COLLOQUIA: LaThuile13

Neutrino mixing: Theoretical overview

G. ALTARELLI

Dipartimento di Matematica e Fisica, Università di Roma Tre and
INFN, Sezione di Roma Tre - Roma, Italy and
CERN, Department of Physics, Theory Unit - CH-1211 Geneva 23, Switzerland

Summary. — We present a concise review of the recent important experimental developments on neutrino mixing (hints for sterile neutrinos, large θ_{13}, possible non-maximal θ_{23}, approaching sensitivity on δ_{CP}) and their implications on models of neutrino mixing. The new data disfavour many models but the surviving ones still span a wide range going from Anarchy (no structure, no symmetry in the lepton sector) to a maximum of symmetry, as for the models based on discrete non-abelian flavour groups that can be improved following the indications from the data.

PACS `11.15.-q` – Gauge field theories.
PACS `12.38.-t` – Quantum chromodynamics.

1. – Introduction

On the experimental side the main recent developments on neutrino mixing [1] were the results on θ_{13} from T2K [2], MINOS [3], DOUBLE CHOOZ [4], RENO [5] and especially DAYA BAY [6]. The different experiments are in good agreement and the most precise is DAYA BAY with the result $\sin^2 2\theta_{13} = 0.0890 \pm 0.0112$ (equivalent to $\sin^2 \theta_{13} = 0.023 \pm 0.003$ or $\theta_{13} = (8.7 \pm 0.6)^o$). A summary of recent global fits to the data on oscillation parameters is presented in table I [7-9]. The combined value of $\sin^2 \theta_{13}$ is by now about 10 σ away from zero and the central value is rather large, close to the previous upper bound. In turn a sizable θ_{13} allows to extract an estimate of θ_{23} from accelerator data like T2K and MINOS. There are now solid indications of a deviation of θ_{23} from the maximal value, probably in the first octant [7]. In addition, some tenuous hints that $\cos \delta_{CP} < 0$ are starting to appear in the data.

A hot issue is the possible existence of sterile neutrinos: a number of hints have been recently reported (for a review, see [10]). They do not make yet an evidence but certainly pose an experimental problem that needs clarification (see, for example, ref. [11]).

Cosmological data allow for one single sterile neutrino but more than one are disfavoured by the stringent bounds arising form nucleosynthesis (assuming that they are

TABLE I. – *Fits to neutrino oscillation data. For* $\sin^2\theta_{23}$ *from ref.* [8] *only the absolute minimum in the first octant is shown*

Quantity	Ref. [7]	Ref. [8]
Δm^2_{sun} ($10^{-5}\,\mathrm{eV}^2$)	$7.54^{+0.26}_{-0.22}$	7.50 ± 0.185
Δm^2_{atm} ($10^{-3}\,\mathrm{eV}^2$)	$2.43^{+0.06}_{-0.10}$	$2.47^{+0.069}_{-0.067}$
$\sin^2\theta_{12}$	$0.307^{+0.018}_{-0.016}$	0.30 ± 0.013
$\sin^2\theta_{23}$	$0.386^{+0.024}_{-0.021}$	$0.41^{+0.037}_{-0.025}$
$\sin^2\theta_{13}$	0.0241 ± 0.025	0.023 ± 0.0023

thermalized) [12]. Actually the recently published Planck data [13] on the cosmic microwave background (CMB) are completely consistent with no sterile neutrinos (they quote $N_{eff} = 0.30 \pm 0.27$). The sum of all (quasi) stable (thermalized) neutrino masses is also bounded by cosmology. The South Pole Telescope, also using data on CMB and on the baryon acoustic oscillations (BAO) has claimed the result $\sum m_\nu = 0.32 \pm 0.11 \pm$? [14], where the question mark refers to the dependence on the assumed priors. The Planck experiment, also using the WMAP 9 and BAO data, finds the limit $\sum m_\nu \leq 0.23$ at 95% CL. From a completely different domain of physics, the search for neutrino-less double beta decay ($0\nu\beta\beta$), the EXO-200 experiment [15] gives an independent input on the absolute scale of neutrino masses: depending on the assumed nuclear matrix elements, they find $m_{\beta\beta} \leq 0.14$–$0.38\,\mathrm{eV}$.

