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Helicity and chirality precessions of maximally accelerated fermions

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Summary. — Caianiello's model is used to calculate the effect of maximal acceleration on the time evolutions of helicity and chirality for spin-(1/2) particles. The corresponding transition amplitudes may lead to significant effects on the cooling of supernovae and baryogenesis in the early Universe.

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1. – Introduction

This paper presents the calculation of the helicity and chirality precessions of maximally accelerated fermions according to the model of Caianiello and collaborators [1, 2]. The view frequently held [3, 4] that the proper acceleration of a particle has an upper limit finds in this model a geometrical interpretation epitomized by the line element

$$d\tilde{s}^2 = \tilde{g}_{\mu\nu}dx^\mu dx^\nu = \left(1 - \frac{|\ddot{x}|^2}{\mathcal{A}_m^2}\right)ds^2 = \sigma^2(x)ds^2 \quad (1.1)$$

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experienced by the accelerating particle along its worldline. In (1.1) $\mathcal{A}_m = 2mc^3/\hbar$ is the proper maximal acceleration (MA) of the particle of mass m, $\ddot{x}^\mu = \mathrm{d}^2x^\mu/\mathrm{d}s^2$ its acceleration and $\mathrm{d}s^2 = g_{\mu\nu}\mathrm{d}x^\mu\mathrm{d}x^\nu$ is the metric due to a background gravitational field. In the absence of gravity, $g_{\mu\nu}$ is replaced by the Minkowski metric tensor $\eta_{\mu\nu}$. Similar results have also been obtained in the context of Weyl space [5] and of a geometrical analogue of Vigier's stochastic theory [6].

Equation (1.1) has several implications for relativistic kinematics [7], the energy spectrum of a uniformly accelerated particle [8], the periodic structure as a function of momentum in neutrino oscillations [8], the Schwarzschild horizon [9], the expansion of the very early universe [10] and the mass of the Higgs boson [11,12]. It also makes the metric observer-dependent, as conjectured by Gibbons and Hawking [13], and leads in a natural way to hadron confinement [14].

The extreme large value that $\mathcal{A}_m$ takes for all known particles makes a direct test of eq. (1.1) very difficult. It is nonetheless imperative that a comparison of theory with experiment be carried out, if at all possible, lest the theory remain nothing but an interesting idea. Recently some realistic tests have been suggested [15-17] and a detailed comparison of MA corrections with experimental data has been carried out in the case of the Lamb shift of hydrogenic atoms with very encouraging results. It is however desirable that the search for physical consequences stemming from eq. (1.1) be expanded. It is with this intent that we analyze here the behaviour of helicity and chirality of particles subject to the MA constraint.

The outline of the paper is as follows. In sect. **2** we shortly derive the Dirac equation for a conformally flat metric, where the conformal factor is given by (1.1), and write down the Dirac Hamiltonian for a fermion accelerating in an electromagnetic field. Sections **3** and **4** are devoted to the study of the helicity evolution of maximally accelerated fermions. In sect. **5** we analyze the chirality precession. Section **6** contains the conclusions.

