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Experimental prospects to observe the $g-2$ muon anomaly in the electron sector

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Summary. — The long-standing difference between the experimental measurement and the standard model prediction for the muon's anomalous magnetic moment, $a_\mu = (g_\mu - 2)/2$, can be due to new particles flowing in loop contributions: such discrepancy might thus signal the presence of new physics at the TeV scale. The vast majority of models explaining the muon discrepancy in terms of new physics (NP) predict sizable effects in $a_e = (g_e - 2)/2$, too. We discuss the experimental prospects to reach sub-ppb precision on a_e and test the NP origin of the muon anomaly in its electron counterpart.

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1. – Introduction

The magnetic moment of the muon, $\vec{\mu}_\mu = \frac{e}{2m_\mu}(1 + a_\mu)\vec{\sigma}$, is a key Standard Model (SM) observable and one of the most precisely measured quantities in physics. It is also an important ingredient to electroweak precision tests [1]. Since 1960, the experimental precision in a_μ has been improved by more than five orders of magnitude and further improvements are expected soon.

The experimental value of the anomalous contribution a_μ from the latest and most precise E821 experiment at BNL [2] differs from the Standard Model prediction by more than three standard deviations. This discrepancy could be due to theory or experimental nuisance. It could be related to a statistical fluctuation, an overlooked systematic effect

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in a_μ^{exp} or to a genuine new physics effect. In this case, loop corrections from new particles beyond the SM could produce a difference

$$\Delta a_\mu = a_\mu^{exp} - a_\mu^{\mathrm{SM}} = 2.90\,(90) \times 10^{-9}, \tag{1}$$

explaining the discrepancy observed at BNL.

The experimental value will soon be cross-checked by the E989 experiment at Fermilab and the planned $g-2$/EDM experiment at J-PARC in about five years. A NP explanation of the muon puzzle remains a viable option and should the new experiments confirm the findings of E821, independent searches of these NP effects by direct production of new particles at colliders or by the exploitation of other high-precision observables will play a key role.

The vast majority of the models explaining the muon puzzle in term of new physics, predicts sizable effects in the electron sector, too. This is due to the fact that the size of loop contributions generally scales as the squared ratio of the lepton masses. Hence, NP effects are:

- Enhanced by $(m_\tau/m_\mu)^2$ in the tau counterpart of a_μ. This enhancement has not been exploited yet due to the experimental challenges related with the tiny τ lifetime.

- Suppressed by $(m_e/m_\mu)^2$ in the electron counterpart of a_μ. In spite of the superior experimental accuracy of a_e compared with a_μ, NP effects become invisible in the electron sector due to $(m_e/m_\mu)^2 \simeq 2 \times 10^{-5}$.

This natural suppression factor ("naive scaling"- NS) can be sidestepped in specific models due to either non-universality of the couplings or of the NP mass spectrum [3]. It has been noted in 2012 [3] that a combination of naive scaling violation and the higher accuracy in a_e could make NP effects visible to experiments in the electron sector. In fact [4], improvements in the electron sector measurements that can be achieved employing atom interferometry are so significant that they can attain the level of precision needed to test NP effects even in the naive scaling hypothesis (0.06 ppb in a_e). These results will likely be achieved on a timescale comparable to next generation $g-2$ muon experiments at Fermilab and JPARC and, hence, will play a major role in clarifying the origin of the muon anomaly.

2. – The measurement of a_e and the fine structure constant

The best measurements of a_e have been obtained from single electrons in Penning traps [5]. In the 70's research groups at the University of Mainz and the University of Washington (UW) developed methods to measure the electron magnetic moment using a large number of electrons stored in a Penning trap. Finally, they were able to confine and detect a single electron in the trap reaching an uncertainty of 4 ppt in $g/2$ (3.5 ppb in a_e) in 1987. This result led the CODATA fits [6] for nearly 20 years. A major breakthrough occurred in 2006 thanks to the development of cylindrical Penning traps. This device sets up boundary conditions that produce a controllable radiation field within the trap cavity. Spontaneous emission is strongly reduced at the same time as corresponding shifts of the electron's oscillation frequencies are avoided.

The most recent Harvard measurement of a_e with cylindrical Penning trap achieved a 0.24 ppb accuracy, *i.e.* a factor of four larger than the precision needed to see the new physics responsible for the muon $g-2$ in the occurrence of naive scaling. This measurement (a_e^{exp}) would already be able to constrain specific models that break NS and enhance the new physics contributions in a_e with respect to a_μ. However, the role of a_e as a probe for NF becomes quite marginal once we remove the correlation between a_e^{exp}, which is commonly used to extract α, and its theory expectation a_e^{SM} that strongly depends on α (a_e^{SM} is $\alpha/2\pi$ at leading order). If we use a fully independent determination of α based on cold atom techniques, the accuracy on the theory prediction for a_e (a_e^{SM}) grows up to 0.66 ppb.

This considerations strengthened the standard approach of exploiting the electron $g-2$ for the determination of α and resorting to the muon (or tau) $g-2$ to seek for NP effects in loop contributions.

