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# Neutrino mixing: A theoretical overview

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**Summary.** — We present a concise review of the recent idea formulated to explain the values of the masses and mixing of neutrinos. Models based on discrete non-Abelian flavour groups are compared to the ones bases on the simpler $U(1)$ mechanism, showing that the description of the neutrino data is still possible for both approaches.

PACS 11.15.-q – Gauge field theories.
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## 1. – Description

The main relevant developments on neutrino mixing involved the results on $\theta_{13}$ from T2K [1], Minos [2], Double Chooz [3], Reno [4] and Daya-Bay [5]. Recent global fits to the data on oscillation parameters [6-8], a couple of them summarized in table I, show that the combined value of $\sin^2\theta_{13}$ is about $10\,\sigma$ away from zero and that its central value is rather large, close to the previous upper bound. There are also solid indications of a deviation of $\theta_{23}$ from the maximal value, probably in the first octant [6] and, thanks to the combined T2K and Daya-Bay data, a tenuous hints for non-zero $\delta_{CP}$ is starting to appear from the data. Looking at the results of table I, one is still tempted to recognize some special mixing patterns as good first approximations to describe the data, the most famous ones being the Tri-Bimaximal (TB [9]), the Golden Ratio (GR [10]) and the Bi-Maximal (BM) mixing. The corresponding mixing matrices all have

$$\sin^2\theta_{23} = 1/2, \qquad \sin^2\theta_{13} = 0 \tag{1}$$

and differ by the value of the solar angle $\sin^2\theta_{12}$, which is

$$\sin^2\theta_{12} = 1/3 \text{ for TB}, \quad \sin^2\theta_{12} = \frac{2}{5+\sqrt{5}} \sim 0.276 \text{ for GR}, \quad \sin^2\theta_{12} = \frac{1}{2} \text{ for BM}. \tag{2}$$

 

TABLE I. – *Fits to neutrino oscillation data. For* $\sin^2\theta_{23}$ *from ref.* [7] *only the absolute minimum in the first octant is shown.*

| Quantity | ref. [6] | ref. [7] |
|---|---|---|
| $\Delta m^2_{sun}$ ($10^{-5}\,\mathrm{eV}^2$) | $7.54^{+0.26}_{-0.22}$ | $7.45^{+0.19}_{-0.16}$ |
| $\Delta m^2_{atm}$ ($10^{-3}\,\mathrm{eV}^2$) | $2.43^{+0.06}_{-0.10}$ | $2.417 \pm 0.014$ |
| $\sin^2\theta_{12}$ | $0.307^{+0.018}_{-0.016}$ | $0.306 \pm 0.012$ |
| $\sin^2\theta_{23}$ | $0.386^{+0.024}_{-0.021}$ | $0.446 \pm 0.008$ |
| $\sin^2\theta_{13}$ | $0.0241 \pm 0.025$ | $0.0231 \pm 0.0019$ |

Being a leading-order approximations (LO), all previous patterns need corrections (for example, from the diagonalization of charged leptons) to describe the current mixing angles. In particular, the relatively large value of the reactor angle requires sizable corrections of the order of the Cabibbo angle $\lambda_C$, for all three patterns; on the other hand, the deviations from the LO values of $\sin^2\theta_{12}$ must be small enough in the TB and GR cases but as large as $\lambda_C$ for the BM pattern. Finally, corrections not too much larger than $\lambda_C^2$ can be tolerated by $\sin^2\theta_{23}$. Since the corresponding mixing matrices have the form of rotations with special angles, discrete flavour groups naturally emerge as good candidates. The most studied groups have been the permutation groups of four object, $S_4$ and $A_4$, see ref. [11] for an exhaustive review. The important point for model building is that these symmetries must be broken by suitable scalar fields $\varphi$ that take a vacuum expectation value (vev) at large scale $\Lambda$, so they generally provide a new adimensional parameter $\xi = \langle\varphi\rangle/\Lambda$. The breaking must preserve different subgroups in the charged lepton and neutrino sectors, otherwise the neutrino mixing matrix would be the identity matrix and no mixing will be generated. The desired directions in flavor space are generally difficult to achieve, so consistent models are those where the vevs of the scalar fields can be naturally obtained from the minimization of the scalar potential.

In a tipical $A_4$ model for TB mixing [12], the LO neutrino mass matrix reads:

$$m = \begin{pmatrix} x & y & y \\ y & x+v & y-v \\ y & y-v & x+v \end{pmatrix}, \tag{3}$$

where $x$, $y$ and $z$ are generic complex parameters, and leads to exact TB mixing, with no free parameters in the neutrino mixing matrix. Having specified the field content of the theory and the assignment of matter and scalar fields to suitable irreducible representations of the discrete non-abelian group, the non-leading corrections to TB mixing arise from a number of higher-dimensional effective operators (with unknown coefficients) and, in the absence of specific dynamical tricks, all three mixing angles receive corrections of the same order of magnitude. Indicating with $c_{12}^{\nu,e}$, $c_{13}^{\nu,e}$ and $c_{23}^{\nu,e}$ the entries of the unitary matrices that diagonalize the charged and neutrino mass matrices, the NLO expressions