2. – The Dirac Hamiltonian

MA corrections due to the metric (1.1) appear directly in the Dirac equation written in covariant form [18] and referred to a local Minkowski frame by means of the vierbein field ${e_\mu}^a(x)$. We follow the notations and derivation of ref. [16]. From (1.1) one finds ${e_\mu}^a = \sigma(x){\delta_\mu}^a$, where Latin indices refer to the local inertial frame and Greek indices to a generic non-inertial frame. The covariant matrices $\gamma^\mu(x)$ satisfy the anticommutation relations $\{\gamma^\mu(x), \gamma^\nu(x)\} = 2\tilde{g}^{\mu\nu}(x)$, while the covariant derivative $\mathcal{D}_\mu \equiv \partial_\mu + \omega_\mu$ contains the total connection $\omega_\mu = \frac{1}{2}\sigma^{ab}\omega_{\mu ab}$, where $\sigma^{ab} = \frac{1}{4}[\gamma^a, \gamma^b]$, ${\omega_\mu}^a{}_b = (\Gamma^\lambda_{\mu\nu}\, {e_\lambda}^a - \partial_\mu {e_\nu}^a){e^\nu}_b$ and $\Gamma^\lambda_{\mu\nu}$ represents the usual Christoffel symbols. For conformally flat metrics ω_μ takes the form $\omega_\mu = (1/\sigma)\sigma^{ab}\eta_{a\mu}\sigma_{,b}$. By using the transformations $\gamma^\mu(x) = {e^\mu}_a(x)\gamma^a$ so that $\gamma^\mu(x) = \sigma^{-1}(x)\gamma^\mu$, where γ^μ are the usual constant Dirac matrices, the Dirac equation can be written in the form

$$\left[i\hbar\gamma^\mu\left(\partial_\mu + i\frac{e}{\hbar c}A_\mu\right) + i\frac{3\hbar}{2}\gamma^\mu(\ln\sigma)_{,\mu} - mc\sigma(x)\right]\psi(x) = 0\,. \tag{2.1}$$

From (2.1) one obtains the Hamiltonian operator

$$H = -i\hbar c\vec{\alpha}\cdot\vec{\nabla} + e\gamma^0\gamma^\mu A_\mu(x) - i\frac{3\hbar c}{2}\gamma^0\gamma^\mu(\ln\sigma)_{,\mu} + mc^2\sigma(x)\gamma^0\,, \tag{2.2}$$

which is in general non-Hermitean [18]. If σ varies slowly in time, or is time-independent as in the present case, this term can be neglected and hermiticity is recovered.

For convenience, let us rewrite the Hamiltonian (2.2) of a maximally accelerated fermion as follows:

$$H = H_0 + H' \,, \tag{2.3}$$

where

$$H_0 \equiv c\boldsymbol{\alpha} \cdot \boldsymbol{p} + \beta mc^2 \tag{2.4}$$

is the Hamiltonian for a free Dirac particle and

$$H^{'} \equiv -i\frac{3\hbar c}{2}\gamma^0\gamma^\mu(\ln\sigma)_{,\mu} + mc^2[\sigma(x)-1]\gamma^0 + e\gamma^0\gamma^\mu A_\mu(x) \tag{2.5}$$

is the MA Hamiltonian in the presence of an electromagnetic field. We treat $H^{'}$ as a perturbation of the free Dirac Hamiltonian.

For example, in the case of a charged particle in the central field, discussed in ref. [16], the conformal factor $\sigma(x)$ can be written as

$$\sigma(r) = \sqrt{1-\left(\frac{r_0}{r}\right)^4}\,, \quad r_0 \equiv \left(\frac{KZe^2}{m\mathcal{A}}\right)^{1/2} . \tag{2.6}$$

If the field is not central, $\sigma(x)$ is given by the corresponding accelerating field according to eq. (1.1).

3. – Helicity precession of maximally accelerated fermions

The helicity and chirality precessions of maximally accelerated fermions are calculated by using the Hamiltonian (2.2). In all calculations the chiral representation of the Dirac matrices is used.

The helicity operator h is defined as

$$h = \frac{\boldsymbol{\Sigma}\cdot\boldsymbol{p}}{|\boldsymbol{p}|} \,. \tag{3.1}$$

$\boldsymbol{p}$ is the momentum operator and $\boldsymbol{\Sigma}$ is given by

$$\boldsymbol{\Sigma} = \begin{pmatrix} \boldsymbol{\tau} & 0 \\ 0 & \boldsymbol{\tau} \end{pmatrix} , \tag{3.2}$$

where $\boldsymbol{\tau}$ are the Pauli matrices.

