IL NUOVO CIMENTO Vol. 121 B, N. 1 Gennaio 2006
DOI 10.1393/ncb/i2005-10155-7

Composite fluid models for rotating neutron stars

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(ricevuto il 30 Agosto 2005; revisionato il 23 Novembre 2005; approvato il 10 Gennaio 2006)

Summary. — A technique for constructing fluid models for rotating neutron stars is presented. The technique is applied to construct composite fluid sources for an exterior stationary-axisymmetric field given by Gutsunaev and Manko. Matter constructing the source is shown to satisfy energy conditions and hence the source is physically reasonable.

PACS 04.20.-q – Classical general relativity.

1. – Introduction

The discovery of pulsars and their subsequent interpretation as rotating neutron stars have prompted attention towards the study of both equilibrium configurations and stationary solutions in general relativity. These investigations have greatly advanced our understanding of the final state of matter in evolving stars. Stationary axisymmetric fields are intimately related to the investigation of rotating stars: their constructions, gravitational fields and gravitational collapse, whence they derive their extreme importance.

The case of slow rotation was treated for the first time by Hartle and Thorne [1, 2]. The analytic structure of the space-time outside a slowly rotating star, and its relation to the Kerr metric, have been well understood since their seminal work. In 1997, Darke *et al.* [3] applied the Newman-Janis algorithm to obtain interior rotating sources of the Kerr metric from non rotating ones. On the other hand, since 2003, thanks to Stergioulas [4], numerical solutions of the Einstein equations for stars rotating up to the mass-Shedding limit are now routinely obtained with a number of different methods. In 2004, Stergioulas and Berti [5] matched approximately the analytic and the numerical solutions for rapidly rotating Neutron stars. After a few months, Berti *et al.* [6] compared three different models of rotating stars space-time: the Hartle-Thorne model, the exact analytic solution of Gutsunaev and Manko [7], and a numerical solution of the full Einstein equations.

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The plan of the paper is as follows: in sect. **2** we describe the vacuum solution which was obtained by Gutsunaev and Manko. In sect. **3**, we construct the interior solution which consists of a slowly rotating core and a rotating spherical shell. In sect. **4**, we investigate physical properties of the interior and present plots for pressures, density and angular velocity. Section **5** is devoted to a discussion of the results.

2. – The Gutsunaev-Manko metric

The complex Ernst potential is defined by the relations

$$\begin{aligned} \epsilon &= \beta + i\Phi, \qquad \epsilon^* = \beta - i\Phi, \\ \Phi_{,z} &= \rho^{-1}\beta^2 w_{,\rho}, \quad \Phi_{,\rho} = -\rho^{-1}\beta^2 w_{,z}\,. \end{aligned} \tag{1}$$

The functions β and w are the coefficients in the Papapetrou line element written in canonical Weyl coordinates as

$$\mathrm{d}s^2 = \beta\left(\mathrm{d}t - w\mathrm{d}\phi\right)^2 \;+\; \beta^{-1}[j\left(\mathrm{d}\rho^2 + \mathrm{d}z^2\right) + \rho^2\mathrm{d}\phi^2]\,.$$

The asymptotic behavior of the functions β and Φ in the spherical coordinates (r,θ) is the following:

$$\beta = 1 - \frac{2m}{r} + o(r^{-3}), \qquad \Phi = 2J\frac{\cos\theta}{r^2} + o(r^{-3}),$$

m and J being the total mass and angular momentum respectively, given by

$$\begin{aligned} m &= \frac{[(M-k)(1-a^2) + k(1+a^2)]}{(1-a^2)}\,, \\ J &= \frac{2ak[2M(1-a^2) - k(1-3a^2)]}{(1-a^2)}\,, \end{aligned}$$

where M and k are real constants and a is a very small real constant.

