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Strategies towards a precise measurement of the top-quark mass

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Summary. — In this talk I discuss the importance of precise top-quark mass determinations. Some conceptual aspects of top-quark mass measurements as well as methods used/proposed in the past are presented. In addition an entirely new method, which has been proposed very recently by S. Alioli *et al.*, is reviewed.

PACS 14.65.Ha – top quarks.
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1. – Introduction

The top quark is the heaviest elementary fermion discovered so far. In the Standard Model (SM) its mass is generated by the Higgs mechanism. While convincing evidence for the existence of a Higgs boson has been experimentally established recently, the Higgs mechanism to generate fermion masses is not yet experimentally confirmed. A precise determination of the top-quark mass and an independent measurement of the top-quark Yukawa coupling will shed light on the mechanism generating the fermion masses in the Standard Model. Furthermore the top-quark mass is a crucial input for consistency checks of the Standard Model or possible supersymmetric extensions. In the SM the Higgs boson mass, the top-quark mass and the W-boson mass are not independent of each other. A precise measurement of the three masses thus provides an important test of the underlying theory. This is illustrated in fig. 1 where for a given range of the Higgs mass the mass of the W-boson is plotted as a function of the top-quark mass. In addition the current measurements of the W-mass and the top-quark mass are shown as grey ellipse. Given the present measurements the SM as well as its supersymmetric extension are compatible with the data. From fig. 1 it is obvious that a significant reduction of the uncertainty in the W-boson mass would provide a major step forward to discriminate between the SM and possible extensions. Since the predictions are rather flat as a function of the top-quark mass the naive expectation is, that a reduction in the top-quark mass uncertainty would require at the same time a significant reduction in the W-boson mass. However, it is conceivable that together with a reduced uncertainty in the top-quark mass also the central value may shift to larger or smaller values. A shift of about minus

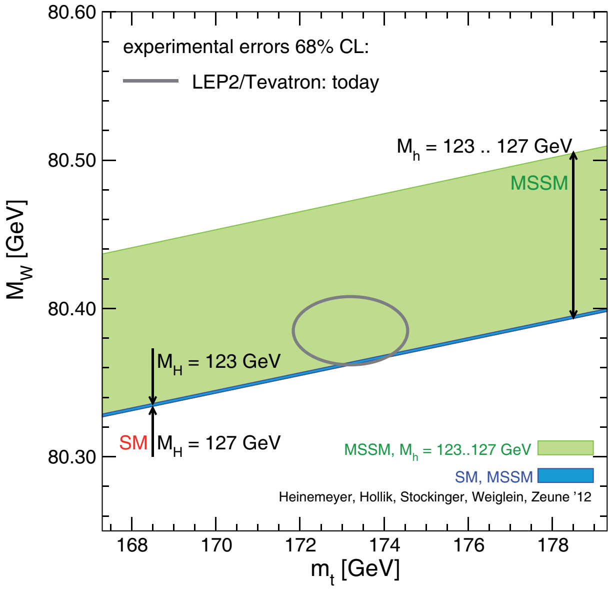

Fig. 1. – Standard model prediction for the mass of W-boson as function of the top-quark mass. (Plot prepared by S. Heinemeyer, G. Weiglein and L. Zeune, see also ref. [1].)

one GeV—which would be perfectly consistent with the present measurements—could result for example in tensions with the SM.

Assuming the SM as *the* theory of elementary particle physics one may also ask about the consequences of the measured parameters on the evolution of the universe. To study this question the stability of the vacuum has been investigated as a function of the Higgs mass and the top-quark mass (see for example [2-6] and the talk by J. R. Espinosa at this conference). Evolving the Standard Model parameters up to the Planck scale, the parameter space can be sub divided into regions where the vacuum is stable, metastable or unstable. The present values for the Higgs mass and the top-quark mass are in the metastable region however very close to the boarder to the stable region. A more precise measurement of the top-quark mass could thus confirm the metastability or shift the prediction to the stable region. The aforementioned examples arguing for a more precise top-quark mass measurement may appear of only academic interest. However the top-quark mass enters also in many predictions we are currently using to confront measurements with theory. A prominent example is the total cross section for top-quark pair production in hadronic collisions. After tremendous work over the past 25 years the accuracy of the theoretical predictions excluding the uncertainties of the input parameters have been reduced below 5% [7-10] (see also A. Mitov's talk at this conference). However a one per cent uncertainty in the top-quark mass leads roughly to a five per cent uncertainty of the total cross section. This is precisely the accuracy which will be achieved by the LHC measurements. To make optimal use of the available data it is thus important to reduce the uncertainties of the top-quark mass as far as possible.