According to the Heisenberg equation of motion, the helicity precession is determined by

$$\dot{h} = \frac{1}{i\hbar}[h, H] \,, \tag{3.3}$$

where h is defined in terms of eq. (3.1) and H is the Hamiltonian operator (2.2).

By calculating the commutator one gets

$$\dot{h} = -\frac{e}{2|\boldsymbol{p}|}\Sigma^i\gamma^0\gamma^\alpha A_\alpha(x)_{,i} - \frac{e}{|\boldsymbol{p}|\hbar}\varepsilon_{ijk}\alpha^k A^j p^i + \frac{3\hbar c}{4|\boldsymbol{p}|}i\Sigma^i\gamma^0\gamma^\alpha(\ln\sigma)_{,\alpha i} + \\ + \frac{3c}{2|\boldsymbol{p}|}i\varepsilon_{jik}\alpha^k(\ln\sigma)^{,i}p^j - \frac{mc^2}{2|\boldsymbol{p}|}\Sigma^i\sigma(x)_{,i}\gamma^0. \tag{3.4}$$

According to eq. (3.4), the helicity precession of a spin-(1/2) particle depends on its mass, the electromagnetic field and the conformal factor σ. Two distinct cases can be discussed, corresponding to the possibilities $\sigma \approx 0$ and $\sigma \approx 1$. The first case implies that the acceleration of the particle is extremely large. This could apply to fermions in accelerators or to some special astrophysical situations. When σ is almost zero, the third and fourth terms in eq. (3.4) dominate the helicity precession and we can omit the first two terms if the electromagnetic field is sufficiently weak. The ratio of the third and fourth terms to the fifth one are calculated as follow:

$$\frac{\text{third term}}{\text{fifth term}} = \frac{3i\hbar\Sigma^i\gamma^0\gamma^\alpha}{-2mc\Sigma^j\gamma^0}\frac{(\ln\sigma)_{,\alpha i}}{\sigma(x)_{,j}} = -\frac{3i\hbar\Sigma^i\gamma^0\gamma^\alpha}{2mc\Sigma^j\gamma^0}\frac{1}{\sigma(x)}\left(\frac{\sigma_{,\alpha i}}{\sigma_{,j}} - \frac{\sigma_{,\alpha}\sigma_{,i}}{\sigma\sigma_{,j}}\right), \tag{3.5}$$

$$\frac{\text{fourth term}}{\text{fifth term}} = \frac{3i\varepsilon_{jik}\alpha^k p^j}{-mc\Sigma^l\gamma^0}\frac{(\ln\sigma)^{,i}}{\sigma(x)_{,l}} = -\frac{3i\varepsilon_{jik}\alpha^k p^j}{mc\Sigma^l\gamma^0}\frac{1}{\sigma}\frac{\sigma^{,i}}{\sigma_{,l}}. \tag{3.6}$$

If the derivatives of σ remain finite, the fifth term is negligible compared to the third and fourth terms because σ appears in the denominator of both eqs. (3.5) and (3.6). Then the helicity precession depends only on σ and has the same value for any spin-(1/2) particle with the same acceleration. On the other hand, when the acceleration is extremely small, $\sigma \sim 1$ and the MA contribution vanishes. This may apply, for instance, to the bound electron in a hydrogenic atom because the electron has only a small probability of getting an extreme acceleration. In fact the probability distribution of the electron peaks at $r \approx a_0$ where the acceleration of the electron $a \approx \frac{1}{4\pi\epsilon_0}\frac{e^2}{a_0^2 m_e} \sim 10^7\mathrm{m/s}^2$, while its maximal acceleration is $\sim 10^{29}\mathrm{m/s}^2$. According to eq. (3.4) the helicity precession also depends on the electromagnetic field. When $A_\mu = 0$, the first two terms also vanish and the helicity is constant as expected [19].

