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Advantages of two-photon processes in quantum batteries

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Summary. — We consider a Dicke quantum battery made of N two-level systems embedded into a microwave cavity. We assume a matter-cavity radiation characterized by both single- and two-photon processes. In the $N = 1$ case we analyze how the performances of the device are affected by the initial state of the cavity. By increasing N, we demonstrate that the two-photon interaction allows an improvement of the averaged charging power with respect to the single-photon one.

1. – Introduction

Batteries have been at the core of the technological development over the past decades [1]. An important step in their future evolution involves the possibility of storing energy in miniaturized devices. Here, a relevant role could be played by genuine quantum phenomena such as coherence and entanglement. In this direction, the concept of quantum battery (QB) has been introduced [2] to define a device able to store, transfer and release energy exploiting quantum features and outperforming its classical counterparts. This opened the way to a new and fast developing branch of research [3-7].

Currently, the most promising ways to implement such nanodevices are based on superconducting qubit [8], semiconducting quantum dots [9] and circuit-QED architectures [10] where Dicke QBs, made of N two-level systems (TLSs) interacting with a cavity through a dipolar interaction, have been recently proposed [11-13]. Geometries devoted to suppress the dominant dipole contribution in order to exploit more exotic interactions have also been discussed [14, 15].

Motivated by this, in our work we present a two-photon Dicke QB [16] and we compare its performances concerning the average charging power to the ones of a conventional single-photon Dicke QB. We consider the possibility of engineering the initial state of the cavity, focusing on Fock and coherent states, showing that at $N = 1$ and in the rotating-wave approximation (RWA) [17], the former states are characterized by better performances [18]. For a generic N we observe a collective advantage, namely a superlinear scaling in the ratio between charging power obtained in the Dicke model and the one obtained when each TLS is coupled to its own cavity, which is stronger in the two-photon case ($\propto N$) with respect to the single-photon one ($\propto \sqrt{N}$) [12, 19].

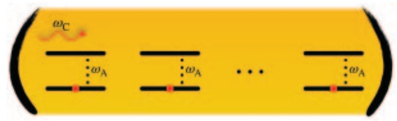

Fig. 1. – Scheme of a Dicke QB, where N TLSs, with energy separation ω_{A}, are coupled to a single cavity mode of frequency ω_{C}, either via a single- or two-photon interaction.

2. – Model

We describe the QB as a collection of N identical TLSs coupled to a unique cavity mode (see fig. 1). This system is described by the Dicke model [11], where we assume that coupling between the TLSs and the single cavity mode is given by either the single-photon interaction [12], or by the two-photon one [14, 15]. The Hamiltonians describing these situations are given respectively by (hereafter we set $\hbar = 1$)

$$H_{\mathrm{1ph}} = \omega_{\mathrm{C}} a^\dagger a + \omega_{\mathrm{A}} J_z + \theta(t) g_1 J_x (a^\dagger + a), \tag{1}$$

$$H_{\mathrm{2ph}} = \omega_{\mathrm{C}} a^\dagger a + \omega_{\mathrm{A}} J_z + \theta(t) g_2 J_x [(a^\dagger)^2 + (a)^2], \tag{2}$$

where ω_{C} is the frequency of the cavity photons, ω_{A} is the energy separation between the ground state $|0\rangle$ and the excited state $|1\rangle$ of the TLSs, g_1 and g_2 are the coupling strength of the single- and two-photon interactions respectively. Here a (a^2) annihilates one (two) photon in the cavity, while $J_{x,z} = \frac{1}{2}\sum_i^N \sigma^i_{x,z}$ represent the components of the pseudospin in terms of the Pauli matrices. Notice that the $\theta(t)$ function indicates a charging protocol where the interaction is switched on at $t = 0$. We emphasize that it is possible to only consider the two-photon coupling in eq. (2), suppressing the dipolar interaction, as recently proved in refs. [14, 15].

In the following we will only consider the resonant regime, where $\omega_{\mathrm{A}} = \omega_{\mathrm{C}}$ for the single-photon and $\omega_{\mathrm{A}} = 2\omega_{\mathrm{C}}$ for the two-photon Dicke model, since it allows the best energy transfer between the photons in the cavity and the TLSs [5, 6]. Moreover, we will choose as initial state $|\psi(0)\rangle = \sum_n \alpha_n |n\rangle \otimes |0, \ldots, 0\rangle$, where at $t = 0$ all the N TLSs are in the ground states. For the cavity, with N_{ph} photons inside it, we consider a Fock (F) and a coherent (C) state whose probability amplitudes α_n are $\alpha_n^{(F)} = \delta_{n,N_{ph}}$ ($\alpha_n^{(F)} = \delta_{n,2N_{ph}}$), $\alpha_n^{(C)} = e^{-\frac{N_{ph}}{2}} \frac{N_{ph}^{\frac{n}{2}}}{\sqrt{n!}}$ ($\alpha_n^{(C)} = e^{-N_{ph}} \frac{2N_{ph}^{\frac{n}{2}}}{\sqrt{n!}}$), for the single- (two)-photon coupling case.