This article is organised as follows. In the next section a few theoretical aspects relevant for a precise mass measurement are summarised. We will also comment briefly on existing measurements. In the next section alternative methods are presented and discussed in detail. In the last section we will present our conclusions.

2. – Preliminaries

As a matter of fact, we do not observe free quarks in nature. Our understanding is, that the coloured quarks are bound due to confinement into colourless hadrons. However, since the mechanism which binds quarks and gluons into hadrons involves non-perturbative physics, a precise theoretical understanding of confinement is difficult and still lacking. A related question is, what precisely do we mean, when we talk about a quark mass? Form a formal point of view, a quark mass is just a parameter of QCD (or in more general the SM) which we believe to be the theory of strong interactions. We determine the parameters of the underlying theory by a detailed comparison of theoretical predictions with the measurements. Evidently, the theoretical predictions used in this comparison should be as precise as possible, to allow a consistent description of the data by fitting the parameters of the underlying theory. Large unknown corrections of the theoretical predictions would otherwise be compensated by a shift of the measured parameters with respect to their nominal values. In QCD this means that in most cases at least next-to-leading order (NLO) corrections need to be taken into account. The requirement to include higher-order corrections is also obvious from a different argument: In principle there is some freedom how precisely the theory parameters are connected to the measurements. In theoretical predictions this freedom is reflected in the choice of a specific renormalisation scheme. In the case of quark masses commonly used schemes are the so-called on-shell scheme/pole mass scheme and the minimal subtraction scheme. In the former the mass is defined as the location of the pole of the "full" quark propagator while in the later the scheme is artificially defined by the requirement that the UV renormalisation is minimal—the renormalisation constants just cancel the ultraviolet (UV) singularities in the transition from the UV divergent quantities before renormalisation to the UV finite quantities after renormalisation. The renormalisation procedure which appears first in next-to-leading order is thus crucial for the precise definition of the parameters. In a leading-order calculation different renormalisation schemes cannot be distinguished. This becomes immediately clear when we study the perturbative relation between parameters defined in different renormalistion schemes. For example the relation between the $\overline{\mathrm{MS}}$ mass $\overline{m}_t(\mu)$ and the pole mass m_t reads

$$m_t = \overline{m}_t(\mu)\left\{1+\frac{\alpha_s}{\pi}C_F\left[1-\frac{3}{4}\ln\left(\frac{\overline{m}_t(\mu)^2}{\mu^2}\right)\right]\right\}, \tag{1}$$

where α_s denotes the coupling constant of the strong interaction, $C_F = \frac{4}{3}$ for $SU(3)$ and μ is the renormalisation scale. Since the difference between m_t and $\overline{m}_t(\mu)$ is formally of order α_s, at least a NLO calculation is required to distinguish the two schemes. If we would be able to compute the observables to all orders in the coupling constants different schemes are equivalent. At finite order however different renormalisation schemes may differ in the size of higher order corrections. It is thus preferred to use a scheme where uncalculated higher order corrections are expected to be small. In specific circumstance for example some logarithmic corrections may be resummed to all orders by using the $\overline{\mathrm{MS}}$ mass, which would thus lead to an improved behaviour of the perturbative expansion. It is also well known that the on-shell mass suffers from renormalon contributions leading to an intrinsic uncertainty of the order of Λ_{QCD} [11,12]. From what has been discussed above we conclude, that precise measurements can be expected if 1) at least NLO predictions are available, 2) theoretical predictions have small uncertainties, 3) the observable shows a good sensitivity to the top-quark mass. The most precise measurements are currently obtained using either the template method or the matrix element method [13-16]. At present mostly leading order predictions in combination with Monte Carlo simulations are used. It is questionable whether this is sufficient to reach an accuracy at the sub-percent level. In principle both methods may be improved. In case of the template method one could generate the templates with next-to-leading order Monte Carlo programs including shower predictions like for example MC@NLO or POWHEG. It could also be useful to include additional distributions in the analysis on top of what is currently done, since the invariant mass distribution is intrinsically difficult to describe with high accuracy due non-perturbative effects like for example colour reconnection. In case of the matrix-element method the extension to NLO is currently investigated [17, 18].