Gutsunaev and Manko [7] consider a special case of solution arising from the choice of constants in the form $k = -l, M = l$. In the prolate ellipsoidal coordinates (x,y)

$$\rho = l\sqrt{(x^2-1)(1-y^2)}, \;\; z = lxy,$$

the solution is given by

$$\begin{aligned} \mathrm{d}s^2 &= \beta(\mathrm{d}t - W\mathrm{d}\phi)^2 - \\ &- \frac{l^2}{\beta}\left[\mathrm{d}^2\phi^2\left(-1+x^2\right)\left(1-y^2\right) + j\left(x^2-y^2\right)\left\{\frac{\mathrm{d}x^2}{x^2-1} + \frac{\mathrm{d}y^2}{1-y^2}\right\}\right], \end{aligned} \tag{2}$$

where

$$\begin{aligned} \beta &= \frac{v\,(x-1)}{u\,(x+1)}, \\ W &= \frac{-4ael\left(1-y^2\right)}{(1-a^2)\,v}, \end{aligned}$$

$$
\begin{aligned}
j &= \frac{v\left(x^2-1\right)}{\left(1-a^2\right)^2\left(x^2-y^2\right)^9},\\
u &= 4a^2(x+1)^2\left(x^2-1\right)^2 y^2\left(-4x^3+x^4-4xy^2+6x^2y^2+y^4\right)^2+\\
&\quad+\left[-a^2(x+1)^2\left(x^2-1\right)^3+\left(x^2-y^2\right)^4\right]^2,\\
e &= -a^2(x+1)^3\left(x^2-y^2\right)^5\left(1-3x+3x^2+y^2\right)+\left(x^2-y^2\right)^5\left(x^3+3x^4+3x^5+3xy^2+\right.\\
&\quad\left.+6x^2y^2+x^3y^2-y^4\right)+a^4(1+x)^5\left(x^2-1\right)^3\left(1-5x+10x^2-10x^3+5x^4+y^2-\right.\\
&\quad\left.-5xy^2+10x^2y^2+y^4\right)-a^2\left(x^2-1\right)^3\left(x^5+5x^6+10x^7+8x^8+5x^9+10x^3y^2+\right.\\
&\quad+45x^4y^2+76x^5y^2-x^6y^2+10x^7y^2+5xy^4+15x^2y^4+10x^3y^4-145x^4y^4+\\
&\quad\left.+x^5y^4-y^6-51x^2y^6-3y^8\right),\\
v &= 4a^2\left(x^2-1\right)^3\left(-1+y^2\right)\left(x^4+6x^2y^2+y^4\right)^2+\left[-a^2\left(-1+x^2\right)^4+\left(x^2-y^2\right)^4\right]^2.
\end{aligned}
$$

This solution can also be obtained using the HKX transformations [8]. Only in [9], it was noticed that the asymptotic flatness can be achieved by a simple choice of integration constant a without performing an additional Ehlers transformation. The quadrupole moment is

$$
Q = 4a^2 l^3\left(3-2a^4-3a^2\right)\left(1-a^2\right)^{-3}.
$$

In the static limit, $a=0$, the multipole moment vanishes and the metric reduces to the Schwarzschild solution. It has been shown that the above solution possesses an event horizon defined by the hypersurface $x=1$, which turns out to be singular only at the poles ($y=\pm 1$). The required commuting Killing vector fields are $\xi^a = \partial/\partial t$, $\eta^a = \partial/\partial\phi$.

Performing in (2) the coordinate transformation

$$
x = -1+\frac{r}{m},\quad y=\cos\theta,\quad l=\frac{\left(1-a^2\right)m}{1-3a^2},
$$

we get, similar to the Kerr solution, two physical parameters representing the total mass m and the angular momentum (determined by a). Then one may come to the Schwarzschild metric at $J=0$ (*i.e.* $a=0$).

3. – Composite fluid models for rotating neutron stars

We construct composite fluid sources for the above vacuum solution by using the technique described in [10], but we replace the static core by a slowly rotating one. The new configuration would be more physically reasonable.

Let the space-time be divided into two material regions, $A(r<c)$ and $B(c<r<R)$, and a vacuum region $E(r>R)$; with the spheres $S_A(r=c)$ and $S_E(r=R)$ separating them. c and R are constants such that $0<c<R$. $g(a)$ be the vacuum rotating metric with $g(0)$ the slowly rotating metric which has a given source g_0 inside a region $I(r<R)$, then we may construct a source for $g(a)$ inside the same region I as follows.