3. – A new role for the electron $g-2$

In the next few years the role of a_e will likely change, moving from the key measurement for the determination of α to a powerful observable to seek for physics beyond the standard model [4] (as it is the case now for a_μ and several classes of rare decays). This is mostly due to three reasons that are briefly recalled in the following.

– *Experimental determination of* a_e. Cylindrical Penning traps have not reached their limiting systematics, yet and additional improvements can be envisaged. The original 2006 Harvard measurement [7] was mostly dominated by cavity shift modeling. In cylindrical Penning traps the interaction of the trapped electron and the cavity modes shifts the cyclotron frequency and the shift has to be properly modeled to extract a_e. This shift cannot be avoided if we want to reduce spontaneous emission of synchrotron radiation. Cavity shift is a source of systematics that is intrinsic to the technology of cylindrical Penning traps. It can, however, be reduced below 0.08 ppb for a_e (< 0.1 ppt for $g/2$).

– *Independent determination of* α. The outstanding precision reached in the measurement of the Rydberg constant offers a different venue to determine α. This new approach (see below) neither depends on the measurement of a_e nor on the high-order perturbative calculation of QED corrections.

– *Theory uncertainty on* a_e. The uncertainty in the theoretical determination of a_e^{SM} is appropriate to check the a_μ anomaly at NS level. The uncertainty is mostly due to the numerical approximations employed for the evaluation of four and five loop QED contributions (0.06 ppb). It is worth mentioning that, unlike a_μ, the hadronic term uncertainty (0.02 ppb) is negligible if we aim at observing new physics in the NS scenario. Even now the overall theory uncertainty is within the error budget for NS (0.06 ppb) and further improvements are possible [3].

4. – An independent determination of α

Having a ppb measurement of α independent of a_e is a realistic goal thanks to the improved measurement of the Rydberg constant. The optical comb generators allowed the measurement of the narrow (1.3 Hz) 1S-2S two-photon resonant line of the hydrogen with a relative precision of 3.4×10^{-13} [8]. It corresponded to an improvement of two orders of magnitude with respect to previous measurements. The new data on hydrogen spectroscopy resulted in a measurement of the Rydberg constant with a precision better than 0.01 ppb (7×10^{-12} [6]). Since $R_\infty = m_e\alpha^2 c/2h$, the outstanding precision on R_∞ connects α to the evaluation of the quotient h/m_e. For a given atom X,

$$\alpha^2 = \frac{2R_\infty}{c}\frac{m_X}{m_e}\frac{h}{m_X} = 2\frac{R_\infty}{c}\frac{m_X}{m_u}\frac{m_u}{m_e}\frac{h}{m_X}\,. \tag{2}$$

Equation (2) opened many opportunities for an independent determination of α based on cold atom interferometers, which are particularly well suited to determine the h/M_X ratio. The latter is also interesting to metrologists for the redefinition of the kilogram. On the other hand, using eq. (2) to extract α pays the penalty of two new systematic sources: the systematics due to the knowledge of the mass (M_X/m_u) of the atom employed to measure h/M_X and the uncertainty on the electron mass expressed in atomic mass unit([1]).

5. – The nuisance parameters M_X/m_u and $A(m_e)$

Atom interferometers (see below) exploit mostly hydrogen-like atoms, *i.e.* atoms with only one electron in partially filled shells. Alkali metals thus play a special role among possible atom candidates. For alkali metals, the atomic masses of the isotopes relevant for the determination of α have been measured with high precision employing orthogonally compensated Penning traps. The most recent results [9] from Washington Univ. and Florida State University significantly improve the AME2012 evaluation for several atomic masses, including ^{23}Na, ^{39,41}K, ^{85,87}Rb and ^{133}Cs. The most plausible candidates for atom interferometers are measured with a typical uncertainty of 0.1 ppb. This accuracy is still too low for new physics tests in the NS framework. Fortunately, reaching precisions well below 0.05 ppb for candidates as Rb, Cs or non-alkali atoms like Sr is possible improving the experiments that are currently operating [10] (double Penning traps with multiply-charged ions) and we do not expect the uncertainty on M_X/m_u to dominate the knowledge of α in the years to come. ^{4}He is worth a special mention since its atomic mass is known with outstanding accuracy (0.016 ppb). ^{4}He is not hydrogen-like but its metastable state is often employed in atom interferometers, making ^{4}He* a specially suitable candidate for an α-oriented measurement of $h/M_{^4\mathrm{He}}$.