3. – Alternative methods

3·1. *The top-quark mass from the total cross section.* – As explained in the previous section any observable can be in principle used to determine the top-quark mass as long as precise theoretical predictions are available. Given the theoretical progress achieved in the recent past in the prediction of the total top-quark pair cross section $\sigma_{t\bar{t}+X}$, (for an overview see again A.Mitovs talk at this conference) the total cross section seems particular well suited. Since higher order corrections are included, the renormalisation scheme is unambiguously fixed. Recently, approximate next-to-next-to-leading order predictions have been formulated using the running mass $\overline{m}_t(\mu)$ [19]. In ref. [19] these results have been used to determine directly the running top-quark mass. A more sophisticated analysis has been presented later by the experimental collaborations. Given however the weak sensitivity of the cross section with respect to the top-quark mass, the achievable precision is limited: A one per cent variation of the top-quark mass leads only to a five per cent variation of the cross section. A determination of the top-quark mass with an accuracy below one per cent would thus require to measure the total cross section with an accuracy significantly below five per cent. Despite these limitations the method is nevertheless interesting since in principle different renormalisation schemes can be used and the consistency can be checked.

3·2. *The top-quark mass from* $M_{B\ell}$. – In ref. [20] the possibility to use the decay $t \to Wb \to \ell\nu J/\psi X$ to measure the top-quark mass has been investigated (see also refs. [21-24]). More precisely, the dependence of the decay on $M_{J/\psi\ell}$ has been used. A clean event sample is assured by requiring that the J/ψ from the b decay as well as the W-boson decay leptonically. The second top in the $t\bar{t}$ sample is assumed to decay purely hadronically. Evidently, the very specific final state leads to significant reduction of the event sample. About 100/fb of integrated luminosity is required to compensate the rareness of the final state. This is the reason why this method has not been used so far in practice. However, Monte Carlo studies [20-24] show, that an accuracy of the measured top-quark mass at the level of 1 GeV might be feasible. In a recent work [25] it has been questioned whether the accuracy of the Monte Carlo tools—usually considered to be accurate only at the level of a few per cent—are precise enough to support this claim—in particular since no NLO corrections are taken into account. To assess this question the authors of ref. [25] studied the $M_{B\ell}$ distribution where B denotes the B-meson taking into account the NLO corrections in the decay. The outcome is, that in principle the Monte Carlo tools give a realistic description of the decay and of the possible accuracy of the top-quark mass determination achievable in the measurement. The NLO corrections lead to a reduced uncertainty of the theoretical predictions. In addition depending on

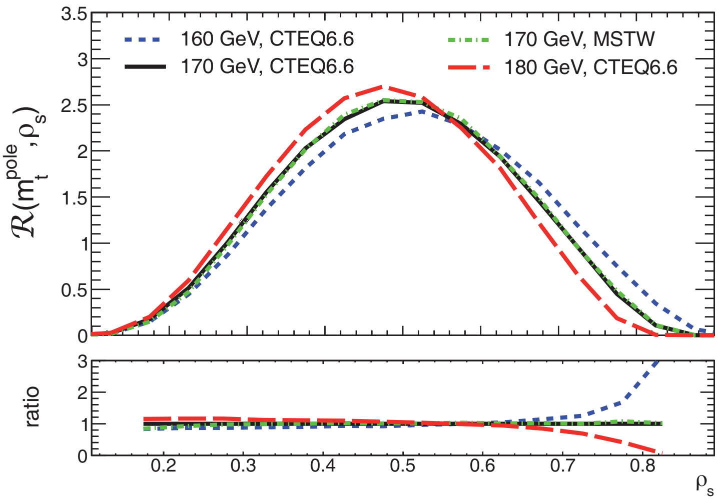

Fig. 2. – $\mathcal{R}(m_t, \rho_s)$ calculated at NLO accuracy for different masses $m_t = 160$, 170 and 180 GeV. For $m_t = 170$ GeV also the scale and PDF uncertainties are shown. The ratio with respect to the result for $m_t = 170$ GeV is shown in the lower plot [26].

the precise definition of the observable they lead to a shift of the predictions which needs to be taken into account in future measurement. These investigations clearly proof the high potential of the method, however, they also show that more studies are required before an uncertainty below 1 GeV can be claimed in a future measurement.