Let $A\subset I$ with boundary S_A. Assume that g_0 is valid in A, *i.e.*

$$
g_A = g_0\,.
$$

In the shell $B = I - A$ construct the metric

$$\mathrm{d}s^2 = B_t \mathrm{d}t^2 + B_r \mathrm{d}r^2 + B_\theta \mathrm{d}\theta^2 + B_\phi \mathrm{d}\phi^2 + 2B_{\phi t}\mathrm{d}\phi \mathrm{d}t,$$

where

$$g_B = g_0 + [g(a) - g(0)]P, \qquad c \leq r \leq R. \tag{3}$$

$P = P(r) \in C^2[c, R]$ is a dimensionless *matching function* satisfying the boundary conditions

$$P(c) = P'(c) = P'(R) = 0, \quad P(R) = 1. \tag{4}$$

Otherwise, P is arbitrary.

Extensions of the work of Brill and Cohen [11, 12] show that the field equations for the case of slow rotation assumes a rather simple form. If the rotation rate is slow, deformations away from spherical can be ignored and the relevant equations are the structure equations for static spherical symmetry and a new equation expressing the dragging of inertial frames, Ω_{in}, in terms of solutions to the static field.

The condition of slow rotation leads to the metric

$$\mathrm{d}s^2 = \gamma^2(r)\mathrm{d}t^2 - \tau^{-1}(r)\mathrm{d}r^2 - r^2[\mathrm{d}\theta^2 + \sin^2\theta(\mathrm{d}\phi - \Omega_{\text{in}}(r)\mathrm{d}t)^2]\,. \tag{5}$$

In this expression, $\gamma(r)$ and $\tau(r)$ are solutions of the equation [13-15]

$$\tau'(r) - \frac{2(\gamma + r\gamma' - r^2\gamma'')}{r(\gamma + r\gamma')}\tau(r) = \frac{-2\gamma}{r(\gamma + r\gamma')},$$

which represent two of the field equations for static spherical symmetry (obtained by assuming an isotropic pressure). The remaining equations are

$$\begin{aligned} 8\pi r^2\mu(r) &= 1 - \tau(r) - r\tau'(r), \\ 8\pi r^2 p(r) &= [\gamma(r) + 2r\gamma'(r)][\tau(r)/\gamma(r)] - 1, \end{aligned}$$

which can be taken as definitions of the density ρ and the pressure p of the fluid.

The new equation which depicts the case of slow rotation is

$$\tau(r)\left(\Omega''_{\text{in}} + 4\frac{\Omega'_{\text{in}}}{r}\right) = 4\pi r\,(p(r) + \mu(r))\left(\Omega'_{\text{in}} + \frac{4(\Omega_{\text{in}} - \omega)}{r}\right),$$

where ω is the angular velocity of the rotating coordinate system, and Ω_{in} is the dragging of inertial frames.

Let the material core region, $\overline{A} = A \cup S_A$, have the slowly rotating metric [16]

$$\begin{aligned} \gamma &= 1 - \frac{m^2}{R^2} - \frac{3m}{2R} + \left(\frac{m^2}{2R^4} + \frac{m}{2R^3}\right) r^2 + O_3, \\ \tau &= 1 - \frac{2m^2r^2}{R^4} - \frac{2mr^2}{R^3} + \frac{2m^2r^4}{R^6} + O_3, \\ \Omega_{\text{in}} &= \frac{-20ma}{7R^3} + \frac{6mar^7}{7R^{10}} + O_3\,. \end{aligned}$$

At the boundary $r = R$, this metric *matches smoothly* onto the exterior slowly rotating metric

$$\gamma_0 = \sqrt{1 - \frac{2m}{R}},$$
$$\tau_0 = 1 - \frac{2m}{r},$$
$$\Omega_0 = \frac{-2ma}{r^3}\,.$$

We assume that the Gutsunaev-Manko metric (2) is valid in the region $\overline{E} = E \cup S_E$.

