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UV completion and not quite black holes

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Summary. — When it comes to black holes, the Einstein action gives us little choice. But the Einstein action should eventually be subsumed into a UV complete theory of gravity, and such a theory may provide an alternative description for what we know of as black holes. We consider the horizonless solutions called 2-2-holes that emerge from quadratic gravity. Within a Planck length of the would-be horizon, strong gravity and high curvatures quickly turn on. These solutions are very close to being completely black, but not quite. An ideal probe to test for not quite black holes are the low frequency gravitational waves that are excited in and around them when they are newly formed, as in the merger events observed by LIGO. We identify the generic features of the resulting gravitational wave echoes. We also consider how it is that quadratic gravity can provide a local quantum field theory for gravity.

1. – Outline

The original outline of the talk had three parts.

1) We will address the question of why a UV completion of gravity needs to be considered in the context of understanding black holes. In particular how is it that UV degrees of freedom can affect our description of macroscopically large black holes?

2) We then describe the UV completion of interest. This is quantum quadratic gravity (QQG) as proposed by Stelle in 1977 [1]. We are finding it worthwhile to think more about the interpretation of this theory. One way to gain understanding of a theory is simply to do calculations, and so we give some examples of calculating cross sections.

3) With greater appreciation for this theory as a possible UV complete theory of quantum gravity, we can then turn to how it is suggesting that there is a horizonless replacement for black holes. This replacement comes with an observable gravitational wave signature.

Part 2) described work that was ongoing at the time. I discussed how QQG has unitarity without positivity. The unusual minus signs that appear in the theory turn out

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not to lead to physical inconsistency, due to a mismatch between the naive perturbative degrees of freedom and the physical degrees of freedom. Given this understanding it then made sense to calculate cross sections in QQG. Of interest is the high energy behavior of these cross sections, but at the time of the workshop it did not look like good high energy behavior could be achieved. My conclusion was that it might be achieved if the Planck mass was dynamically generated, so that the coefficient of the Einstein term was effectively not a constant. Then the high energy behavior would be controlled, and thus improved, by the presence of a form factor.

My continued work on this topic after the workshop changed my conclusions. The result was the paper [2]. This paper supersedes those calculations I presented the workshop for part 2), and so I will not reproduce those here. In [2] we discuss how the relevant cross sections for high energy scattering are inclusive, and due to very intricate cancellations, these turn out to be sensible with good high energy behavior. The naive perturbative degrees of freedom provide a suitable basis for calculating inclusive cross sections, even though the massive gravi-particle states, in particular the ghosts, are not the true asymptotic states. This is completely analogous to the focus on quark and gluon states in perturbative QCD. In the end the unusual minus signs in QQG are crucial for obtaining the result.

Classical quadratic gravity shall be the focus for the rest of this report. Unlike the quantum theory, the classical theory suffers from possible instabilities. We shall assume here that the *static* solutions of the classical theory provide a window onto states of the quantum theory that are of large size. In addition the classical theory may provide an adequate description of high curvatures because the quantum theory is asymptotically free in the high curvature and short distance limit.

2. – The role of UV completion

To describe why it is that a UV completion may be necessary to understand macroscopically large solutions of a theory, and why the low energy theory may be inadequate, we turn to an example from QCD. QCD may have ground states in the form of quark matter (depending on the strange quark mass), and these states can include large objects in the form of quark matter stars.

The basic description of quark matter is that a large enough density of quarks effectively punches a hole in the chiral condensate. The energy saving occurs because the quarks are now effectively massless inside the hole. This phenomenon is captured by a linear sigma model,

$$L = \partial_\mu \Phi^\dagger_{ij} \partial^\mu \Phi_{ij} - V(\Phi) - \Phi_{ij} \bar{q}_i q_j. \tag{1}$$

With $i, j = u, d, s$ being flavor indices, this is a model of mesons, in particular the lightest scalar and pseudoscalar nonets. At the minimum of the potential $V(\Phi)$, Φ_{ij} represents the chiral condensate. Because of the last term in L, a large enough quark density can push Φ_{uu} and Φ_{dd} close to zero [3]. The increase in potential energy is more than balanced by the decrease in quark mass energy.