3·3. *The top-quark mass from Jetrates.* – In ref. [26] a new method using jet rates has been presented. The basic idea is very similar to the measurement of the b-quark mass at LEP [27-29]. More precisely the cross section for the production of a top-quark pair in association with an additional jet is studied. Theoretically this process is very well studied. In refs. [30,31] the NLO QCD corrections have been calculated. The "inclusive" jet cross section $\sigma_{t\bar{t}+1\text{-Jet}+X}$ receives only small corrections in NLO. In ref. [32] the NLO corrections have been included in the POWHEG framework allowing a consistent matching of the fixed order NLO calculation with the parton shower. Investigating the mass sensitivity of $\sigma_{t\bar{t}+1\text{-Jet}+X}$ it turns out, that a similar behaviour as for the total cross section $\sigma_{t\bar{t}+X}$ is observed. To enhance the mass sensitivity one needs to focus on kinematical regions where large effects can be expected. Defining $\rho_s = m_0/\sqrt{s_{t\bar{t}j}}$ where m_0 denotes an arbitrary mass scale of the order of the top-quark mass and $s_{t\bar{t}j}$ is the invariant mass of the final state a particular useful distribution to study is defined by

$$\mathcal{R}(m_t, \rho_s) = \frac{1}{\sigma_{t\bar{t}+1\text{-Jet}+X}} \frac{\mathrm{d}\sigma_{t\bar{t}+1\text{-Jet}+X}}{\mathrm{d}\rho_s}(m_t, \rho_s). \tag{2}$$

In fig. 2 the quantity $\mathcal{R}$ is shown for a top-quark mass of 160, 170 and 180 GeV. In the threshold region ($\rho_s \approx 1$) an increased top-quark mass leads to a suppression of $\mathcal{R}$. Since $\mathcal{R}$ is normalised the opposite is true in the high energy region ($\rho_s \approx 0$). At $\rho_s \approx 0.55$ the different curves cross and the sensitivity to the top-quark mass is lossed. In fig. 3 the

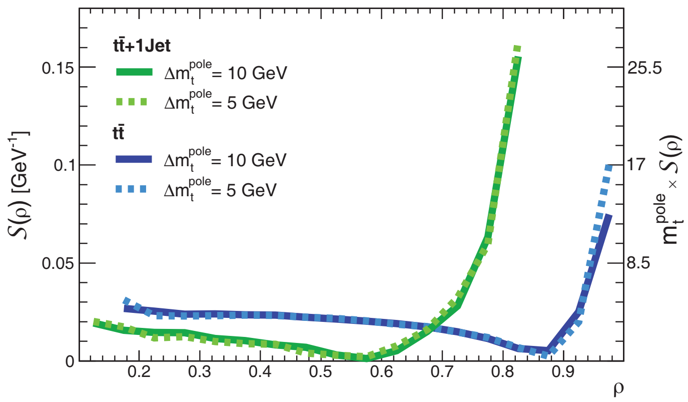

Fig. 3. – The sensitivity $\mathcal{S}(\rho_s)$ of $\mathcal{R}$ with respect to the top-quark mass as defined in eq. (3) [26].

sensitivity $\mathcal{S}$ defined by

$$\mathcal{S}(\rho_s) = \sum_{\Delta=\pm 5(10)\,\mathrm{GeV}} \frac{|\mathcal{R}(170\,\mathrm{GeV}, \rho_s) - \mathcal{R}(170\,\mathrm{GeV} + \Delta, \rho_s)|}{2|\Delta|\mathcal{R}(170\,\mathrm{GeV}, \rho_s)} \tag{3}$$

is shown. Compared to the total cross section an increased sensitivity is observed. For example for $\rho_s \approx 0.8$ a one per cent variation of the top-quark mass leads roughly to a 17% variation of $\mathcal{R}$. Since $\mathcal{R}$ is defined as a ratio, one can expect that many theoretical and experimental uncertainties cancel. Using the sensitivity shown in fig. 3 it is possible to translate experimental or theoretical uncertainties into an uncertainty of the extracted top-quark mass. For PDF and scale uncertainties this is shown in fig. 4. Restricting the analysis to $\rho_s > 0.7$ the impact of the PDF uncertainties is below 100 MeV. The impact of the scale variation is slightly larger but still below 500 MeV.