In the shell $B = I - A$ construct the metric

$$ds^2 = B_t dt^2 + B_r dr^2 + B_\theta d\theta^2 + B_\phi d\phi^2 + 2B_{\phi t} d\phi dt,$$

where

$$\begin{aligned} B_t &= 1 - \frac{3m}{R} + \frac{m^2}{4R^2} + \frac{mr^2}{R^3} - \frac{m^2r^2}{2R^4} + \frac{m^2r^4}{4R^6} + \frac{4Pa^2m^2\sin\theta^2}{r^4} - \frac{400a^2m^2r^2\sin\theta^2}{49R^6} + \\ &\quad + \frac{240a^2m^2r^9\sin\theta^2}{49R^{13}} - \frac{36a^2m^2r^{16}\sin\theta^2}{49R^{20}} + O_3, \\ B_r &= -1 - \frac{2m\,r^2}{R^3} - \frac{2m^2r^2}{R^4} - \frac{2m^2r^4}{R^6} - \frac{4Pa^2m^2\sin\theta^2}{r^2} + O_3, \\ B_\theta &= 4P\,a^2m^2 - r^2 - 4Pa^2m^2\sin\theta^2 + O_3, \\ B_\phi &= 4P\,a^2m^2\sin\theta^2 - r^2\sin\theta^2 + O_3, \\ B_{\phi t} &= \frac{-2P\,a\,m\sin\theta^2}{r} + \frac{12P\,a\,m^2\sin\theta^2}{r} + \frac{20a\,m\,r^2\sin\theta^2}{7R^3} - \frac{6a\,m\,r^9\sin\theta^2}{7R^{10}} + O_3. \end{aligned}$$

It is seen that the metric functions are regular everywhere for $c > 0$.

4. – Physical properties of the source

We note that the source body is deformed with the boundary not being a proper sphere. For the whole interior region $I = A \cup B$ the excess of the proper equatorial radius R_e over the proper polar radius R_p is given by

$$\Delta R = R_e - R_p = 4a^2m^2 \int_0^R \frac{P(r)}{r^2} dr + O_3.$$

Then $\Delta R > 0$ and the source body is oblate.

Many examples of the *matching functions* $P(r)$ can be constructed. The polynomial of least degree satisfying the boundary conditions (3) has the form

$$P_1 = (R-c)^{-3}[c^2(3R-c) - 6cRr + 3(c+R)r^2 - 2r^3].$$

The truncated Fourier series of least multiple is given by

$$P_2 = \frac{1}{2} - \frac{1}{2}\cos\left(\frac{r-c}{R-c}\right)\pi = \sin^2\left(\frac{r-c}{R-c}\right)\frac{\pi}{2}\,.$$

If we impose the additional condition $P''(c) = P''(R) = 0$, the corresponding polynomial and trigonometric functions will, respectively, have the forms

$$\begin{aligned}
P_3 &= (R-c)^{-5}[-c^3(c^2-5cR+10R^2)+30c^2R^2r-30cR(c+R)r^2+ \\
&\quad +10(c^2+4cR+R^2)r^3-15(c+R)r^4+6r^5], \\
P_4 &= \frac{1}{2}-\frac{9}{16}\cos\left(\frac{r-c}{R-c}\right)\pi+\frac{1}{16}\cos^3\left(\frac{r-c}{R-c}\right)\pi = \\
&= \frac{1}{2}-\frac{3}{4}\cos\left(\frac{r-c}{R-c}\right)\pi+\frac{1}{4}\cos^3\left(\frac{r-c}{R-c}\right)\pi.
\end{aligned}$$

Another simple trigonometric function is given by

$$P_5 = 1-\cos^3\left(\frac{r-c}{R-c}\right)^3\frac{\pi}{2}.$$

Such functions will ensure the additional continuity of the tangential pressures.

In principle, a direct substitution of the metric tensor into the field equations yields exact expressions for the stress-energy tensor and hence for the fluid variables.