4. – Helicity evolution

In the previous section, we have derived the helicity rate of change $\dot{h}$. In this section, we evaluate the evolution of helicity over time when all corrections to the Dirac Hamiltonian can be considered as perturbations. In general one has

$$h(t) = h(0) + \int_0^t \dot{h}(t')\mathrm{d}t', \tag{4.1}$$

where $h(0)$ is the helicity state at time $t = 0$, and t is the time over which the particle's acceleration differs from zero. An exact evaluation of eq. (4.1) is obviously very difficult. In order to calculate the effect of MA on the helicity evolution, we use a perturbation method to get a first-order solution.

Starting from the zeroth-order time derivative of an arbitrary operator F

$$\dot{F} = \frac{i}{\hbar}[H_0, F], \tag{4.2}$$

we obtain the following equations:

$$\dot{p}_i = 0, \quad \dot{x}_i = c\alpha_i, \quad \dot{A}_\mu(x) = c\alpha^i A_\mu(x)_{,i}\,, \quad \dot{\sigma}(x) = c\alpha^i \sigma(x)_{,i}\,, \tag{4.3}$$

$$\dot{\alpha}_i = -\frac{2i}{\hbar}(\alpha_i H_0 - cp_i), \quad \dot{\gamma}^0 = -\frac{2i}{\hbar}(\gamma^0 H_0 - mc^2), \quad \dot{\Sigma}_i = -\frac{2c}{\hbar}\varepsilon_{ijk}\alpha^k p^j\,. \tag{4.4}$$

By integrating eqs. (4.3), (4.4), we get the zeroth-order solutions

$$p_i(t) = p_i(0) = p_i, \tag{4.5}$$

$$\alpha_i(t) = cp_i(0)H_0^{-1} + [\alpha_i(0) - cp_i(0)H_0^{-1}]e^{-2iH_0 t/\hbar}, \tag{4.6}$$

$$\begin{aligned} x_i(t) = {} & x_i(0) - \frac{i\hbar c}{2}[\alpha_i(0)H_0^{-1} - cp_i(0)H_0^{-2}] + \\ & + c^2 p_i(0)H_0^{-1}t + \frac{i\hbar c}{2}[\alpha_i(0)H_0^{-1} - cp_i(0)H_0^{-2}]e^{-2iH_0 t/\hbar}, \end{aligned} \tag{4.7}$$

$$\gamma^0(t) = -mc^2 H_0^{-1} + [\gamma^0(0) + mc^2 H_0^{-1}]e^{-2iH_0 t/\hbar}, \tag{4.8}$$

$$\Sigma_i(t) = \Sigma_i(0) + ic\varepsilon_{ijk}\alpha^j(0)p(0)^k H_0^{-1}[1 - e^{-2iH_0 t/\hbar}], \tag{4.9}$$

where H_0 is the eigenvalue of the operator (2.4). We assume for simplicity that $A_\mu(x)$ and $\sigma(x)$ are independent of t. Substituting the above equations into eq. (3.4), we obtain

$$\dot{h} = C_1 + C_2 e^{-2iH_0 t/\hbar}, \tag{4.10}$$

where

$$\begin{aligned} C_1 = {} & -\frac{e}{2|\boldsymbol{p}|}A_0(x)_{,i}\Sigma^i(0) - \frac{iec}{2|\boldsymbol{p}|}A_0(x)^{,i}\varepsilon_{ijk}\alpha^j(0)p^k(0)H_0^{-1} - \\ & -\frac{ec}{2|\boldsymbol{p}|}A_j(x)_{,i}\Sigma^i(0)p^j(0)H_0^{-1} - \frac{ec}{|\boldsymbol{p}|\hbar}\varepsilon_{ijk}A^j p^i(0)p^k(0)H_0^{-1} + \\ & + \frac{3i\hbar c^2}{4|\boldsymbol{p}|}(\ln\sigma)_{,\,ji}\{\Sigma^i(0)p^j + i\varepsilon_{ilk}\alpha^l(0)p^k\alpha^j(0)\}H_0^{-1} + \\ & + \frac{mc^3}{2|\boldsymbol{p}|}\sigma(x)^{,i}\{\Sigma_i(0)mc - i\varepsilon_{ijk}\alpha^j(0)p^k\gamma^0(0)\}H_0^{-1}, \end{aligned} \tag{4.11}$$