The main point of this example is that the radius of such an object is arbitrarily large compared to the length scale of QCD, $1/\Lambda_{QCD}$. Despite this, this object cannot be understood from the dynamics of the low energy theory. The latter is described by the chiral Lagrangian, where by definition $\Phi^\dagger \Phi = 1$. To describe quark matter we need to properly describe the relevant dynamics of QCD, as captured by (1). Both GR and

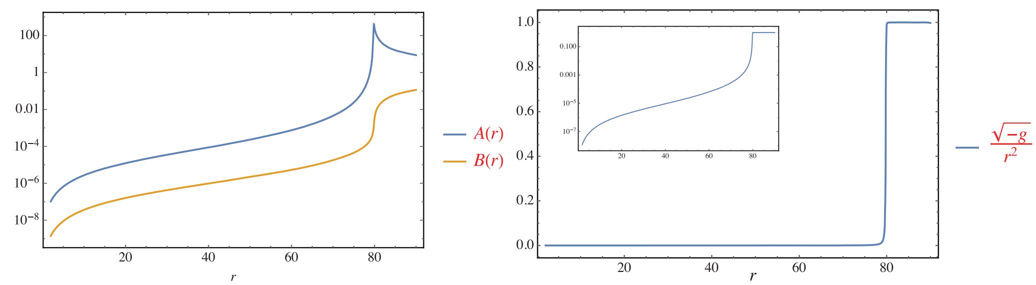

Fig. 1. – Metric functions on log scale, left, and the volume element on both linear and log scales, right.

the chiral Lagrangian are low energy theories, suggesting that both may be missing some essential dynamics to describe macroscopically large solutions of the full theories.

Given the quark matter picture of a hole in a condensate, we might then wonder whether something similar could occur in gravity. Could a large enough density of matter punch a hole in spacetime? We take this to mean a hole in the volume of spacetime. Consider $\sqrt{-g}/r^2 = \sqrt{A(r)B(r)}$ for the metric $\mathrm{d}s^2 = -B(r)\mathrm{d}t^2 + A(r)\mathrm{d}r^2 + r^2\mathrm{d}\Omega^2$. We can define a hole to be a region inside which $\sqrt{-g}/r^2 \to 0$. What does GR say about such holes? Not much. Black hole solutions have $A(r)B(r) = 1$, which we see is the analog of $\Phi^\dagger\Phi = 1$. In the presence of matter, such as for a star, $B(r)$ can get small, but $A(r) \geq 1$ and $A(0) = 1$. We argue that for classical quadratic gravity, something quite different happens for a sufficiently large matter density. This is the 2-2-hole.

Figure 1(left) shows the metric functions $A(r)$ and $B(r)$ for a 2-2-hole on a log scale. We see that both functions quickly become small just inside the 2-2-hole radius. In fig. 1(right) we show the quantity $\sqrt{-g}/r^2$, both on a linear scale and a log scale. The volume element collapses inside the hole. In particular the collapsing spatial volume means a much larger matter density and pressure. This suggests how matter can resist collapse into the time-like singularity at $r = 0$, where $A(r)$ and $B(r)$ vanish.

3. – Brief story of 2-2-holes

This story has a long history with a relatively low rate of activity. Soon after he proposed QQG, Stelle (1978) [4] turned to the classical theory and categorized solutions of quadratic gravity via the leading powers of r for $A(r)$ and $B(r)$ at origin, (0, 0), (1, −1), (2, 2). The first is nonsingular at the origin, the second is the black hole and possible variants, and the third corresponds to 2-2-holes, where $A(r)$ and $B(r)$ vanish like r^2. A numerical example of a vacuum solution of this type was found in 2002 [5], which connected the (2, 2) behavior at the origin to the exterior Schwarzschild solution. It was found that these (2, 2) solutions have 5 parameters (not including the overall normalization of $B(r)$ which is fixed at large r). Lu, Perkins, Pope, Stelle [6] provided a general discussion of both the (1, −1) and (2, 2) solutions and their possible matter sources. Holdom and Ren (2017) [7] found numerical 2-2-hole solutions having a matter shell source. The transition between (0, 0) and (2, 2) solutions was studied as the radius of the shell was decreased. The numerical solutions were sufficient to find how the interior 2-2-hole solution scales with M. In 2019 [8] 2-2-hole solutions with a more realistic relativistic gas source were found. They were studied further in Ren (2019) [9]. Aydermir, Holdom, Ren (2020) [10] studied 2-2-hole remnants as dark matter, and damping of gravitational waves in 2-2-holes was discussed in [11].