In the slowly rotating core A, the principal isotropic pressures p_A , the energy density μ_A and the angular velocity Ω_A have the form

$$\begin{aligned}
\kappa p_A^{(r)} = \kappa p_A^{(\theta)} = \kappa p_A^{(\phi)} &= m^2\left(\frac{3}{R^4}-\frac{3r^2}{R^6}\right)+O_3\,, \\
\kappa\mu_A &= \frac{6m}{R^3}+m^2\left(\frac{6}{R^4}-\frac{10r^2}{R^6}\right)+O_3\,, \\
\kappa\Omega_A &= \frac{-5ar^5}{R^7}+am\left(\frac{-20}{7R^3}+\frac{5r^5}{2R^8}+\frac{137r^7}{21R^{10}}\right)+ \\
&\quad + am^2\left(\frac{10}{7R^4}-\frac{10r^2}{3R^6}-\frac{5r^5}{4R^9}-\frac{29r^7}{84R^{11}}+\frac{137r^9}{18R^{13}}\right)+O_3\,.
\end{aligned}$$

In region B the derived results have the form

$$\begin{aligned}
&\kappa p_B^{(r)} = m^2\left(\frac{3}{R^4}-\frac{3r^2}{R^6}\right)+a^2m^2\,\cdot \\
&\cdot\Big[\frac{6P}{r^6}+\frac{6P^2}{r^6}-\frac{120P}{7R^3r^3}-\frac{P''}{r^2}+\frac{36Pr^4}{7R^{10}}+\Big(\frac{-5P'}{r^5}-\frac{10PP'}{r^5}+ \\
&+\frac{2P'^2}{r^4}+\frac{P''}{r^4}+\frac{2PP''}{r^4}+\frac{180P}{7R^3r^3}-\frac{20P'}{7R^3r^2}-\frac{20P''}{7R^3r}+\frac{240Pr^4}{7R^{10}}+ \\
&+\frac{90P'r^5}{7R^{10}}+\frac{6P''r^6}{7R^{10}}\Big)\sin\theta^2\Big]+O_3, \\
&\kappa p_B^{(\theta)} = m^2\left(\frac{3}{R^4}-\frac{3r^2}{R^6}\right)+a^2m^2\,\cdot \\
&\cdot\left[\frac{6P}{r^6}+\frac{6P^2}{r^6}-\frac{120P}{7R^3r^3}-\frac{P''}{r^2}+\frac{36Pr^4}{7R^{10}}+\left(\frac{-5P'}{r^5}-\frac{10PP'}{r^5}+\right.\right.
\end{aligned}$$