Until 2014, the most important nuisance parameter of eq. (2) was the limited knowledge of the electron mass expressed in atomic mass unit. According to CODATA2010 [6], the relative atomic mass of the electron $A_r(m_e) \equiv m_e/m_u$ is

$$A_r(m_e) = 5.4857990946(22) \times 10^{-4} \quad (0.4\,\mathrm{ppb}). \tag{3}$$

This estimate is not the outcome of a direct measurement of electrons in Penning traps compared with the reference atom ^{12}C because direct measurements do not reach the $\mathcal{O}(1\,\mathrm{ppb})$ precision. The estimate of eq. (3) is obtained from a_e in bound-state QED. For a bound electron in hydrogen-like systems of nuclear charge Z, the electron anomalous magnetic moment g_b is different with respect to the free-particle value. At tree level, $g_b = g_b^{Breit} \equiv 2/3(1 + 2\sqrt{1 - Z^2\alpha^2})$. The measurement of $a_e^b \equiv (g_b - g_b^{Breit})/g_b^{Breit}$ done in Penning traps is not competitive with a_e (1.9 ppm versus 0.24 ppb) but can be used to evaluate $A_r(m_e)$. The most relevant drawback is that this determination heavily

([1]) Here, m_u is the mass of 1/12 the mass of ^{12}C.

relies on theory calculations in bound-QED. However, calculations have been checked against several atoms and these checks strengthen our confidence on theory modeling. Until 2014, bound QED provided $A_r(m_e)$ with an accuracy of 0.4 ppb: this uncertainty propagates to m_e/m_{Rb} bringing the error on this ratio to 0.44 ppb. A much improved accuracy (a factor of 13 better) has been obtained in 2014: details and implications for NP searches can be found in [11] and [4], respectively.

6. – Atom interferometers

Atom interferometers were first demonstrated in early 90's and in the last 20 years advances in atom interferometry led to the development of several innovative techniques for fundamental physics experiments and for applications (see [12] for an overview). In particular, atom interferometers are currently employed for precision measurements of gravity acceleration, rotations, for the determination of the Newtonian constant G, for testing general relativity and quantum gravity models, and for possible applications in geophysics.

The above-mentioned quotient h/M_X results from the measurement of the recoil velocity v_r of an atom X when it absorbs a photon of frequency ν ($v_r = h\nu/M$). The measurement is performed by combining a Ramsey-Bordé atom interferometer with the Bloch oscillations technique. Bloch oscillations can be observed in atomic systems when the atoms are shed with two counterpropagating laser beams whose frequency difference is swept linearly. This setup produce oscillations that resemble the oscillations experienced by an electron inside a solid in the presence of an electric field ("Bloch oscillations"). During the frequency sweep, the internal state of the atom is unchanged, the atom is accelerated and its velocity increases by $2v_r$ per oscillation. The Doppler shift due to this velocity variation is compensated by the frequency sweep itself. This technique is extremely effective in terms of photon momentum transfer and it has soon become the reference method for the measurement of h/M_X.

The most relevant atom candidates and the perspective for improvement of atom interferometers for the measurement of h/M_X are discussed in [4] and briefly summarized in the following.

Cesium: The exploitation of eq. (2) to link α with the quotient h/M_X has been introduced by the group of S. Chu at Stanford University and implemented using ^{133}Cs [13]. The value of α was measured with a precision $\sigma_\alpha/\alpha = 7.4 \times 10^{-9}$. The most recent experiments achieved a relative uncertainty for α of ~ 2 ppb [14], mainly limited by the statistical error. Implementation of the Bloch oscillation technique in this scheme is expected to reduce the overall uncertainty well below ppb [15].

Rubidium: Measurement based on rubidium and performed in Paris currently provide the best measurement of h/M_X. These experiments [16] exploit atom interferometers based on a combination of a Ramsey-Bordé interferometer with Bloch oscillations. The most precise value obtained so far by atom interferometers is for ^{87}Rb [16] and it corresponds to $h/M_{Rb} = 4.5913592729(57) \times 10^{-9}\,\mathrm{m^2 s^{-1}}$ (1.24 ppb in h/M, *i.e.* 0.62 ppb in α). The precision in [16] is mostly limited by laser beams alignment, wave front curvature and Gouy phase effects and the current upgrades are aimed to a precision in α of about 0.1 ppb.

Strontium: Sr-based atom interferometers with long-lived Bloch oscillations have already been developed and they are currently employed for gravity measurements at small spatial scales [17,18]. These interferometers can also provide a measurement of the h/M_{Sr} ratio with systematics and potential upgrades comparable to Rb [19].

Helium: Helium is not hydrogen-like but can be successfully used for atom interferometry thanks to its long-lived metastable states. An experiment on He was started in Amsterdam using ^{4}He in a 1D-lattice to perform Bloch oscillations and velocity measurement with an atom interferometer [20]. In principle, helium can achieve an accuracy comparable (or better than) Rb and, as noted above, the helium mass is known with outstanding precision (relative uncertainty of 0.015 ppb).

7. – Conclusions

For many decades, the electron $g-2$ has been considered an ideal tool for QED tests and for metrology, with very limited sensitivity to new physics in loop corrections. The remarkable progress in the technology of the cylindrical Penning traps, in the determination of the Rydberg constant and in the knowledge of the electron mass opens new venues in this field. In particular, the accuracy that can be reached in a_e^{exp} and α is such that NP effects that might cause the $g-2$ muon discrepancy will be evident even in the occurrence of naive scaling.

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