$$\begin{aligned} C_2 = {} & -\frac{e}{2|\boldsymbol{p}|}A_j(x)_{,i}\Sigma^i(0)\alpha^j(0) - \frac{e}{|\boldsymbol{p}|\hbar}\varepsilon_{ijk}A^j p^i(0)\alpha^k(0) + \\ & + \frac{3i\hbar c}{4|\boldsymbol{p}|}(\ln\sigma)_{,ji}\Sigma^i(0)\alpha^j(0) + \frac{3ic}{2|\boldsymbol{p}|}\varepsilon_{ijk}(\ln\sigma)^{,i}p^j\alpha^k(0) - \\ & - \frac{mc^2}{2|\boldsymbol{p}|}\sigma(x)_{,i}\Sigma^i(0)\gamma^0(0) + \frac{ec}{\hbar|\boldsymbol{p}|}\varepsilon_{ijk}A^j p^i(0)p^k(0)H_0^{-1} + \\ & + \frac{iec}{2|\boldsymbol{p}|}A_0(x)^{,i}\varepsilon_{ijk}\alpha^j p^k(0)H_0^{-1} + \frac{ec}{2|\boldsymbol{p}|}A_j(x)_{,i}\Sigma^i(0)p^j(0)H_0^{-1} - \end{aligned} \tag{4.12}$$

$$-\frac{3i\hbar c^2}{4|\boldsymbol{p}|}(\ln\sigma)^{,ji}\{p_j\Sigma_i(0)+i\varepsilon_{ilk}\alpha^l(0)\alpha_j(0)p^k\}H_0^{-1}-$$
$$-\frac{mc^2}{2|\boldsymbol{p}|}\sigma(x)^{,i}\{\Sigma_i(0)mc^2-ic\varepsilon_{ijk}\alpha^j(0)p^k\gamma^0(0)\}H_0^{-1}.$$

The integration of eq. (4.10) gives the solution

$$h(t)=B_0+B_1t+B_2e^{-2iH_0t/\hbar}, \tag{4.13}$$

where t is again the time during which the particle accelerates and

$$B_0=h(0)+\frac{ie}{2|\boldsymbol{p}|}\varepsilon_{ijk}A^jp^i(0)\alpha^k(0)H_0^{-1}+ \tag{4.14}$$
$$+\frac{3\hbar^2c}{8|\boldsymbol{p}|}(\ln\sigma)_{,ji}\Sigma^i(0)\alpha^j(0)H_0^{-1}+\frac{3\hbar c}{4|\boldsymbol{p}|}\varepsilon_{ijk}(\ln\sigma)^{,i}p^j\alpha^k(0)H_0^{-1},$$

$$B_1=-\frac{iec}{2|\boldsymbol{p}|}A_0(x)^{,i}\varepsilon_{ijk}\alpha^j(0)p^k(0)H_0^{-1}+ \tag{4.15}$$
$$+\frac{3i\hbar c^2}{4|\boldsymbol{p}|}(\ln\sigma)^{,ji}\{\Sigma_i(0)p_j+i\varepsilon_{ilk}\alpha^l(0)\alpha_j(0)p^k(0)\}H_0^{-1}-$$
$$-\frac{imc^3}{2|\boldsymbol{p}|}\sigma(x)^{,i}\varepsilon_{ijk}\alpha^j(0)p^k\gamma^0(0)H_0^{-1},$$

$$B_2=-\frac{ie}{2|\boldsymbol{p}|}\varepsilon_{ijk}A^jp^i(0)\alpha^k(0)H_0^{-1}- \tag{4.16}$$
$$-\frac{3\hbar^2c}{8|\boldsymbol{p}|}(\ln\sigma)_{,ji}\sigma^i(0)\alpha^j(0)H_0^{-1}-\frac{3\hbar c}{4|\boldsymbol{p}|}\varepsilon_{ijk}(\ln\sigma)^{,i}p^j\alpha^k(0)H_0^{-1}.$$

