

Doubly heavy baryons in a quark model with AdS/QCD inspired potential

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(ricevuto il 19 Settembre 2009; pubblicato online il 24 Novembre 2009)

Summary. — The spectrum of doubly heavy baryons, hadrons made up of two heavy quarks and one light quark, is computed through a potential model with relativistic kinematics. The expression for the $Q\bar{Q}$ potential comes from the AdS/QCD correspondence.

PACS 12.39.Pn – Potential models.

PACS 14.20.Lq – Charmed baryons.

PACS 14.20.Mr – Bottom baryons.

In 2002 the Selex Collaboration observed the baryon Ξ_{cc}^+ (dcc) in the decay $\Xi_{cc}^+ \rightarrow \Lambda_c K^- \pi^+$ [1] and later in $\Xi_{cc}^+ \rightarrow p D^+ K^-$ [2]. The mass of the state was measured to be

$$(1) \quad M_{\Xi_{cc}} = 3518.9 \pm 0.9 \text{ MeV.}$$

This is the only observed baryon comprising two heavy quarks: no other experiment has observed these states so far. However the quark model predicts their existence and is able to estimate their masses.

Baryons can be studied by potential models in two different ways. The first one consists in studying the three-body problem, in which the potential term is the sum of the interactions between each pair of quarks. The second one consists in studying the two-body problem, in which one quark interacts with a bound state of the other two quarks, in such a way that the problem becomes as simple as studying a meson. In this paper the second approach is followed [3], supposing that the two heavy quarks are close enough to be seen as one particle (diquark) by the light quark. The relativistic kinematics is introduced by the Salpeter equation for S wave hadrons:

$$(2) \quad \left(\sqrt{m_1^2 - \nabla^2} + \sqrt{m_2^2 - \nabla^2} + V(r) \right) \psi(\mathbf{r}) = M \psi(\mathbf{r})$$

while the potential term results from a recent computation of the quark-antiquark potential within the AdS/QCD correspondence [4], a recently developed approach to the non-perturbative regime of QCD, inspired by the AdS/CFT correspondence. It is expressed as a parametric equation, in which the energy and the distance between the

quark and the antiquark are functions of a parameter λ ($\lambda \in [0, 2]$):

$$(3) \quad \begin{cases} V_{\text{AdS/QCD}}(\lambda) = \frac{g}{\pi} \sqrt{\frac{c}{\lambda}} \left(-1 + \int_0^1 dv v^{-2} \left[e^{\lambda v^2/2} \left(1 - v^4 e^{\lambda(1-v^2)} \right)^{-1/2} - 1 \right] \right), \\ r(\lambda) = 2 \sqrt{\frac{\lambda}{c}} \int_0^1 dv v^2 e^{\lambda(1-v^2)/2} \left(1 - v^4 e^{\lambda(1-v^2)} \right)^{-1/2}. \end{cases}$$

The use of this potential term represents the main novelty of this computation. For comparison, one can consider the results found by other potential models [5-8], QCD Sum Rules [9] or lattice QCD [10].

A spin interaction is also considered, having the form

$$(4) \quad V_{\text{spin}}(r) = A \frac{\tilde{\delta}(r)}{m_1 m_2} \mathbf{S}_1 \cdot \mathbf{S}_2 \quad \text{with} \quad \tilde{\delta}(r) = \left(\frac{\sigma}{\sqrt{\pi}} \right)^3 e^{-\sigma^2 r^2},$$

with A and σ parameters. To avoid a divergence in the wave function at $r = 0$, coming from the divergent potential, a cut-off is added, in order to make the potential constant at small distances, namely for $r < r_M = 4\pi\Lambda/(3M)$ ($\Lambda = 1$ if $m_1 = m_2$). The Salpeter equation (2) is solved using a numerical method, the Mulhopp method [11]. All the parameters are fixed fitting the spectrum of mesons comprising two heavy quarks or one heavy and one light quark. The parameters that better reproduce meson spectrum are [12]: $c = 0.4 \text{ GeV}^2$, $g = 2.50$, $V_0 = -0.47 \text{ GeV}$ (constant term) for the AdS/QCD inspired potential (3); $\sigma = 0.47 \text{ GeV}$, $A_c = 14.56$, $A_b = 6.49$ for the spin term (4) (two different parameters have been introduced to describe the spin splitting of mesons with charm and bottom quark); $\Lambda = 0.5$; the constituent quark masses $m_q = 0.34 \text{ GeV}$ ($q = u, d$), $m_s = 0.48 \text{ GeV}$, $m_c = 1.59 \text{ GeV}$ and $m_b = 5.02 \text{ GeV}$.