The MiniBooNE experiment published [16] a combined analysis of ν_e appearance in a ν_μ beam together with $\bar\nu_e$ appearance in a $\bar\nu_\mu$ beam. They observe an excess of events from neutrinos over expected background in the low energy (below $500\,\mathrm{MeV}$) region of the event spectrum. In the most recent data the shapes of the neutrino and antineutrino spectra appear to be consistent with each other, showing excess events below $500\,\mathrm{MeV}$ and data consistent with background in the high energy region. The allowed region from MiniBooNE antineutrino data has some overlap with the parameter region preferred by LSND. Recently the ICARUS experiment at Gran Sasso has published the results of a search for electrons produced by the CERN neutrino beam [17]. No excess over the background was observed. As a consequence a large portion of the region allowed by LSND, MiniBooNE. KARMEN... is now excluded.

Then there are $\bar\nu_e$ disappearance experiments: in particular, the reactor and the gallium anomalies. A reevaluation of the reactor flux [18] produced an apparent gap between the theoretical expectations and the data taken at small distances from reactors ($\leq 100\,\mathrm{m}$). A different analysis confirmed the normalization shift [19]. Similarly the gallium anomaly [20] depends on the assumed cross-section which could be questioned.

Even leaving cosmology aside, if all the indications listed above were confirmed (it looks unlikely), then adding one or more sterile neutrinos would probably not be enough to satisfactorily describe all the oscillation data (see, for example ref. [21]). There is in fact a strong tension between appearance (LSND, MiniBooNE, ICARUS) and disappearance (reactors, Ga anomaly) data. Thus the situation is at present confused but the experimental effort should be continued because establishing the existence of sterile neutrinos would be a great discovery (an experiment to clarify the issue of sterile neutrinos is proposed on the CERN site [22]). In fact a sterile neutrino is an exotic particle

not predicted by the most popular models of new physics. As only a small leakage from active to sterile neutrinos is allowed by present neutrino oscillation data (see, for example, refs. [21, 23-25] and references therein), in the following we restrict our discussion to 3-neutrino models.

The rather large measured value of θ_{13}, close to the old CHOOZ bound and to the Cabibbo angle, and the indication that θ_{23} is not maximal both go in the direction of models based on Anarchy [26, 27], *i.e.* the ansatz that perhaps no symmetry is needed in the leptonic sector, only chance (this possibility has been recently reiterated, for example, in ref. [28]). Anarchy can be formulated in a $SU(5) \otimes U(1)_{FN}$ context by taking different Froggatt-Nielsen [29] charges only for the $SU(5)$ tenplets (for example $10 \sim (a, b, 0)$, where $a > b > 0$ is the charge of the first generation, b of the second, zero of the third) while no charge differences appear in the $\bar{5}$ (*e.g.* $\bar{5} \sim (0, 0, 0)$). The observed fact that the up-quark mass hierarchies are more pronounced than for down-quark and charged leptons is in agreement with this assignment. The $SU(5)$ generators act vertically inside one generation, whereas the $U(1)_{FN}$ charges are different horizontally from one generation to the other. If, for a given interaction vertex, the $U(1)_{FN}$ charges do not add to zero, the vertex is forbidden in the symmetric limit. However, the $U(1)_{FN}$ symmetry (that one can assume to be a gauge symmetry) is spontaneously broken by the VEVs v_f of a number of flavon fields with non-vanishing charge and GUT-scale masses. Then a forbidden coupling is rescued but is suppressed by powers of the small parameters $\lambda = v_f/M$, with M a large mass, with the exponents larger for larger charge mismatch. Thus the charges fix the powers of λ, hence the degree of suppression of all elements of mass matrices, while arbitrary coefficients k_{ij} of order 1 in each entry of mass matrices are left unspecified (so that the number of parameters exceeds the number of observable quantities). A random selection of these k_{ij} parameters leads to distributions of resulting values for the measurable quantities. For anarchy (A) the mass matrices in the leptonic sector are totally random, while in the presence of unequal charges different entries carry different powers of the order parameter and thus some hierarchies are enforced. The embedding of Anarchy in the $SU(5) \otimes U(1)_{FN}$ context allows to implement a parallel treatment of quarks and leptons. Within this framework there are many variants of these models: fermion charges can all be nonnegative with only negatively charged flavons, or there can be fermion charges of different signs with either flavons of both charges or only flavons of one charge. In models with no See-Saw, the $\bar{5}$ charges completely fix the hierarchies (or anarchy, if the case) in the neutrino mass matrix. If Right-Handed (RH) neutrinos are added, they transform as $SU(5)$ singlets and can in principle carry $U(1)_{FN}$ charges, which also must be all equal in the anarchy case. With RH neutrinos the see-saw mechanism can take place and the resulting phenomenology is modified. In ref. [30], given the new experimental results, we have made a reappraisal of Anarchy and its variants within the (SUSY) $SU(5) \times U(1)_{\text{FN}}$ GUT framework. Based on the most recent data we argue that the Anarchy ansatz is probably oversimplified and, in any case, not compelling. In fact, suitable differences of $U(1)_{FN}$ charges, if also introduced within pentaplets and singlets, lead to distributions that are in better agreement with the data with the same number of random parameters as for Anarchy. The hierarchy of quark masses and mixing and of charged lepton masses in all cases impose a hierarchy defining parameter of the order of $\lambda_C = \sin \theta_C$, with θ_C being the Cabibbo angle. The weak points of Anarchy (A) are that with this ansatz all mixing angles should be of the same order, so that the relative smallness of $\theta_{13} \sim o(\lambda_C)$ is not automatic. Similarly the smallness of $r = \Delta m^2_{solar}/\Delta m^2_{atm}$ is not easily reproduced: with no See-Saw r is of $o(1)$, while in the See-Saw version of Anarchy the problem is only partially alleviated by