$$+\frac{2P'^2}{r^4}+\frac{P''}{r^4}+\frac{2PP''}{r^4}+\frac{180P}{7R^3r^3}-\frac{20P'}{7R^3r^2}-\frac{20P''}{7R^3r}+\frac{240Pr^4}{7R^{10}}+$$
$$+\frac{90P'r^5}{7R^{10}}+\frac{6P''r^6}{7R^{10}}\Bigg)\sin\theta^2\Bigg]+O_3,$$
$$\kappa p_B^{(\phi)}=m^2\left(\frac{3}{R^4}-\frac{3r^2}{R^6}\right)+a^2m\cdot$$
$$\cdot\left(\frac{-2P'^2R^3}{3r^6}+\frac{2P'P''R^3}{3r^5}-\frac{P''^2R^3}{6r^4}+\frac{20P'r^3}{R^7}-\frac{10P''r^4}{R^7}-\frac{150r^{12}}{R^{17}}\right)\cdot$$
$$\cdot\sin\theta^2+a^2m^2\Bigg[\frac{4P}{r^6}+\frac{4P^2}{r^6}+\frac{4P'}{r^3}-\frac{80P}{7R^3r^3}-\frac{2P''}{r^2}+\frac{24Pr^4}{7R^{10}}+$$
$$+\Bigg(\frac{24P}{r^6}-\frac{3P^2}{r^6}-\frac{P'^2R^2}{r^6}+\frac{8P'^2R^3}{r^6}-\frac{14P'}{r^5}-\frac{4PP'}{r^5}+\frac{P'P''R^2}{r^5}-$$
$$-\frac{8P'P''R^3}{r^5}+\frac{8P'^2}{9r^4}+\frac{2P''}{r^4}+\frac{PP''}{r^4}-\frac{P''^2R^2}{4r^4}+\frac{2P''^2R^3}{r^4}-\frac{6P'}{r^3}-\frac{8P'P''}{9r^3}+$$
$$+\frac{120P}{7R^3r^3}+\frac{2P''}{r^2}+\frac{17P''^2}{36r^2}+\frac{40P'}{7R^3r^2}-\frac{40P''}{7R^3r}+\frac{30P'r^3}{R^8}-\frac{120P'r^3}{R^7}+$$
$$+\frac{132Pr^4}{7R^{10}}-\frac{15P''r^4}{R^8}+\frac{60P''r^4}{R^7}-\frac{596P'r^5}{21R^{10}}+\frac{694P''r^6}{21R^{10}}-\frac{225r^{12}}{R^{18}}+$$
$$+\frac{488r^{14}}{R^{20}}\Bigg)\sin\theta^2\Bigg]+O_3,$$
$$\kappa\mu_B=\frac{6m}{R^3}+m^2\left(\frac{6}{R^4}-\frac{10r^2}{R^6}\right)+a^2m\cdot$$
$$\cdot\left(\frac{-2P'^2R^3}{3r^6}+\frac{2P'P''R^3}{3r^5}-\frac{P''^2R^3}{6r^4}+\frac{20P'r^3}{R^7}-\frac{10P''r^4}{R^7}-\frac{150r^{12}}{R^{17}}\right)\cdot$$
$$\cdot\sin\theta^2+a^2m^2\Bigg[\frac{-4P'}{r^3}+\frac{4P''}{r^2}+\Bigg(\frac{-9P^2}{r^6}-\frac{P'^2R^2}{r^6}+\frac{8P'^2R^3}{r^6}+\frac{12PP'}{r^5}+$$
$$+\frac{P'P''R^2}{r^5}-\frac{8P'P''R^3}{r^5}-\frac{10P'^2}{9r^4}-\frac{3PP''}{r^4}-\frac{P''^2R^2}{4r^4}+\frac{2P''^2R^3}{r^4}+$$
$$+\frac{6P'}{r^3}-\frac{8P'P''}{9r^3}-\frac{2P''}{r^2}+\frac{17P''^2}{36r^2}+\frac{30P'r^3}{R^8}-\frac{120P'r^3}{R^7}-$$
$$-\frac{72Pr^4}{R^{10}}-\frac{15P''r^4}{R^8}+\frac{60P''r^4}{R^7}-\frac{116P'r^5}{3R^{10}}+\frac{94P''r^6}{3R^{10}}-$$
$$-\frac{225r^{12}}{R^{18}}+\frac{506r^{14}}{R^{20}}\Bigg)\sin\theta^2\Bigg]+O_3,$$
$$\kappa\Omega_B=a\left(\frac{P'R^3}{3r^4}-\frac{P''R^3}{6r^3}-\frac{5r^5}{R^7}\right)+am\cdot$$
$$\cdot\Bigg(\frac{-20}{7R^3}-\frac{P'R^2}{6r^4}-\frac{2P'R^3}{r^4}+\frac{P}{2r^3}+\frac{P''R^2}{12r^3}+\frac{P''R^3}{r^3}+\frac{2P'}{9r^2}+$$