Since baryons are studied as bound states of a light quark and a heavy diquark, it is necessary to start computing diquark masses. They are obtained solving the Salpeter equation (2) for the interaction of two heavy quarks: m_1 and m_2 are the quark masses, ψ and M are the wave function and the mass of the diquark, respectively, and $V(r)$ is the potential of interaction of two quarks. In the one-gluon-exchange approximation two quarks can attract each other forming a diquark in the $\bar{3}$ representation of $SU(3)_c$ and the energy of the interaction is one half of the quark-antiquark one. So, $V(r)$ in the case of diquarks is one half of the sum of (3) and (4). This fixes diquark masses; the computed values are reported in table I.

Diquark masses are used as input to compute baryon masses. The starting point is again eq. (2): when applied to baryons, m_1 is the mass of the light quark, m_2 is the mass of the diquark, ψ and M are the wave function and the mass of the baryon, respectively, $V(r)$ is the energy of the interaction between a quark and a diquark. Since the diquark is in the $\bar{3}$ representation of the $SU(3)_c$ group, its interaction with a quark is analogous to the one between a quark and an antiquark, the only difference arising from the fact that the diquark is not a pointlike object. So, the diquark-quark potential is again related to the potential sum of (3) + (4), but it is modified by an interpolation with the diquark wave function to account for its finite size (fig. 1):

$$(5) \quad \tilde{V}(R) = \frac{1}{N} \int d\mathbf{r} |\psi_d(\mathbf{r})|^2 V(|\mathbf{R} + \mathbf{r}|)$$

with N a normalization constant.

TABLE I. – Diquark masses in GeV. Curly (respectively, squared) brackets indicate a spin 1 (respectively, spin 0) diquark; n is the radial number.

Diquark	State	Mass
$\{cc\}_{nS}$	$1S$	3.238
	$2S$	3.589
$[bc]_{nS}$	$1S$	6.558
	$2S$	6.882
$\{bc\}_{nS}$	$1S$	6.562
	$2S$	6.883
$\{bb\}_{nS}$	$1S$	9.871
	$2S$	10.165

TABLE II. – Masses (GeV) of baryons comprising a diquark $\{cc\}_{1S}$ and a light quark (q or s).

Particle	State	J^P	q - d content	Results
Ξ_{cc}	$1S$	$\frac{1}{2}^+$	$q\{cc\}_{1S}$	3.547
	$2S$			4.183
	$3S$			4.640
Ξ_{cc}^*	$1S$	$\frac{3}{2}^+$	$q\{cc\}_{1S}$	3.719
	$2S$			4.282
	$3S$			4.719
Ω_{cc}	$1S$	$\frac{1}{2}^+$	$s\{cc\}_{1S}$	3.648
	$2S$			4.268
	$3S$			4.714
Ω_{cc}^*	$1S$	$\frac{3}{2}^+$	$s\{cc\}_{1S}$	3.770
	$2S$			4.334
	$3S$			4.766

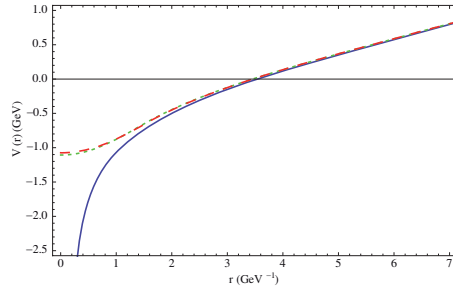
Fig. 1. – Full $Q\bar{Q}$ potential from eqs. (3), (4) (solid line); modified potential in eq. (5) through the wave function of $\{cc\}_{1S}$ (dashed line) and $\{cc\}_{2S}$ (dot-dashed line).