the spreading of the neutrino mass distributions that follows from the product of three matrix factors in the See-Saw formula. An advantage is already obtained if Anarchy is only restricted to the 23 sector of leptons as in the $A_{\mu\tau}$ model (in the notation of ref. [30]). In this case, with or without See-Saw, θ_{13} is naturally suppressed and, with a single fine tuning one gets both θ_{12} large and r small (this model was also recently rediscussed in ref. [31]). Actually in ref. [30] we have shown that, in the no See-Saw case, a very good performance is observed in a new model, the H model, where Anarchy is also relaxed in the 23 sector. In the H model, by taking a relatively large order parameter, one can reproduce the correct size for all mixing angles and mass ratios. Alternatively, in the See-Saw case, we have shown that the freedom of adopting RH neutrino charges of both signs, as in the $PA_{\mu\tau}$ model, can be used to obtain a completely natural model where all small quantities are suppressed by the appropriate power of λ. In this model a lopsided Dirac mass matrix is combined with a generic Majorana matrix to produce a neutrino mass matrix where the 23 subdeterminant is suppressed and thus r is naturally small with unsuppressed θ_{23}. In addition θ_{12} is large while θ_{13} is suppressed. We stress again that the number of random parameters is the same in all these models: one coefficient of $o(1)$ for every matrix element. Moreover, with an appropriate choice of charges, it is not only possible to reproduce the charged fermion hierarchies and the quark mixing, but also the order of magnitude of all small observed parameters can be naturally guaranteed. In conclusion, we agree that models based on chance are still perfectly viable, but we consider Anarchy a particularly simple choice perhaps oversimplified and certainly not compelling and we have argued in favour of less chaotic solutions.

Anarchy and its variants, all sharing the dominance of randomness in the lepton sector, are to be confronted with models with a richer dynamical structure, some based on continuous groups [32] but in particular those based on discrete flavour groups (for reviews, see, for example, refs. [33-35]). After the measurement of a relatively large value for θ_{13} there has been an intense work to interpret these new results along different approaches and ideas. Examples are suitable modifications of the minimal models [36,37] (we discuss the Lin model of ref. [37] in the following), larger symmetries that already at LO lead to non vanishing θ_{13} and non-maximal θ_{23} [38], smaller symmetries that leave more freedom [39], models where the flavour group and a generalised CP transformation are combined in a non trivial way [40] (other approaches to discrete symmetry and CP violation are found in refs. [41]).