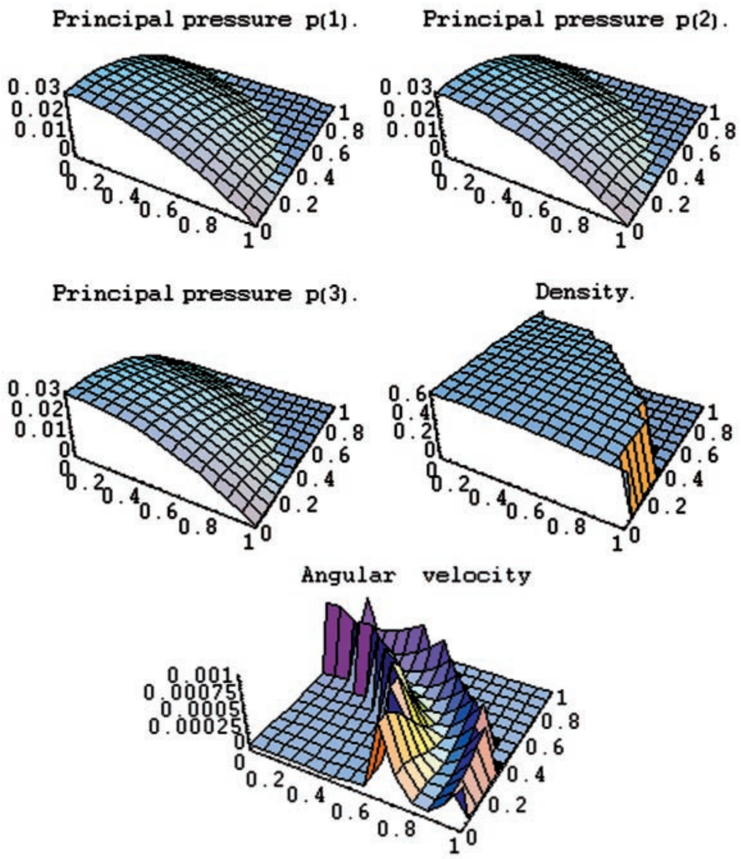

Fig. 1.

$$+\frac{5P''}{36r}+\frac{5r^5}{2R^8}+\frac{137r^7}{21R^{10}}\Bigg)+am^2\Bigg(\frac{10}{7R^4}+\frac{7P'}{27R^3}+\frac{P'R}{12r^4}+\frac{P'R^2}{r^4}-\frac{P}{4Rr^3}-\frac{P''R}{24r^3}-$$
$$-\frac{P''R^2}{2r^3}-\frac{7P'}{3r^2}-\frac{11P'}{36Rr^2}+\frac{7P''}{6r}+\frac{7P}{12R^3r}+\frac{P''}{36Rr}+\frac{35P''r}{216R^3}-$$
$$-\frac{10r^2}{3R^6}-\frac{5r^5}{4R^9}-\frac{29r^7}{84R^{11}}+\frac{137r^9}{18R^{13}}\Bigg)+O_3.$$

O_n denotes terms of order $\geq n$ in m/R and a.

This is an anisotropic fluid with variable density in differential rotation. Formulas in the slowly rotating core A are obtained by setting $a^2 = 0$.

As an illustration, we consider a strong field specified by the following values for the parameters (in geometric units):

$$m = 0.1, \quad a = -0.02, \quad c = 0.6, \quad R = 1.$$

Using the simple *matching function* $P = P_5$, we produced plots for the fluid variables through the source region $I = A \cup B$. The output is shown in fig. 1.

It can be seen that the density, the principal pressures and the angular velocity are all positive everywhere inside the source. Furthermore, $\mu > \Sigma p^{(i)}$ hold and hence the energy conditions [17] are satisfied. Besides, the pressures are monotonically decreasing outward, and hence our model, so constructed, is physically reasonable.

5. – Conclusion

A global solution describing a rigidly rotating disk of dust [18] could be considered a first step towards the exact description of rotating stellar models.

In this work we presented a technique for constructing families of composite fluid models for rotating neutron stars, each model corresponding to a particular choice of the matching function P. The technique has been successful in constructing physically reasonable models for rotating neutron stars in this work and, previously, for sources of the rotating Curzon metric [10] and the Kerr metric [16].

Contrary to the fluid sources of the Kerr metric [16], the angular velocity is always positive for the source of the Gutsunaev-Manko metric. This absence of counter-rotating part should be stressed as a nice feature from an astrophysical point of view. Our source body is not a proper sphere but it is oblate.

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