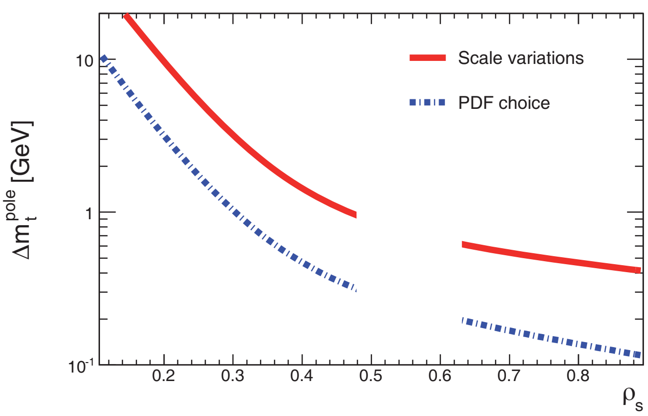

Fig. 4. – Expected impact on the top-quark mass measurement due to scale (solid line) and PDF (dashed line) uncertainties [26].

In ref. [26] a variety of different possible uncertainties have been investigated such as colour reconnection, uncertainties of the jet energy scale and uncertainties due to the misinterpretation of the reconstructed jets. The largest uncertainty originates from an imperfect determination of the jet energy scale (JES). Assuming an accuracy of $\pm 3\%$ leads to an uncertainty of about 0.8–1 GeV of the determined top-quark mass. Estimating the impact of colour reconnection (CR) by switching CR on and off in Phythia6 and Pythia8 an effect below 400 MeV is found in ref. [26]. In both cases JES and CR the estimate is considered to be rather conservative. Most likely in-situ calibration of the JES will reduce the uncertainties below 3%. In case of CR a more realistic estimate could be obtained by comparing different model's of CR instead of looking at the extreme situation (on/off). Combining the different uncertainties it is argued in ref. [26] that top-quark mass measurement with an uncertainty below 1 GeV might be possible. Taking into account that the method unambiguously fixes the renormalisation scheme it is certainly worth to study the approach in more detail.

4. – Conclusions

In this paper we argue that a precise measurement of the top-quark mass is very important. The interest in the top-quark mass is motivated by consistency checks of the SM but also by the aim of making precise predictions in the SM. Since the top-quark does not appear as free particle a detailed comparison of measurements and theoretical predictions is necessary to determine the top-quark mass. A meaningful determination of the mass parameter thus requires reliable theoretical predictions including in particular NLO corrections to fix unambiguously the renormalisation scheme. Using the total $t\bar{t}$ cross section is theoretically well motivated. However, the reachable precision is limited by the weak sensitivity on the mass parameter. The $M_{J/\psi\ell}$ distribution in the decay $t \to \ell\nu J/\psi X$ offers an interesting alternative when large data samples are available. In ref. [26] the top-quark mass determination from jet-rates has been investigated. This seems to be a very attractive method allowing a measurement in the sub GeV range without requiring a particular large data sample. The method fixes the renormalisation scheme and is only mildly affected by colour reconnection. It will be highly interesting to see whether the promising accuracy can indeed be achieved in the experimental analysis.

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REFERENCES

[1] Heinemeyer S., Hollik W., Stockinger D., Weber A. and Weiglein G., *JHEP*, **08** (2006) 052, DOI 10.1088/1126-6708/2006/08/052.
[2] Casas J., Espinosa J. and Quiros M., *Phys. Lett. B*, **342** (1995) 171, DOI 10.1016/0370-2693(94)01404-Z.
[3] Espinosa J. and Quiros M., *Phys. Lett. B*, **353** (1995) 257, DOI 10.1016/0370-2693(95)00572-3.
[4] Elias-Miro J., Espinosa J. R., Giudice G. F., Isidori G., Riotto A. *et al.*, *Phys. Lett. B*, **709** (2012) 222, DOI 10.1016/j.physletb.2012.02.013.
[5] Degrassi G., Di Vita S., Elias-Miro J., Espinosa J. R., Giudice G. F. *et al.*, *JHEP*, **08** (2012) 098, DOI 10.1007/JHEP08(2012)098.

