

Quantum spin Hall based heterostructures

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Summary. — A summary of my presentation at the Congresso Nazionale SIF 2021 is given. The potential of quantum spin Hall based heterostructures is briefly reviewed with particular emphasis on the role of the possible gap opening mechanisms: magnetic barriers, superconducting barriers, strong interactions, constriction potentials.

1. – Introduction

The discovery of the quantum Hall effect (QHE), in 1980, marked the beginning of the era of topological effects on electronic matter [1, 2]. From the point of view of the main possible technological applications, the QHE can be summarized as follows: if one takes a clean two-dimensional electron gas, at very low temperature, and applies a strong perpendicular magnetic field, the system becomes insulating in the bulk but develops chiral metallic channels at its physical boundaries. Crucially, the metallic channels are ballistic even in the presence of impurities. The clear appeal of the discovery is somehow diminished by the need of strong magnetic fields. However, in 1988, Haldane discovered that time-reversal symmetry breaking, instead of the magnetic field, is the essential ingredient [3]. His model (the Haldane model) is based on a free electron model on the honeycomb lattice and reproduces the QHE physics without a net magnetic flux. Unfortunately, no material was known to realize such a model. A breakthrough happened in 2005, when Kane and Mele discovered that one sheet of graphene, with spin-orbit interaction, realizes a time-reversal invariant extension of the Haldane model [4, 5]. More specifically, their proposal is equivalent to two copies of the Haldane model, one per chirality. Moreover, electrons with opposite momentum have opposite spin projection. Kramers' theorem then implies that there is no elastic backscattering between them, unless time-reversal symmetry is broken. The system is called quantum spin Hall effect (QSHE) and is the first example of a fermionic time-reversal invariant topological phase [6]. Unfortunately, it was soon noticed that spin-orbit coupling in graphene is indeed too weak to allow for

the detection of the QSHE. However, in 2006 Bernevig, Hughes and Zhang found a feasible alternative [7], based on CdTe-HgTe heterostructures. The experimental realization came soon after, in 2007, from the Molenkamp group in Würzburg [8]. From then on, the research in the field moved essentially in two directions: finding QSHE in different materials [9-11], with the main aim of making the effect observable at room temperature, and create functional nanostructures exploiting the properties of the edges for applications in spintronics, superconducting spintronics and topological quantum computation. From the point of view of the nanostructuring, which is the focus of this article, four main ingredients are beneficial: contacts, proximitizing superconductors, ferromagnetic barriers and constrictions between the edges. Contacting the QSHE is rather natural. Indeed, contacts were already present in the original experiment [8]. Proximity-induced superconductivity has also been achieved [12]. On the other hand, up to date, ferromagnetic barriers could not be successfully implanted. Finally, a single experiment, in 2020, has shown signatures of a constriction between the edges. This experiment also showed the importance of electronic interactions, that further enlarge the possible applications of QSHE based systems [13].

In this article I will briefly summarize the Hamiltonians and the applications of the various ingredients. The rest of the article is divided as follows. In sect. **2** I will describe the models employed for the QSHE-based nanostructures and the main gap-opening mechanisms, and finally in sect. **3** I will draw the conclusions.

2. – Metallic edges and gap-opening mechanisms

A single edge of QSHE consists of two counter-propagating modes with opposite spin. The corresponding Hamiltonian H_0 is given by [4]

$$(1) \quad H_0 = \sum_{s=\pm} \int_0^L dx \psi_s^\dagger(x) (-i s v_F \partial_x) \psi_s(x).$$

Here, $x = 0$ and $x = L$ are the extrema of the sample, v_F the Fermi velocity, $s = \pm$ the spin projection and $\psi_s(x)$ the Fermi operator of an electron with spin projection s at position x . The Hamiltonian sheds light on two aspects that have not been discussed so far: the dispersion relation is linear, with the crossing point that can be accessed by gating (in narrow samples); the single edge has half of the degrees of freedom with respect to usual spinful systems. These two aspects make the edge very intriguing, as it represents half of a Dirac fermion in one dimension.