2·1. *Average charging power and collective advantage.* – To study the performances of the QB we introduce the average charging power at a given time t [5]

$$P(t) \equiv \omega_{\mathrm{A}} \frac{[\langle\psi(t)|J_z|\psi(t)\rangle - \langle\psi(0)|J_z|\psi(0)\rangle]}{t}, \tag{3}$$

with $|\psi(t)\rangle$ as the time evolved state of the system according to the proper Hamiltonian in eq. (1) and (2). In the following we will also consider the maximum of the charging power, occurring at time t_P, namely $P_{\max} \equiv \max_t[P(t)] \equiv P(t_P)$. Moreover, we introduce the collective advantage defined as [3]

$$\Gamma = \frac{P_{\max}}{P^{ind}_{\max}}, \tag{4}$$

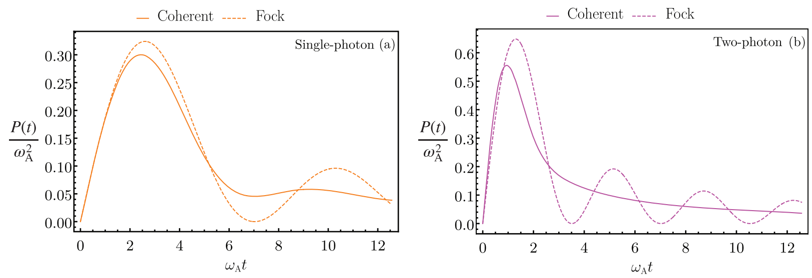

Fig. 2. – Average charging power $P(t)$ in unit of ω_A^2 as function of $\omega_A t$ for the single-photon model (a) and the two-photon one (b). Full curves represent coherent states, while dashed curves represent Fock states. Other parameters are $g_1 = g_2 = 0.2\ \omega_A$ and $N_{ph} = 6$.

where $P_{\max}^{ind} \propto N$ is the power in the individual case, where each TLS is coupled to its own cavity, for both the single- and two-photon models.

3. – Advantages of the two-photon model

3˙1. *Initial state of the cavity.* – First of all, we consider the Hamiltonians in eq. (1) and (2) when $N = 1$ and in the RWA [17], analyzing the dependence on the cavity initial state.

As we can see from fig. 2, both in the single- and two-photon model a Fock state is characterized by a higher average charging power with respect to a coherent state with the same average number of photons. Moreover, one notices that the two-photon coupling leads to a shorter time t_P to reach this maximum power.

3˙2. *Collective advantage.* – In fig. 3 we report the behaviour of the charging power as function of number N of TLSs.

This allows us to define the collective advantage of the Dicke model compared to the case of parallel charging of N independent TLSs each one coupled to a different cavity. We can see that both the single- and two-photon models approach a steady value for large N. This indicates the scaling laws

$$P_{\max}^{1ph} \propto N\sqrt{N}, \qquad P_{\max}^{2ph} \propto N^2, \tag{5}$$

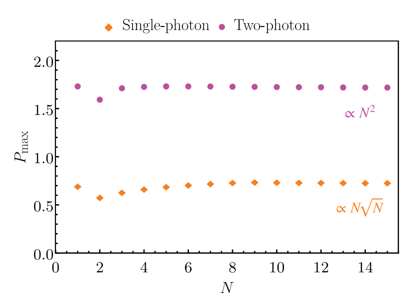

Fig. 3. – Maximum average charging power $P_{\max}$ as a function of N for the single-photon interaction in unit of $g_1\sqrt{N}N\omega_A$ (orange diamonds) and for the two-photon interaction in unit of $g_1 N^2\omega_A$ (magenta circles). Other parameters are $g_1 = g_2 = 0.2\ \omega_A$.

and shows that, for large N, the two-photon Dicke model presents a greater collective advantage $\Gamma^{2ph} \propto N$ compared to the single-photon one ($\Gamma^{1ph} \propto \sqrt{N}$), proving again its better performances.

4. – Conclusion

We have considered a Dicke QB where N TLSs are coupled to a cavity with a single photonic mode by either a single- or two-photon interaction. Focusing on the case $N = 1$ and in the RWA, we have determined that a Fock initial condition for the cavity is more suitable for engineering a QB. Then we have demonstrated that the two-photon interaction leads to better performances concerning the charging power, where we found a collective advantage that scales, for a large number N of TLSs, as $\propto N$ compared to the one obtained in the single-photon model ($\propto \sqrt{N}$). This is a promising result in view of future implementation of QBs.

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