Among the models with a non trivial dynamical structure those based on discrete flavour groups were motivated by the fact that the data suggest some special mixing patterns as good first approximations like Tri-Bimaximal (TB) or Golden Ratio (GR) or Bi-Maximal (BM) mixing, for example. The corresponding mixing matrices all have $\sin^2 \theta_{23} = 1/2$, $\sin^2 \theta_{13} = 0$, values that are good approximations to the data (although less so since the most recent data), and differ by the value of the solar angle $\sin^2 \theta_{12}$. The observed $\sin^2 \theta_{12}$, the best measured mixing angle, is very close, from below, to the so called Tri-Bimaximal (TB) value [42] of $\sin^2 \theta_{12} = 1/3$. Alternatively, it is also very close, from above, to the Golden Ratio (GR) value [43] $\sin^2 \theta_{12} = \frac{1}{\sqrt{5}\,\phi} = \frac{2}{5+\sqrt{5}} \sim 0.276$, where $\phi = (1+\sqrt{5})/2$ is the GR (for a different connection to the GR, see refs. [44]). On a different perspective, one has also considered models with Bi-Maximal (BM) mixing, where at leading order (LO), before diagonalization of charged leptons, $\sin^2 \theta_{12} = 1/2$, *i.e.* it is also maximal, and the necessary, rather large, corrective terms to θ_{12} arise from the diagonalization of the charged lepton mass matrices (a list of references can be found in ref. [33]). Thus, if one or the other of these coincidences is taken seriously, models

where TB or GR or BM mixing is naturally predicted provide a good first approximation (but these hints cannot all be relevant and it is well possible that none is). As the corresponding mixing matrices have the form of rotations with fixed special angles one is naturally led to discrete flavour groups.

In the following we will mainly refer to TB or BM mixing which are the most studied first approximations to the data. A simplest discrete symmetry for TB mixing is A_4 while BM can be obtained from S_4. Starting with the ground breaking paper in ref. [45], A_4 models have been widely studied (for a recent review and a list of references, see ref. [50]). At LO the typical A_4 model (like, for example, the one discussed in in ref. [46]) leads to exact TB mixing. In these models the starting LO approximation is completely fixed (no chance), but the Next to LO (NLO) corrections still introduce a number of undetermined parameters, although in general much less numerous than for $U(1)_{FN}$ models. These models are therefore more predictive and in each model, one obtains relations among the departures of the three mixing angles from the LO patterns, restrictions on the CP violation phase δ_{CP}, mass sum rules among the neutrino mass eigenvalues, definite ranges for the neutrinoless beta decay effective Majorana mass and so on. Given the set of flavour symmetries and having specified the field content, the non-leading corrections to TB mixing, arising from higher dimensional effective operators, can be evaluated in a well-defined expansion. In the absence of specific dynamical tricks, in a generic model all three mixing angles receive corrections of the same order of magnitude. Since the experimentally allowed departures of θ_{12} from the TB value, $\sin^2 \theta_{12} = 1/3$, are small, numerically not larger than $\mathcal{O}(\lambda_C^2)$ where $\lambda_C = \sin \theta_C$, it follows that both θ_{13} and the deviation of θ_{23} from the maximal value are also expected to be typically of the same general size. This generic prediction of a small θ_{13}, numerically of $\mathcal{O}(\lambda_C^2)$, is at best marginal after the recent measurement of θ_{13}.