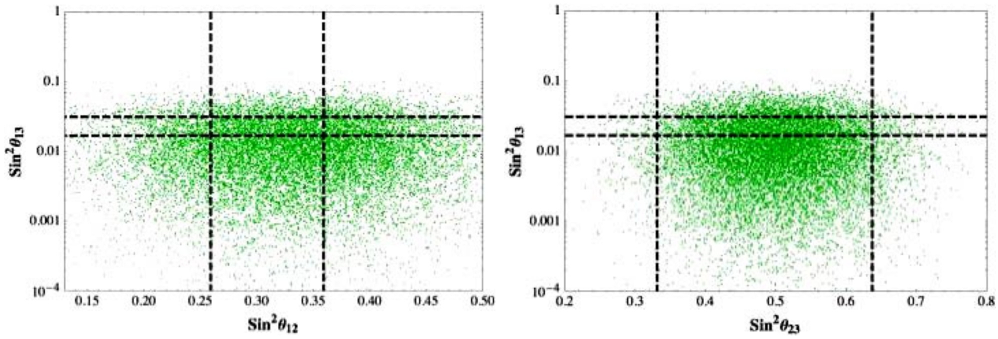


Fig. 1. – On the left (right), correlation between $\sin^2\theta_{13}$ and $\sin^2\theta_{12}$ ($\sin^2\theta_{23}$), following eq. (4). The dashed-black lines represent the $3\sigma$ values for the mixing angles from the Fogli *et al.* fit [6]. Only the NH data sets is shown. Figure from ref. [13].

of the mixing angles read [13]

$$
\begin{aligned}
\sin^2\theta_{23} &= \frac{1}{2} + \mathcal{R}e(c^e_{23})\xi + \frac{1}{\sqrt{3}}\left(\mathcal{R}e(c^\nu_{13}) - \sqrt{2}\mathcal{R}e(c^\nu_{23})\right)\xi,\\
\sin^2\theta_{12} &= \frac{1}{3} - \frac{2}{3}\mathcal{R}e(c^e_{12}+c^e_{13})\xi + \frac{2\sqrt{2}}{3}\mathcal{R}e(c^\nu_{12})\xi,\\
\sin\theta_{13} &= \frac{1}{6}\left|3\sqrt{2}\,(c^e_{12}-c^e_{13}) + 2\sqrt{3}\left(\sqrt{2}c^\nu_{13}+c^\nu_{23}\right)\right|\xi,
\end{aligned}
\tag{4}
$$

where $c^{\nu,e}_{ij}$ are unspecified complex parameters of order one in absolute value. The previous equations, depending on the same parameters, indicate that a form of correlation should be present when the mixing angles are computed at the NLO. This has been numerically proved in [13], where the $c^{e,\nu}_{ij}$ parameters are treated as random complex numbers with absolute values following a Gaussian distribution around 1 with variance 0.5; the value of $\xi$ has been kept fixed to $\xi = 0.076$, which maximizes the success rate to reproduce all the three mixing angles inside the corresponding $3\sigma$ ranges. The results are reported in fig. 1, where the plot on the left (right) shows the correlation between $\sin^2\theta_{13}$ and $\sin^2\theta_{12}$ ($\sin^2\theta_{23}$) implied by eq. (4) for the normal ordering only.

These results can be easily explained: in fact, $\sin^2\theta_{13}$ and $\sin^2\theta_{12} - 1/3$ increase with $\xi$, so the requirement of having a reactor angle inside its $3\sigma$ error range forces the solar angle to spans all the $3\sigma$ experimental error bar.

Since the corrections of $\theta_{13}$ are the most difficult ones to achieve, one can try a different approach, in which the corrections to the charged lepton and the neutrino sectors are kept separated not only at LO but also at NLO. An explicit realization of this idea was provided in [14], where the contribution from the diagonalization of the charged leptons is expected to be of $\mathcal{O}(\lambda_C^2)$ and to correct the TB prediction for the solar angle, while those in the neutrino sector make $\theta_{13} \sim \mathcal{O}(\lambda_C)$. For the atmospheric angle, the relation $\sin^2\theta_{23} = 1/2 + 1/\sqrt{2}\cos\delta_{CP}|\sin\theta_{13}|$ holds, with $\delta_{CP}$ being the CKM-like $CP$-violating phase of the lepton sector. If the (weak) indication for a non-vanishing CP phase around $\delta_{CP} \sim 3\pi/2$ will be confirmed by future neutrino experiments [7], sizable deviations of $\sin^2\theta_{13} - 1/2$ from zero will invalidate the previous construction.