[6] Alekhin S., Djouadi A. and Moch S., *Phys. Lett. B*, **716** (2012) 214, DOI 10.1016/j.physletb.2012.08.024.
[7] Czakon M. and Mitov A., *JHEP*, **01** (2013) 080, DOI 10.1007/JHEP01(2013)080.
[8] Czakon M. and Mitov A., *JHEP*, **12** (2012) 054, DOI 10.1007/JHEP12(2012)054.
[9] Baernreuther P., Czakon M. and Mitov A., *Phys. Rev. Lett.*, **109** (2012) 132001, DOI 10.1103/PhysRevLett.109.132001.
[10] Czakon M., Fiedler P. and Mitov A. *Phys. Rev. Lett.*, **110** (2013) 252004.
[11] Bigi I. I. Y., Shifman M. A., Uraltsev N. G. and Vainshtein A. I., *Phys. Rev. D*, **50** (1994) 2234, DOI 10.1103/PhysRevD.50.2234.
[12] Beneke M. and Braun V. M., *Nucl. Phys. B*, **426** (1994) 301, DOI 10.1016/0550-3213(94)90314-X.
[13] Aaltonen T. *et al.*, *Phys. Rev. D*, **86** (2012) 092003, DOI 10.1103/PhysRevD.86.092003.
[14] Aad G. *et al.*, *Eur. Phys. J. C*, **72** (2012) 2046, DOI 10.1140/epjc/s10052-012-2046-6.
[15] Chatrchyan S. *et al.*, *Eur. Phys. J. C*, **72** (2012) 2202, DOI 10.1140/epjc/s10052-012-2202-z.
[16] Chatrchyan S. *et al.*, *JHEP*, **12** (2012) 105, DOI 10.1007/JHEP12(2012)105.
[17] Campbell J. M., Giele W. T. and Williams C., *JHEP*, **11** (2012) 043, DOI 10.1007/JHEP11(2012)043.
[18] Campbell J. M., Ellis R. K., Giele W. T. and Williams C. (2013) DOI 10.1103/PhysRevD.87.073005.
[19] Langenfeld U., Moch S. and Uwer P., *Phys. Rev. D*, **80** (2009) 054009, DOI 10.1103/PhysRevD.80.054009.
[20] Kharchilava A., *Phys. Lett. B*, **476** (2000) 73, DOI 10.1016/S0370-2693(00)00120-9.
[21] Beneke M., Efthymiopoulos I., Mangano M. L., Womersley J., Ahmadov A. *et al.*, hep-ph/0003033, CERN-TH-2000-100.
[22] Grenier P., ATL-PHYS-2001-016, ATL-COM-PHYS-2001-027, CERN-ATL-PHYS-2001-016.
[23] Nekrasov M., *Eur. Phys. J. C*, **44** (2005) 233, DOI 10.1140/epjc/s2005-02362-2.
[24] Chierici R. and Dierlamm A., CERN-CMS-NOTE-2006-058.
[25] Biswas S., Melnikov K. and Schulze K., *JHEP*, **08** (2010) 048, DOI 10.1007/JHEP08(2010)048.
[26] Alioli S., Fernandez P., Fuster J., Irles A., Moch S. O. *et al.*, *Eur. Phys. J. C*, **73** (2013) 2438.
[27] Rodrigo G., Santamaria A. and Bilenky M. S., *Phys. Rev. Lett.*, **79** (1997) 193, DOI 10.1103/PhysRevLett.79.193.
[28] Abreu P. *et al.*, *Phys. Lett. B*, **418** (1998) 430, DOI 10.1016/S0370-2693(97)01442-1.
[29] Bilenky M. S., Caberera S., Fuster J., Marti S., Rodrigo G. *et al.*, *Phys. Rev. D*, **60** (1999) 114006, DOI 10.1103/PhysRevD.60.114006.
[30] Dittmaier S., Uwer P. and Weinzierl S., *Phys. Rev. Lett.*, **98** (2007) 262002, DOI 10.1103/PhysRevLett.98.262002.
[31] Dittmaier S., Uwer P. and Weinzierl S., *Eur. Phys. J. C*, **59** (2009) 625, DOI 10.1140/epjc/s10052-008-0816-y.
[32] Alioli S., Moch S. O. and Uwer P., *JHEP*, **01** (2012) 137, DOI 10.1007/JHEP01(2012)137.