $(F_s, F_p, G_p, H_p, \delta = \delta_s - \delta_p)$ fits are performed. In the second step, the variation of each fitted parameter with $M_{\pi\pi}$ is used to extract the form factor parameters using the above relations. The Monte Carlo sample contains 25 times larger statistics than the data sample.

The following preliminary results are obtained:

$$\begin{aligned} f'_s/f_s &= 0.158 \pm 0.007_{\mathrm{stat}} \pm 0.006_{\mathrm{syst}}, \\ f''_s/f_s &= -0.078 \pm 0.007_{\mathrm{stat}} \pm 0.007_{\mathrm{syst}}, \\ f'_e/f_s &= 0.067 \pm 0.006_{\mathrm{stat}} \pm 0.009_{\mathrm{syst}}, \\ f_p/f_s &= -0.049 \pm 0.003_{\mathrm{stat}} \pm 0.004_{\mathrm{syst}}, \\ g_p/f_s &= 0.869 \pm 0.010_{\mathrm{stat}} \pm 0.012_{\mathrm{syst}}, \\ g'_p/f_s &= 0.087 \pm 0.017_{\mathrm{stat}} \pm 0.015_{\mathrm{syst}}, \\ h_p/f_s &= -0.402 \pm 0.014_{\mathrm{stat}} \pm 0.008_{\mathrm{syst}}. \end{aligned}$$

The systematic errors are conservatively taken from the published results on the 2003 data sample [5] and are mostly statistically limited. Scattering lengths are extracted from the q^2 dependence of $\delta = \delta_s - \delta_p$ by using numerical solutions of Roy equations [6,7]. The unprecedented precision of NA48/II result on part of the statistics triggered theoretical work on determination of the effect of isospin symmetry breaking on phase shift [8]. The size of the correction on δ is $\sim 10\,\mathrm{mrad}$ and the resulting change of a_0 and a_2 is ~ 2 standard deviations. Using a fit with both a_0 and a_2 as free parameters, the results are (correlation of 96.7%):

$$a_0 = 0.218 \pm 0.013_{\mathrm{stat}} \pm 0.007_{\mathrm{syst}} \pm 0.004_{\mathrm{theo}},$$
$$a_2 = -0.0457 \pm 0.0084_{\mathrm{stat}} \pm 0.0041_{\mathrm{syst}} \pm 0.028_{\mathrm{theo}}.$$

Using the χPT constraint ($a_2 + 0.0444 = 0.236(a_0 - 0.22) - 0.61(a_0 - 0.22)^2 - 9.9(a_0 - 0.22)^2 \pm 0.0008$) leads to the following result for the only free parameter a_0:

$$a_0 = 0.220 \pm 0.005_{\mathrm{stat}} \pm 0.002_{\mathrm{syst}} \pm 0.006_{\mathrm{theo}}.$$

The theoretical error comes from the control of isospin corrections and inputs to the Roy equations. The above results are in excellent agreement with the prediction of χPT [9]: $a_0 = 0.220 \pm 0.005$ and $a_2 = -0.0444 \pm 0.0010$.

3. – The $K^\pm \to \pi^\pm\pi^0\gamma$ decay

The total amplitude of the $K^\pm \to \pi^\pm\pi^0\gamma$ decay is the sum of two terms: the inner bremsstrahlung associated with the $K^\pm \to \pi^\pm\pi^0$ decay with a photon emitted from the outgoing charged pion, and the direct emission in which the photon is emitted at the weak vertex. Using the Low theorem the branching ratio of the IB component can be predicted from that of the $K^\pm \to \pi^\pm\pi^0$ channel, using QED corrections [10, 11].

The DE term has been extensively studied in the framework of χPT [12]. Direct photon emission can occur through both electric and magnetic dipole transitions. The electric dipole transition can interfere with the IB amplitude giving rise to an interference term, which can have CP violating contributions. The magnetic part at order $O(p^4)$ in χPT is the sum of two anomalous amplitudes: one reducible, that can be calculated using the Wess-Zumino-Witten functional, and one direct amplitude, whose size is not model independent but is expected to be small. There is no definite prediction from χPT for the electric transition amplitude, which depends on undetermined constants.