Beside the models based on TB, one can consider models where BM mixing holds in the neutrino sector at LO and the relatively large corrective terms for $\theta_{12}$ and $\theta_{13}$, of

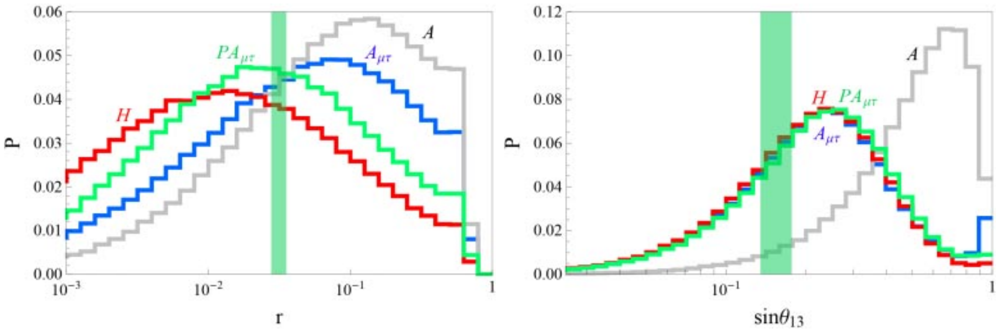


Fig. 2. – Probability distributions of $r$ (left panel) and $\sin\theta_{13}$ (right panel) for the models A and H, in the see-saw case. The modulus (argument) of the complex random coefficients has been generated in the interval $[0.5, 2]$ ($[0, 2\pi]$) with a flat distribution. For $A$, $\lambda = 0.2$ whereas for $H$, $\lambda = 0.4$. The shaded vertical band emphasizes the experimental $2\sigma$ window according to [6]. Figures taken from ref. [19].

$\mathcal{O}(\lambda_C)$, arise from the diagonalization of charged lepton masses; the atmospheric angle, however, should deviate from maximal mixing by quantities not much larger than $\mathcal{O}(\lambda_C^2)$. Explicit models of this type based on the group $S_4$ have been developed in ref. [15].

The relatively large value of $\theta_{13}$ and the fact that $\theta_{23}$ is not maximal both point to the direction of models based on Anarchy [16, 17], that is the assumption that no special symmetry is needed in the leptonic sector, and that the values of neutrino masses and mixing are reproduced by chance. Anarchy can be formulated in a $U(1)$ context a la Froggatt-Nielsen [18]: a mass term is allowed at the renormalizable level only if the $U(1)_{FN}$ charges add to zero. Breaking the $U(1)_{FN}$ symmetry spontaneously by the vevs $v_f$ of a number of flavon fields with non-vanishing charge allows to rescue the forbidden vertex, although suppressed by powers of the small parameters $\lambda = v_f/M$, with $M$ a large mass scale. Since these invariant mass terms appear with arbitrary coefficients of order 1, typically the number of parameters exceeds the number of observable quantities and make this kind of model less predictive that the ones based on non-abelian discrete symmetries.

Opposite to Anarchy, generic $U(1)$ models are characterized by well-defined hierarchies of the neutrino mass matrix elements and are ofter referred to as hierachiacal models. The authors of ref. [19] have performed an updated analysis of the performance of anarchical versus hierarchical models in the $SU(5)\otimes U(1)_{FN}$ context, which also allows to implement a parallel treatment of quarks and leptons. Among the different charge assignments of the 10, $\bar{5}$ and the $SU(5)$ singlet, we focus here on two different realizations:

– Anarchy (A): $10 = (3, 2, 0)$, $\bar{5} = (0, 0, 0)$ $1 = (0, 0, 0)$

$$m_\ell = \begin{pmatrix} \lambda^3 & \lambda^3 & \lambda^3 \\ \lambda^2 & \lambda^2 & \lambda^2 \\ 1 & 1 & 1 \end{pmatrix}, \qquad m_\nu = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}. \tag{5}$$

– Hierarchy (H): $10 = (5, 3, 0)$, $\bar{5} = (2, 1, 0)$ $1 = (2, 1, 0)$

$$m_\ell = \begin{pmatrix} \lambda^7 & \lambda^6 & \lambda^5 \\ \lambda^5 & \lambda^4 & \lambda^3 \\ \lambda^2 & \lambda & 1 \end{pmatrix}, \qquad m_\nu = \begin{pmatrix} \lambda^4 & \lambda^3 & \lambda^2 \\ \lambda^3 & \lambda^2 & \lambda \\ \lambda^2 & \lambda & 1 \end{pmatrix}. \tag{6}$$

The values of the neutrino observables are computed extracting the modulus (argument) of the complex random coefficients in the interval $[0.5, 2]$ ($[0, 2\pi]$) with a flat distribution. Notice that in order to ensure a reasonable hierarchy for charged fermions, the values of $\lambda$ are different in the two cases: $\lambda = 0.2$ for A, $\lambda = 0.4$ for H. In fig. 2 the comparison among A and H (and other models not interesting for this talk) to reproduce the values of the small ratio $r = \Delta m^2_{sun}/\Delta m^2_{atm}$ and $\sin^2\theta_{13}$ is shown. Since the problem with Anarchy is that all mixing angles should be large and of the same order of magnitude, it is quite difficult to reproduce $\theta_{13} \sim \mathcal{O}(\lambda_C)$. In addition, the smallness of $r$ is not easily reproduced, being generically one order of magnitude larger than expected. On the other hand, in the $H$ model one can reproduce the correct size for $r$ and $\sin^2\theta_{13}$, thus making this option preferable over Anarchy.

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