Of course, one can introduce some additional theoretical input to improve the value of θ_{13}. In the case of A_4, one particularly interesting example is provided by the Lin model [37] (see also ref. [36]), formulated before the recent θ_{13} results. In the Lin model the A_4 symmetry breaking is arranged, by suitable additional Z_n parities, in a way that the corrections to the charged lepton and the neutrino sectors are kept separated not only at LO but also at NLO. As a consequence, in a natural way the contribution to neutrino mixing from the diagonalization of the charged leptons can be of $\mathcal{O}(\lambda_C^2)$, while those in the neutrino sector of $\mathcal{O}(\lambda_C)$. Thus, in the Lin model the NLO corrections to the solar angle θ_{12} and to the reactor angle θ_{13} are not necessarily related. In addition, in the Lin model the largest corrections do not affect θ_{12} and satisfy the relation $\sin^2 \theta_{23} = 1/2 + 1/\sqrt{2} \cos \delta_{CP} |\sin \theta_{13}|$, with δ_{CP} being the CKM-like CP-violating phase of the lepton sector. Note that, for θ_{23} in the first octant, the sign of $\cos \delta_{CP}$ must be negative. Alternatively, one can think of models where, because of a suitable symmetry, BM mixing holds in the neutrino sector at LO and the corrective terms for θ_{12}, which in this case are required to be large, arise from the diagonalization of charged lepton masses [47]. These terms from the charged lepton sector, numerically of order $\mathcal{O}(\lambda_C)$, would then generically also affect θ_{13} and the resulting angle could well be compatible with the measured value. An explicit model of this type based on the group S_4 has been developed in ref. [48] (see also refs. [49]). An important feature of this particular model is that only θ_{12} and θ_{13} are corrected by terms of $\mathcal{O}(\lambda_C)$ while θ_{23} is unchanged at this order. This model is compatible with present data and clearly prefers the upper range of the present experimental result for θ_{13}.

In ref. [51] we discuss three possible classes of models: 1) Typical A_4 models where θ_{13} is generically expected to be small, of the order of the observed departures of θ_{12} from

the TB value, and thus with preference for the lower end of the allowed experimental range. 2) Special A_4 models, like the Lin model, where θ_{13} is made independent of the deviation of $\sin^2 \theta_{12}$ from the TB value $1/3$ and can be as large as the upper end of the allowed experimental range. In the same paper we discuss a general characterization of these special A_4 models. 3) Models where BM mixing holds in the neutrino sector and large corrections to θ_{12} and θ_{13} arise from the diagonalization of charged leptons (in this case the value of θ_{13} is naturally close to the present experimental range but θ_{12} is at risk).

In each of the three classes of models the dominant corrections to the LO mixing pattern involve a number of parameters of the same order of magnitude, ξ. We discuss the success rate corresponding to the optimal value of ξ for each model, obtained by scanning the parameter space according to a similar procedure for all three cases. We argue that, while the absolute values of the success rates depend on the scanning assumptions, their relative values in the three classes of models, provide a reliable criterium for comparison. We find that, for reproducing the mixing angles, the Lin type models have the best performance, as expected, followed by the typical A_4 models while the BM mixing models lead to an inferior score, as they can well reproduce the size of θ_{13} but most often fail to reproduce the correct value of θ_{12}. We also discuss the conditions for $\cos \delta_{CP}$ peaking around -1.

In ref. [51] we have also studied the implications for lepton flavour-violating (LFV) processes of the above three classes of possibilities, assuming a supersymmetric context, with or without See-Saw. The present bounds on LFV reactions pose severe constraints on the parameter space of the models (for a recent general analysis of lepton flavour violating effects in the context of flavour models, see ref. [52]). In particular, we refer to the recent improved MEG result [53] on the $\mu \to e\gamma$ branching ratio, $Br(\mu \to e\gamma) \le 5.7 \times 10^{-13}$ at 90% CL and to other similar processes like $\tau \to (e$ or $\mu)\gamma$. In ref. [51] we have studied this issue by adopting the simple CMSSM framework. While this very constrained version of supersymmetry is rather marginal after the results of the LHC searches, more so given that the Higgs mass is around $m_H = 125\,\mathrm{GeV}$, we still believe it can be used for indicative purposes as in this case. We find that the most constrained versions are the models with BM mixing at LO where relatively large corrections directly appear in the off-diagonal terms of the charged lepton mass matrix. The A_4 models turn out to be the best suited to satisfy the experimental bounds, as the non-diagonal charged lepton matrix elements needed to reproduce the mixing angles are quite smaller. An intermediate score is achieved by the models of the Lin type, where the main corrections to the mixing angles arise from the neutrino sector and the non-diagonal charged lepton matrix elements are smaller. As for the regions of the CMSSM parameter space that are indicated by our analysis, the preference is for small $\tan \beta$ and large SUSY masses (at least one out of m_0 and $m_{1/2}$ must be above $1\,\mathrm{TeV}$). Given that SUSY has not been found at the LHC the preference for heavy SUSY masses does not pose a problem to these flavour models, except that, as a consequence, it appears impossible, at least within the CMSSM rigid framework, to satisfy the MEG bound and simultaneously to reproduce the muon $g - 2$ [54] discrepancy [55].