The properties of the $K^\pm \to \pi^\pm\pi^0\gamma$ decay and the components IB, DE and INT, can be conveniently described using the T^*_π and W variables, where T^*_π is the kinetic energy of the charged pion in the kaon rest frame and W is a Lorentz invariant variable given by [10, 11]

$$W^2 = \frac{(P_K \cdot P_\gamma)(P_\pi \cdot P_\gamma)}{(m_K m_\pi)^2},$$

where P_K, P_π, P_γ are the 4 momenta of kaon, charged pion and radiative gamma, respectively. In the NA48/II analysis the main background (BG) sources are $K^\pm \to \pi^\pm\pi^0$ and $K^\pm \to \pi^\pm\pi^0\pi^0$. The first decay needs an accidental photon or a hadronic extra cluster to mimic the signal final state, while the second a lost or two fused γ. The rejection of $K^\pm \to \pi^\pm\pi^0$ relies on the $T^*_\pi < 80\,\mathrm{MeV}$ cut. To suppress $K^\pm \to \pi^\pm\pi^0\pi^0$ BG

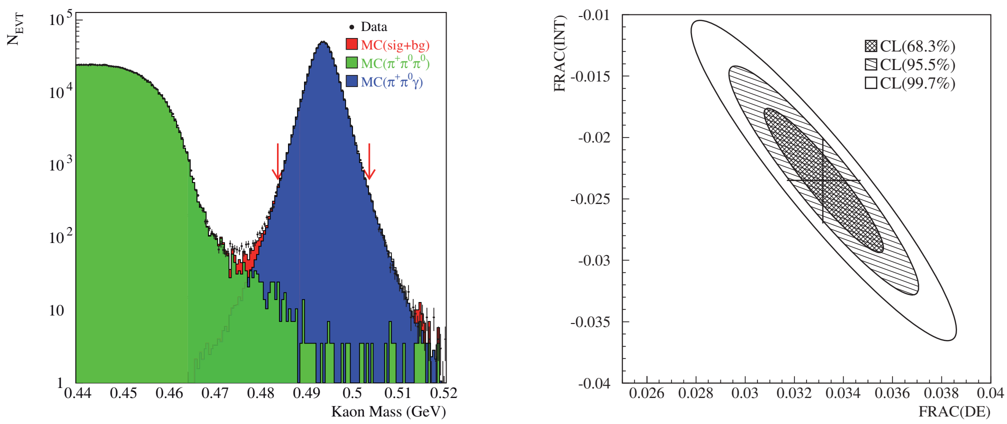

Fig. 1. – Left: kaon mass distribution for $K^\pm \to \pi^\pm\pi^0\gamma$ candidates. Right: contour plot in the DE INT plane.

the very good kaon mass resolution, (2.2 MeV) and the identification of fused gamma events by kinematical constraints have been used. In fig. 1 left the kaon mass spectrum from data is compared with the sum of signal plus background Monte Carlo. To correctly reconstruct the W value it is very important to distinguish the radiative γ from the two coming from the π^0 decay. Using a set of cuts a misidentification probability, computed using MC simulation, lower than the permille for all the components has been achieved. At the end of the selection $\sim 600\,\text{k}$ $K^\pm \to \pi^\pm\pi^0\gamma$ candidates have been identified with a BG contamination $< 1\%$ with respect to the DE component.

The extraction of DE and INT fractions relies on their different W distribution. An algorithm based on extended maximum likelihood assigns weights to MC W distributions of the three components to reproduce data. The fit has been performed in the region $0.2 < W < 0.9$. The following values for the DE and INT fractions have been obtained:

$$\text{Frac(DE)}_{T^*_\pi < 80\,\text{MeV}} = (3.32 \pm 0.15_{\text{stat}} \pm 0.14_{\text{syst}}) \times 10^{-2},$$
$$\text{Frac(INT)}_{T^*_\pi < 80\,\text{MeV}} = -(2.35 \pm 0.35_{\text{stat}} \pm 0.39_{\text{syst}}) \times 10^{-2}.$$

This is the first evidence of a non-vanishing interference term in the $K^\pm \to \pi^\pm\pi^0\gamma$ decay. The contour plot in fig. 1 right shows the very high correlation (-0.93) of the two contributions. In order to compare the NA48/II result with those from previous experiments, the fit has been redone setting the INT term to zero. The corresponding χ^2, 51/13 d.o.f., demonstrates that the data distribution cannot be properly described without an INT term.

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