From the theoretical point of view, the most straightforward term to add is the magnetic barrier. This term shows analogies with the mass in the Dirac equation, but has one degree of freedom more: the direction of the magnetization. Indeed, if magnetization is parallel to the quantization axis used to define s , conventionally the z axis, no gap opens. I will not consider this case. The Hamiltonian considered is hence [14]

$$(2) \quad H_B = \sum_{s,s'=\pm} \int_0^L dx \psi_s^\dagger(x) \left(M_x(x) \sigma_{s,s'}^x + M_y(x) \sigma_{s,s'}^y \right) \psi_{s'}(x).$$

Here, $M_{x/y}(x)$ are the magnetization components in the x and y directions and $\sigma^{x/y}$ are the Pauli matrices. By simply changing the mass to mimic the Jackiw-Rebbi model (or

more precisely the Goldstone-Wilczek model) fractional charges of generic value appear. Moreover, a connection with the chiral anomaly has been delineated [15-18].

Superconducting terms can be added as well by means of the Hamiltonian [19]

$$(3) \quad H_{\Delta} = \int_0^L dx \psi_+^{\dagger}(x) \Delta(x) \psi_-^{\dagger}(x) + \text{h.c.},$$

where $\Delta(x)$ is the superconducting pairing. Intriguing phenomena such as odd frequency pairing and missing Shapiro steps characterize the superconducting QSHE [20]. Moreover, the combination of superconductivity and magnetic barriers leads to Majorana bound states [19, 21] and 4π -periodic Josephson currents [19].

Majorana fermions can be generalized to give more flexible topological bound states: parafermionic states. The QSHE is an optimal playground for Z_4 parafermions, if strong interactions are included. In that case, indeed, a new kind of mass can emerge, given by

$$(4) \quad H_{2p} = \int_0^L dx g_{2p}(x) \psi_+^{\dagger}(x) \partial_x \psi_+^{\dagger}(x) \psi_-^{\dagger}(x) \partial_x \psi_-^{\dagger}(x) + \text{h.c.}.$$

Here, $g_{2p}(x)$ regulates the strength of the coupling. Such term can result, alone, in the appearance of fractional charges and fractional Wigner crystals [22]. In combination with superconductivity, it can lead to parafermionic states [23].

A further mass term is present when two edges (indexed by $\eta = \pm 1$) are brought close to each other [24-28]. In this case, the kinetic energy H_0 becomes $H'_0 = \sum_{\eta} H_0^{(\eta)}$, with $H_0^{(\eta)} = \sum_{s=\pm} \int_0^L dx \psi_s^{\dagger(\eta)}(x) (-i\eta s v_F \partial_x) \psi_s^{(\eta)}(x)$, and $\psi_s^{(\eta)}(x)$ the Fermi operator on the edge indexed by η . The mass term becomes

$$(5) \quad H_{bs} = \sum_{s,\eta} \int_0^L dx g_{bs}(x) \psi_s^{\dagger(\eta)}(x) \psi_s^{(-\eta)}(x),$$

with $g_{bs}(x)$ parametrizing the term. This mass term preserves both time-reversal symmetry and axial spin symmetry. Alone, it is hence of little use. However, it comes together with a forward scattering term of the form

$$(6) \quad H_{fs} = \sum_{s,\eta} \int_0^L dx g_{fs}(x) \psi_s^{\dagger(\eta)}(x) \psi_{-s}^{(-\eta)}(x).$$

Again, $g_{fs}(x)$ is a parameter giving the strength of the term. The forward scattering contribution H_{fs} is also time-reversal invariant. However, it breaks axial spin symmetry. The combination of H_{bs} and H_{fs} is extremely promising for designing heterostructures. Indeed, it makes the two edges equivalent to a spin-orbit coupled quantum wire, with all its intriguing properties in terms of Majorana fermions and parafermions. However, the QSHE has the advantages that cleaner samples can be fabricated and that multi-terminal experiments are more easily designed. Moreover, the constriction potentials are able to make ferromagnetic barriers unessential in superconducting spintronics and topological quantum computation applications. This fact is extremely relevant since, to date, ferromagnetic barriers were never successfully implanted on QSHE systems.

3. – Conclusions

In this short paper, the main mechanisms able to open gaps in QSHE systems were described. They are important in possible applications, since they enable the conception of heterostructures for spintronics, superconducting spintronics and topological quantum computation. Magnetic barriers, superconducting elements, two-particle backscattering due to strong interactions and constriction potentials were discussed.

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