$$* * *$$

REFERENCES

[1] ALTARELLI G. and FERUGLIO F., *New J. Phys.*, **6** (2004) 106, arXiv:hep-ph/0405048; MOHAPATRA R. N. and SMIRNOV A. Y., *Annu. Rev. Nucl. Part. Sci.*, **56** (2006) 569, arXiv:hep-ph/0603118; GRIMUS W., *PoS*, **P2GC** (2006) 001, arXiv:hep-ph/0612311; GONZALEZ-GARCIA M. C. and MALTONI M., *Phys. Rept.*, **460** (2008) 1, arXiv:0704.1800.

[2] ABE K. *et al.* (T2K COLLABORATION), *Phys. Rev. Lett.*, **107** (2011) 041801, arXiv:1106.2822; arXiv:1106.2822.

[3] ADAMSON P. *et al.* (MINOS COLLABORATION), *Phys. Rev. Lett.*, **107** (2011) 181802, arXiv:1108.0015.

[4] ABE Y. *et al.* (DOUBLE-CHOOZ COLLABORATION), arXiv:1207.6632.

[5] AHN J. K. *et al.* (RENO COLLABORATION), arXiv:1204.0626.

[6] AN F. P. *et al.* (DAYA-BAY COLLABORATION), arXiv:1203.1669.

[7] FOGLI G. *et al.*, arXiv:1205.5254.

[8] GONZALEZ-GARCIA M. C., MALTONI M., SALVADO J. and SCHWETZ T., arXiv:1209.3023.

[9] FORERO D., TORTOLA M. and VALLE J., arXiv:1205.4018.

[10] ABAZAJIAN K. *et al.*, arXiv: 1204.5379.

[11] RUBBIA C. *et al.*, arXiv:1304.2047.

[12] GIUSARMA E. *et al.*, arXiv:1102.4774; JOUDAKI S. *et al.*, arXiv:1208.4354.

[13] ADE P. A. R. *et al.* (PLANCK COLLABORATION), arXiv:1303.5076.

[14] HOU Z. *et al.*, arXiv:1212.6267.

[15] AUGER M. *et al.*, arXiv:1205.5608.

[16] AGUILAR-AREVALO A. A. *et al.* (MiniBooNE COLLABORATION), arXiv:1207.4809; arXiv:1303.2588.

[17] ANTONELLO M. *et al.* (ICARUS COLLABORATION), arXiv:1209.0122.

[18] MENTION G. *et al.*, arXiv:1101.2755.

[19] HUBER P., arXiv:1106.0687.

[20] GIUNTI C. and LAVEDER M., arXiv:1006.3244.

[21] KOPP J., MACHADO P. A. N., MALTONI M. and SCHWETZ T., arXiv:1303.3011.

[22] ANTONELLO M. *et al.* (ICARUS and NESSIE COLLABORATION), arXiv:1203.3432.

[23] ARCHIDIACONO M. *et al.*, arXiv:1302.6720.

[24] PALAZZO A., arXiv:1302.1102.

[25] MIRIZZI A. *et al.*, arXiv:1303.5368.

[26] HALL L. J., MURAYAMA H. and WEINER N., *Phys. Rev. Lett.*, **84** (2000) 2572, arXiv:hep-ph/9911341.

[27] DE GOUVEA A. and MURAYAMA H., *Phys. Lett. B*, **573** (2003) 94, arXiv:hep-ph/0301050.

[28] DE GOUVEA A. and MURAYAMA H., arXiv:1204.1249.

[29] FROGGATT C. D. and NIELSEN H. B., *Nucl. Phys. B*, **147** (1979) 277.