For whatever the matter source, the metric near the origin is parameterized by two parameters, $A(r) = a_2 r^2 + \ldots$ and $B(r) = b_2 r^2 + \ldots$. The scaling behavior of these parameters is $a_2, b_2 \sim G/(GM)^4$, and thus they become very small for large M. For the realistic case of a relativistic gas, with $\rho(r) = 3p(r)$, the temperature is $T(r) = T_\infty B(r)^{-1/2}$. The field equations determine $A(r)$ and $B(r)$ as well as a certain combination of parameters: $NT_\infty m_2^2/m_{\mathrm{Pl}}^2$. N counts the particle degrees of freedom and m_2 is the ghost mass. Some exact relations emerge; the total matter energy is

$$U = \frac{3}{8}M \tag{2}$$

and the 2-2-hole entropy S satisfies

$$ST_\infty = S_{\mathrm{BH}} T_{\mathrm{Hawking}}. \tag{3}$$

To show the entropy calculation in more detail, we have

$$S = \frac{(2\pi)^3}{45} N \int T(r)^3 A(r)^{1/2} r^2 \mathrm{d}r = \zeta \frac{\mathrm{Area}}{4G} = \zeta S_{\mathrm{BH}}. \tag{4}$$

The area law emerges due to the scaling behavior of $A(r)$ and $B(r)$ that we have mentioned. The new constant is

$$\zeta \approx 0.75 N^{\frac{1}{4}} \sqrt{m_2/m_{\mathrm{Pl}}}. \tag{5}$$

It can easily be larger than unity, in which case

$$S > S_{\mathrm{BH}} \text{ and } T_\infty < T_{\mathrm{Hawking}}. \tag{6}$$

Thus 2-2-holes can be preferred over black holes by an entropy argument.

Figure 2 shows how two densities behave in the interior of a 2-2-hole. The left plot shows entropy density $s(r)$ defined by $S = \int s(r)\mathrm{d}r$, while the right plot shows the dominant term in the action density. This is the Weyl-squared term, which completely dominates the R and R^2 terms. The corresponding behavior for a black hole is also shown. The 2-2-hole could be considered a ball of Weyl action.

If we consider the wave equation on the 2-2-hole background, we find that the time-like curvature singularity at $r = 0$ has an interesting effect. The resulting wave equation (the

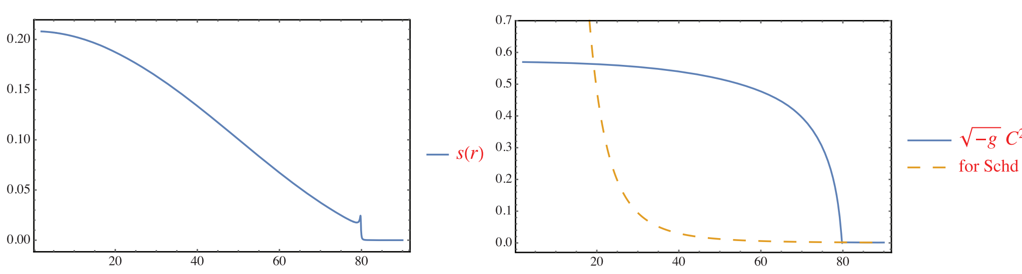

Fig. 2. – Entropy density (left) and action density (right).

radial equation in spherical coordinates) is *less* singular than in flat space! In particular consider the term $\ell(\ell+1)B(r)\phi(r)/r^2$ in the scalar wave equation. For flat space, or for normal stars, this is the singular centrifugal barrier at the origin for all $\ell > 0$. For a 2-2-hole $B(r) \sim r^2$, and thus this barrier does not exist. All waves can penetrate to the origin, just as s-waves do in flat space.

Another object with a similar feature is the truncated black hole, where effectively a wall is placed just slightly outside the horizon. The radial wave equation for both of these objects displays a cavity structure. One end of the cavity occurs at a radius of $r \approx 3GM$, where both objects have the same angular momentum barrier of finite height. Inside this radius, at a finite tortoise distance Δx from the barrier, both objects effectively have a wall that forms the other side of the cavity. For the 2-2-hole this "wall" is due to the s-wave-type boundary condition that is automatically imposed at the origin. The point is that for both objects, there is no centrifugal barrier at the inside wall of the cavity.

The result is a 1D cavity that slightly leaks at one end, and so a disturbance that bounces back and forth in the cavity can produce gravitational wave echoes on the outside. But it is clear that there is an even more generic feature, and that is an evenly spaced resonance spectrum. The resonance spacing is $\Delta f = 1/(2\Delta x)$ and it is small because the cavity size Δx is large due to a large log enhancement,

$$\Delta x \approx 4GM \log\left(\frac{GM}{\ell_{\mathrm{Pl}}}\right). \tag{7}$$

When the 2-2-hole first forms in some merger event, it will be in a perturbed state. When the interior of the cavity is perturbed in some quite random way, more random than just a single pulse, then well-defined echoes may not even exist. But the resonance pattern still exists [12]. This suggests that the resonance pattern should be the focus for the search for gravitational wave echoes [13, 14].

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