TABLE III. – Masses (GeV) of baryons comprising a diquark $\{bb\}_{1S}$ and a light quark (q or s).

Particle	State	J^P	q - d content	Results
Ξ_{bb}	1S	$\frac{1}{2}^+$	$q\{bb\}_{1S}$	10.185
	2S			10.751
	3S			11.170
Ξ_{bb}^*	1S	$\frac{3}{2}^+$	$q\{bb\}_{1S}$	10.216
	2S			10.770
	3S			11.184
Ω_{bb}	1S	$\frac{1}{2}^+$	$s\{bb\}_{1S}$	10.271
	2S			10.830
	3S			11.240
Ω_{bb}^*	1S	$\frac{3}{2}^+$	$s\{bb\}_{1S}$	10.289
	2S			10.839
	3S			11.247

The spectra of baryons comprising two charm, and two bottom quarks are reported in tables II and III, respectively. The mass of Ξ_{cc} predicted in this paper is in agreement with the experimental value (1), assuming that the uncertainty of the evaluation is of the same order as the difference between the experimental and the theoretical value of meson masses in [12].

The excited states of doubly heavy baryons can also be evaluated considering the interaction of a 2S diquark with a quark. The energy level of this configuration can be compared with the energy of the first radial resonance (2S state) of the baryon comprising the same constituent quarks. The first level is higher than the second for baryons comprising one heavy quark, while it becomes lower for baryons comprising two heavy quarks [6]. The masses of the bound states of 2S diquark and a quark are re-

TABLE IV. – Masses (GeV) of baryons comprising a diquark in the 2S state and comparisons with the results of other models.

Baryon	J^P	q - d content	This paper	[6]	[7]	[8]
Ξ_{cc}	$\frac{1}{2}^+$	$q\{cc\}_{2S}$	3.893	4.029	3.910	3.812
Ξ_{cc}^*	$\frac{3}{2}^+$	$q\{cc\}_{2S}$	4.021	4.042	4.027	3.944
Ω_{cc}	$\frac{1}{2}^+$	$s\{cc\}_{2S}$	3.992	4.180	4.075	
Ω_{cc}^*	$\frac{3}{2}^+$	$s\{cc\}_{2S}$	4.105	4.188	4.174	
Ξ_{bb}	$\frac{1}{2}^+$	$q\{bb\}_{2S}$	10.453	10.576	10.441	10.373
Ξ_{bb}^*	$\frac{3}{2}^+$	$q\{bb\}_{2S}$	10.478	10.578	10.482	10.413
Ω_{bb}	$\frac{1}{2}^+$	$s\{bb\}_{2S}$	10.538	10.693	10.610	
Ω_{bb}^*	$\frac{3}{2}^+$	$s\{bb\}_{2S}$	10.556	10.721	10.645	

ported in table IV: the obtained values are comparable with the ones found with other models.

The results presented here can be analysed through HQET, supposing that a $1/m_{\{QQ\}}$ expansion can be performed for the mass of doubly heavy baryons, in analogy with [13]

$$(6) \quad M_{\{QQ\}q} = m_{\{QQ\}} + \bar{\Lambda} + \frac{\lambda_1}{2m_{\{QQ\}}} + A_Q d_H \frac{\lambda_2}{2m_{\{QQ\}}},$$

$m_{\{QQ\}}$ being the mass of the diquark and $d_H = \mathbf{S}_{\{QQ\}} \cdot \mathbf{S}_q$. From (6) the mass splitting between baryons with $J^P = 3/2^+$ and $J^P = 1/2^+$ can be predicted. These predictions are verified by the results found in this paper.

The comparison with the predictions of HQET could be important for testing the validity of the model. However the proof of the existence of doubly heavy baryons can only come from the experimental observations, for which we have to wait for forthcoming experiments.

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I thank P. COLANGELO, F. DE FAZIO and S. NICOTRI for precious suggestions. I also thank M. V. CARLUCCI, M. PELLICORO and S. STRAMAGLIA for collaboration on developing the numerical method used here. This work was supported in part by the EU Contract No. MRTN-CT-2006-035482, "FLAVIANet".

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