[30] ALTARELLI G., FERUGLIO F., MASINA I. and MERLO L., arXiv:1207.0587.

[31] BUCHMULLER W., DOMCKE V. and SCHMITZ K., *JHEP*, **03** (2012) 008, arXiv:1111.3872.

[32] VARZIELAS I. D. M. and ROSS G. G., arXiv:1203.6636; ALONSO R. *et al.*, *JHEP*, **06** (2011) 037, arXiv:1103.5461; ALONSO R., GAVELA M., HERNANDEZ D. and MERLO L., *Phys. Lett. B*, **715** (2012) 194, arXiv:1206.3167; BLANKENBURG G., ISIDORI G. and JONES-PEREZ J., arXiv:1204.0688.

[33] ALTARELLI G. and FERUGLIO F., *Rev. Mod. Phys.*, **82** (2010) 2701, arXiv:1002.0211.

[34] ISHIMORI H. *et al.*, *Prog. Theor. Phys. Suppl.*, **183** (2010) 1, arXiv:1003.3552; KING S. and LUHN C., arXiv:1301.1340.

[35] GRIMUS W. and LUDL P. O., *J. Phys. A*, **45** (2012) 233001, arXiv:1110.6376.

[36] MA E. and WEGMAN D., *Phys. Rev. Lett.*, **107** (2011) 061803, arXiv:1106.4269; KING S. F. and LUHN C., *JHEP*, **09** (2011) 042, arXiv:1107.5332; BAZZOCCHI F., arXiv:1108.2497; DE MEDEIROS VARZIELAS I. and MERLO L., *JHEP*, **02** (2011) 062, arXiv:1011.6662; RODEJOHANN W. and ZHANG H., arXiv:1207.1225; BAZZOCCHI F. and MERLO L., arXiv:1205.5135.

[37] LIN Y., *Nucl. Phys. B*, **824** (2010) 95, arXiv:0905.3534.

[38] DE ADELHART TOOROP R., FERUGLIO F. and HAGEDORN C., *Phys. Lett. B*, **703** (2011) 447, arXiv:1107.3486; *Nucl. Phys. B*, **858** (2012) 437, arXiv:1112.1340; KING S. F., LUHN C. and STUART A. J., arXiv:1207.5741.

[39] GE S.-F., DICUS D. A. and REPKO W. W., *Phys. Lett. B*, **702** (2011) 220, arXiv:1104.0602; *Phys. Rev. Lett.*, **108** (2012) 041801, arXiv:1108.0964; HERNANDEZ D. and SMIRNOV A. Y., *Phys. Rev. D*, **86** (2012) 053014, arXiv:1204.0445.

[40] ECKER G., GRIMUS W. and NEUFELD H., *J. Phys. A*, **20** (1987) L807; *Int. J. Mod. Phys. A*, **3** (1988) 603; GRIMUS W. and LAVOURA L., *Phys. Lett. B*, **579** (2004) 113, arXiv:hep-ph/0305309; KRISHNAN R., HARRISON P. F. and SCOTT W. G., arXiv:1211.2000; MOHAPATRA R. N. and NISHI C. C., *Phys. Rev. D*, **86** (2012) 073007, arXiv:1208.2875; HOLTHAUSEN M., LINDNER M. and SCHMIDT M. A., arXiv:1211.6953; GRIMUS W. and REBELO M. N., *Phys. Rept.*, **281** (1997) 239, arXiv:hep-ph/9506272; FERUGLIO F., HAGEDORN C. and ZIEGLER R., arXiv:1211.5560; arXiv:1303.7178.

