

## Endurance of mesoscopic twin-beam states propagating in noisy channels

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received 31 January 2022

**Summary.** — Quantum resources can improve the quality and security of data transmission. Here we prove, both theoretically and experimentally, that mesoscopic twin-beam states of light are robust against loss and noise sources, thus opening new perspectives in the implementation of Quantum Communication protocols.

### 1. – Introduction

In the field of optical Quantum Communication, quantum resources have the potential to improve both the quality and the security of transmitted information. The standard protocols employ entangled states generated at the single-photon level and the message to be transmitted is encoded in the different degrees of freedom of such states, such as polarization, momentum, time bin and orbital angular momentum [1,2]. As an alternative, in 2002 Grosshans and Grangier proposed a different approach, based on the exploitation of continuous variables instead of discrete ones [3]. In such a case, the detection of light states is no more performed by single-photon detectors, but by means of a homodyne detection scheme.

At variance with these solutions, here we propose a different strategy, which is based on the use of mesoscopic twin-beam (TWB) states and of a commercial class of photon-number-resolving (PNR) detectors. As recently shown in [4,5], we prove that mesoscopic states of light are robust against the possible drawbacks that affect communication channels, namely loss and noise sources [6]. The use of PNR detectors operated at room temperature represents a fundamental step towards the implementation of a communication protocol in a more realistic scenario, as better discussed in the following.

### 2. – Theoretical model

A mesoscopic TWB state is typically multi-mode both in spectrum and space. Assuming that such modes are equally populated and that there is no phase relation among

them, the resulting state can be written as [7]

$$(1) \quad \rho_{\text{TWB},\mu} = \sum_{n=0}^{\infty} P^{\mu}(n) |n, n\rangle \langle n, n|,$$

where  $|n\rangle = \delta(n - \sum_{k=1}^{\mu} n_k) \bigotimes_{k=1}^{\mu} |n_k\rangle_k$ ,  $n_k$  is the number of photons in the  $k$ -th mode and  $n$  is the overall number of photons in the  $\mu$  spatio-spectral modes that impinge on the detector, while  $P^{\mu}(n)$  is the multi-mode thermal distribution [8]

$$(2) \quad P^{\mu}(n) = \frac{(n + \mu - 1)!}{n!(\mu - 1)! (\langle n \rangle / \mu + 1)^{\mu} (\mu / \langle n \rangle + 1)^n},$$

in which  $\langle n \rangle$  is the mean number of photons in the two arms of TWB. The state in eq. (1) is entangled in the number of photons. In order to prove it experimentally, it is convenient to consider the nonclassicality criterion based on the calculation of the noise reduction factor,  $R$ , which is defined as  $R = \sigma^2(n_1 - n_2) / (\langle n_1 \rangle + \langle n_2 \rangle)$ , where  $\sigma^2(n_1 - n_2)$  is the variance of the distribution of the photon-number difference between the two parties, while  $\langle n_1 \rangle + \langle n_2 \rangle$  is the shot-noise-level. The condition  $R < 1$  is a sufficient condition for entanglement. In the case of the multi-mode TWB in eq. (1),  $R$  assumes the analytic form:

$$(3) \quad R = 1 - \frac{2\sqrt{\eta_1\eta_2}\sqrt{\langle m_1 \rangle \langle m_2 \rangle}}{\langle m_1 \rangle + \langle m_2 \rangle} + \frac{(\langle m_1 \rangle - \langle m_2 \rangle)^2}{\mu(\langle m_1 \rangle + \langle m_2 \rangle)}.$$

As anticipated in the Introduction, here we want to prove that mesoscopic TWB states are robust against loss and noise. To quantify their level of robustness, the expression of  $R$  can be re-calculated by assuming that there is an imbalance between the two arms of TWB, and that a noise source with mean value  $\langle m_N \rangle$  and variance  $\sigma_N^2$  is superimposed to one arm

$$(4) \quad R = 1 - \frac{2\eta t \langle m \rangle}{(1+t)\langle m \rangle + \langle m_N \rangle} + \frac{(1-t)^2 \langle m \rangle^2}{\mu[(1+t)\langle m \rangle + \langle m_N \rangle]} + \frac{\sigma^2(m_N) - \langle m_N \rangle}{(1+t)\langle m \rangle + \langle m_N \rangle}.$$

Here  $t$  is the transmission efficiency quantifying the balancing level between the two arms. According to the specific statistics of the noise source, it is interesting to investigate under which conditions it is still possible to obtain  $R < 1$ . As an example, in the following section we experimentally investigate the case of a quasi-single-mode thermal noise source with mean value  $\langle m_N \rangle$  and variance  $\sigma_N^2 = \langle m_N \rangle (\langle m_N \rangle / \mu_N + 1)$  superimposed to one arm of TWB.

### 3. – Experimental results

As shown in fig. 1, mesoscopic TWB states are produced by parametric down conversion in a  $\beta$ -Barium-Borate (BBO) crystal pumped by the fourth harmonics of a Nd:YLF laser regeneratively amplified at 500 Hz. Two portions at frequency degeneracy (523 nm) are spatially (I) and spectrally (BF) selected, focused (L) into two multi-mode fibers having 600- $\mu\text{m}$  core diameter (MF) and delivered to two hybrid photodetectors (HPDs). A quasi-single-mode thermal state is produced by sending the second harmonics of the laser to a rotating ground glass disk (GD) and selecting a single speckle by means of



of TWB, while in panel (b) we plot the same quantity as a function of the mean value of TWB for a fixed mean value of noise, both in the presence (magenta dots) and in the absence (black dots) of the noise source. It is interesting to notice that in both panels there are conditions under which it is still possible to observe nonclassical correlations. Indeed, from eq. (4) we have that  $R < 1$  if

$$(5) \quad \langle m_N \rangle < \sqrt{\mu_N [2\eta t \mu - (1-t)^2 \langle m \rangle]} \frac{\langle m \rangle}{\mu}.$$

Hence, the maximum value of  $\langle m_N \rangle$  satisfying the nonclassicality condition is limited by the mean number of photons per mode of the TWB state, namely  $\langle m \rangle / \mu$ . From inequality (5) we can argue that increasing the mean value of TWB can guarantee the observation of sub-shot-noise correlations even in the presence of a thermal noise source, as far as the detection apparatus is able to properly resolve the number of detected photons.

#### 4. – Conclusions

We have shown that mesoscopic TWB states are good candidates for encoding and transmitting information in Quantum Communication protocols, since they are robust against loss and noise. In particular, we have proved that there are conditions under which a quasi-single-mode thermal state superimposed to one arm of TWB does not prevent the observation of nonclassical correlations. Increasing the mean value of TWB could help even in the presence of a non-negligible noise source. To this aim, the use of Silicon photomultipliers instead of HPDs as PNR detectors could be crucial, since they are characterized by a wide dynamic range.

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The author acknowledges the Project “Investigating the effect of noise sources in the free-space transmission of mesoscopic quantum states of light” supported by the University of Insubria.

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