In deriving B_0, B_1 and B_2 we have used eqs. (4.3), (4.4). In all these calculations, only terms to order H_0^{-1} have been kept. According to eq. (3.1), $0\leq|h|\leq\frac{1}{2}$. This condition defines the limits of validity of eq. (4.13). B_0 is determined by the initial conditions. B_0 and B_2 are small if eA_μ, $\sigma_{,i}$, $\hbar c(\ln\sigma)_{,i}$, and $(\hbar^2c/|\vec{p}|)(\ln\sigma)_{,ij}$ are all small relative to H_0. From eqs. (4.15) and (4.16), we find that B_1t is also small when the length ct is small relative to the distance over which $A_0(x)$ and $\sigma(x)_{,i}$ change. In particular, when $A_\mu=0$, B_1 vanishes for $\sigma\to1$. Hence B_1 is present only as long as as $\sigma\neq1$, that is as long as the particle accelerates ($\sigma\approx1$ and $\sigma_{,i}\neq0$). If the fermion is an electron in the field of the nucleus and the electron is in an eigenstate, then $\Delta E=0$ and the acceleration vanishes leading to $\sigma=1$. Then the contributions of MA to B_1 and B_2 vanish and we find $h(t)=h(0)$, that is the helicity is constant [19]. The exponential term in eq. (4.13) reveals the same time-dependence as in the *Zitterbewegung* of a massive spin-(1/2) particle [20]. Finally, it is interesting to notice that when an electromagnetic field is present, B_0, B_1 and B_2 are all non-vanishing even when $\sigma=1$ rigorously and $\sigma_{,i}=0$ [19].

5. – Chirality precession of maximally accelerated fermions

Let us now discuss the chirality precession induced by the maximal acceleration corrections to the Dirac Hamiltonian. In the chiral representation [21], the Dirac matrices are

$$\gamma_5 = \gamma^5 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}, \quad \gamma^0 = \begin{pmatrix} 0 & -I \\ -I & 0 \end{pmatrix}, \quad \boldsymbol{\alpha} = \begin{pmatrix} \boldsymbol{\tau} & 0 \\ 0 & -\boldsymbol{\tau} \end{pmatrix}.$$

It is easy to calculate the commutator between the γ^5 matrix and the Hamiltonian operator (2.2). It gives

$$i\hbar\dot{\gamma}^5 \equiv [\gamma^5, H] = 2mc^2\sigma(x)\gamma^5\gamma^0\,. \tag{5.1}$$

In the standard case $\sigma \equiv 1$, if the mass of a particle is vanishingly small, eq. (5.1) reduces to

$$\dot{\gamma}^5 \to 0\,, \tag{5.2}$$

which implies conservation of the chirality quantum number. If MA corrections are taken into account, then $\sigma \neq 1$ and the limit $m \to 0$ leads to a divergent conformal factor and the model proposed fails. If the acceleration is small with respect to MA, say $\sigma(x) \approx 1$, then the expectation value $\langle\dot{\gamma}^5\rangle$ is given by

$$\frac{\mathrm{d}}{\mathrm{d}t}\langle\gamma^5\rangle = \frac{i}{\hbar}\langle[H, \gamma^5]\rangle = \frac{2i}{\hbar}mc^2\langle\gamma^0\gamma^5\rangle, \tag{5.3}$$

as follows from (5.1) and the chirality transition probability depends only on the mass of the particle.