[41] BRANCO G. C., GERARD J. M. and GRIMUS W., *Phys. Lett. B*, **136** (1984) 383; DE MEDEIROS VARZIELAS I. and EMMANUEL-COSTA D., *Phys. Rev. D*, **84** (2011) 117901; arXiv:1106.5477; DE MEDEIROS VARZIELAS I., EMMANUEL-COSTA D. and LESER P., *Phys. Lett. B*, **716** (2012) 193; arXiv:1204.3633; DE MEDEIROS VARZIELAS I., *JHEP*, **08** (2012) 055; arXiv:1205.3780; BHATTACHARYYA G., DE MEDEIROS VARZIELAS I. and LESER P., *Phys. Rev. Lett.*, **109** (2012) 241603; arXiv:1210.0545; BABU K. S. and KUBO J., *Phys. Rev. D*, **71** (2005) 056006; arXiv:hep-ph/0411226]; BABU K. S., KAWASHIMA K. and KUBO J., *Phys. Rev. D*, **83** (2011) 095008, arXiv:1103.1664; CHEN M.-C. and MAHANTHAPPA K. T., *Phys. Lett. B*, **681** (2009) 444; arXiv:0904.1721; MERONI A., PETCOV S. T. and SPINRATH M., *Phys. Rev. D*, **86** (2012) 113003, arXiv:1205.5241.

[42] HARRISON P. F., PERKINS D. H. and SCOTT W. G., *Phys. Lett. B*, **530** (2002) 167, arXiv:hep-ph/0202074; HARRISON P. F. and SCOTT W. G., *Phys. Lett. B*, **535** (2002) 163, arXiv:hep-ph/0203209; XING Z.-z., *Phys. Lett. B*, **533** (2002) 85, arXiv:hep-ph/0204049; HARRISON P. F. and SCOTT W. G., *Phys. Lett. B*, **547** (2002) 219, arXiv:hep-ph/0210197; *Phys. Lett. B*, **557** (2003) 76, arXiv:hep-ph/0302025.

[43] KAJIYAMA Y., RAIDAL M. and STRUMIA A., *Phys. Rev. D*, **76** (2007) 117301, arXiv:0705.4559; EVERETT L. L. and STUART A. J., *Phys. Rev. D*, **79** (2009) 085005, arXiv:0812.1057; DING G.-J., EVERETT L. L. and STUART A. J., *Nucl. Phys. B*, **857** (2012) 219, arXiv:1110.1688; FERUGLIO F. and PARIS A., *JHEP*, **03** (2011) 101, arXiv:1101.0393.

[44] RODEJOHANN W., *Phys. Lett. B*, **671** (2009) 267, arXiv:0810.5239; ADULPRAVITCHAI A., BLUM A. and RODEJOHANN W., *New J. Phys.*, **11** (2009) 063026, arXiv:0903.0531.

[45] MA E. and RAJASEKARAN G., *Phys. Rev. D*, **64** (2001) 113012, arXiv:hep-ph/0106291.

[46] ALTARELLI G. and FERUGLIO F., *Nucl. Phys. B*, **741** (2006) 215, arXiv:hep-ph/0512103.

[47] ALTARELLI G., FERUGLIO F. and MASINA I., *Nucl. Phys. B*, **689** (2004) 157, arXiv:hep-ph/0402155.

[48] ALTARELLI G., FERUGLIO F. and MERLO L., *JHEP*, **05** (2009) 020, arXiv:0903.1940.

[49] DE ADELHART TOOROP R., BAZZOCCHI F. and MERLO L., *JHEP*, **08** (2010) 001, arXiv:1003.4502; PATEL K. M., *Phys. Lett. B*, **695** (2011) 225, arXiv:1008.5061; MELONI D., *JHEP*, **10** (2011) 010, arXiv:1107.0221.

[50] ALTARELLI G., FERUGLIO F. and MERLO L., arXiv:1205.5133.

[51] ALTARELLI G., FERUGLIO F., MERLO L. and STAMOU E., *JHEP*, **08** (2012) 021, arXiv:1205.4670.

[52] CALIBBI L., LALAK Z., POKORSKI S. and ZIEGLER R., arXiv:1204.1275.

[53] ADAM J. *et al.* (MEG COLLABORATION), arXiv:1303.0754.

[54] BENNETT G. W. *et al.* (MUON G-2 COLLABORATION), *Phys. Rev. D*, **73** (2006) 072003; ROBERTS B. L., *Chin. Phys. C*, **34** (2010) 741.

[55] HOECKER A. and MARCIANO W., The Muon Anomalous Magnetic Moment, in Particle Data Group, BERINGER J. *et al.*, *Phys. Rev. D*, **86** (2012) 010001.