In order to discuss the chirality transitions in greater detail, we calculate the expectation value of γ^5 between eigenstates $|E_1\rangle$ and $|E_2\rangle$, where $E_2 \approx E_1 \approx E$. From eq. (5.1) we obtain

$$\begin{aligned}\langle\dot{\gamma}^5\rangle \approx \frac{i}{\hbar}\frac{mc^2}{2E^2}\langle\hbar c^2\{2\sigma_,{}^i p^j\varepsilon_{ijk}\gamma^k + \hbar\sigma_{,ij}\alpha^j\gamma^i\gamma^5\} +&\\ +2i\hbar ce\{A_0(x)\sigma_{,i}\gamma^i\gamma^5 + A^j(x)\sigma_,{}^i i\varepsilon_{ijk}\gamma^k\} +&\\ +i\hbar cB(x)_{,i}\gamma^i\gamma^5 + 2eA_0(x)B(x)\gamma^0\gamma^5 +&\\ +3i\hbar^2c^2\varepsilon_{ijk}\sigma^{,i}(\ln\sigma)^{,j}\gamma^k\rangle\,,&\end{aligned} \tag{5.4}$$

where terms containing derivatives of σ with respect to time have been omitted and

$$B(x) \equiv 2eA_0(x)\sigma(x). \tag{5.5}$$

Decomposing the wavefuntion ψ into components of definite chirality

$$\psi = \begin{pmatrix} c_1\psi_+ \\ c_2\psi_- \end{pmatrix}, \tag{5.6}$$

and using the normalization condition

$$|c_1|^2 + |c_2|^2 = 1, \tag{5.7}$$

we find

$$\langle\psi|\gamma^5|\psi\rangle = (c_1^*\psi_+^+, c_2^*\psi_-^+)\begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}\begin{pmatrix} c_1\psi_+ \\ c_2\psi_- \end{pmatrix} = |c_1|^2 - |c_2|^2. \tag{5.8}$$

By differentiating eq. (5.8) and using (5.7), we get

$$\frac{\mathrm{d}|c_1|^2}{\mathrm{d}t} = -\frac{\mathrm{d}|c_2|^2}{\mathrm{d}t} = \frac{1}{2}\langle\dot{\gamma}^5\rangle\,. \tag{5.9}$$

$\dot{\gamma}^5$ is given by (5.4). We thus find that the chirality transition probability depends on $\sigma(x)$ and its derivatives. If $\sigma \to 0$, but its derivatives do not vanish, then chirality transitions take place according to eq. (5.9) even though the effective mass of the particle $m\sigma(x)$ tends to zero. This contrasts with the behaviour of a particle in the absence of MA effects. Chirality transitions occur, in that case, only if the mass of the particle is finite. If, on the other hand, a particle's acceleration is extremely large and the derivatives of σ vanish, then the transition probability tends to zero and the chirality remains constant. In both instances the behaviour of the derivatives of σ plays a predominant role in chirality transitions. Finally, if $\sigma(x) \approx 1$ and its derivatives vanish, then chirality transitions can take place only if $m \neq 0$ (see eq. (5.3)) and at the same time $A_0(x) \neq 0$. If $\sigma(x) \approx 1$, but its derivatives are non-vanishing, then transitions can take place according to eq. (5.9).

6. – Conclusions

We have calculated changes in the helicity and chirality of spin-(1/2) particles induced by MA. These changes are not obvious and represent additional contributions to the usual, known spin-inertia coupling terms [22-24]. Helicity and chirality transitions can occur for both $\sigma \approx 1$ and $\sigma \approx 0$ provided $\sigma_{,i} \neq 0$. This may appear at first surprising. However $\sigma_{,i}$ plays the role of a gravitational field and the latter is known to affect helicity and chirality transitions. The line element (1.1) can therefore lead to a new class of phenomena. Neutrinos appear particularly interesting in this connection, because the transitions discussed can lead to sterile particles with significant consequences for the cooling of supernovae [24, 25]. Baryogenesis in the early Universe would also be affected by the transitions discussed.

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