

On accelerated clocks and the quantum theory (*)

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Summary. — It is shown that the locality hypothesis of relativity breaks down for large proper accelerations which are relevant to semiclassical phenomena. A general modification for the rate of accelerated clocks incorporating the effect of proper acceleration is thus proposed. Connection is made with Caianiello's quantum line element.

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In the construction of the theory of relativity, the locality hypothesis (LH) of Einstein played a fundamental role. This hypothesis states that [1] “an accelerated observer O is locally equivalent to a hypothetical inertial observer $\bar{O}$ which has the same velocity at the given point in space-time”. The physical and operational significance of the LH is clear: It determines the results of measurements performed by accelerated observers.

According to the LH, the acceleration of O with respect to $\bar{O}$ (*i.e.* the proper acceleration of O) at any point in space-time does not affect the results of its measurements performed at that point. In particular, locally,

$$(1) \quad d\sigma = d\tau,$$

where $d\sigma$ ($d\tau$) denotes the rate of the clock carried by O ($\bar{O}$). Relation (1) may be regarded as an alternative (perhaps somewhat weaker) statement of the LH, usually referred to as the ideal-clock hypothesis [2] or the accelerated-clock principle [3]. There has been no concrete observational evidence that causes one to suspect the correctness of this hypothesis. However, alternatives, which assume that the rate of a clock depends on its proper acceleration, have been proposed and investigated [3-7].

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Because of finite precision of real measurements, however, the LH involves a finite local neighborhood in space-time (and not only a point) in which the proper acceleration of O can be neglected. Classically, the more accurately we bin our measurements, the smaller this neighborhood becomes. To see the extent of this local region, let α represent the proper acceleration of O (restricting to one dimension for simplicity) at the point P , the origin of $\bar{O}$, where the velocities of O and $\bar{O}$ coincide. Then, in $\bar{O}$'s experience, after a time interval $\Delta\tau$, O acquires a velocity Δu and a displacement Δx . Clearly, within any desired accuracy, O and $\bar{O}$ can still be regarded as equivalent, provided $\Delta u/c$ lies sufficiently close to zero, *i.e.* provided $\Delta\tau$ and Δx are sufficiently small. In other words, LH requires that $\Delta u/c \ll 1$, yielding

$$(2) \quad \Delta u = \alpha \Delta\tau \ll c, \quad \Delta x = \frac{1}{2} \alpha \Delta\tau^2,$$

where the change in proper acceleration during $\Delta\tau$ is assumed negligible. The open neighborhood associated with the LH thus involves the space-time region $(-\Delta\tau, \Delta\tau)$, $(-\Delta x, \Delta x)$ about the point P . In this neighborhood O and $\bar{O}$ can be regarded as equivalent and the LH operates. Now if Δp denotes the momentum of O (in $\bar{O}$'s experience) acquired during $\Delta\tau$, we obtain from (2) characteristic scales for the domain of validity of the LH, namely

$$(3) \quad \Delta\tau \ll \frac{c}{\alpha}, \quad \Delta x \ll \frac{c^2}{2\alpha}, \quad \Delta p \ll m_0 c,$$

where m_0 is the rest mass of O and the smallness of the above quantities depends, of course, on the desired accuracy. If O is a classical (point-like) object, then ideally, *i.e.* in the limit of infinite resolution of measuring devices, we have $\Delta\tau = \Delta x = \Delta p = 0$ and the LH will hold strictly at P . The local phase-space neighborhood of P , whose volume satisfies $\Delta x \Delta p \ll m_0 c^3 / 2\alpha$ by (3), can therefore be made arbitrarily small by increasing the resolution. However, if O is a quantum object, there will be a lower bound on the volume of this local region even in the ideal limit of infinite resolution. This is because, precisely at P , with the uncertainty principle operating even in the ideal limit, the momentum of O is undefined rendering the equivalent observer $\bar{O}$ undefinable and hence the LH unoperational. We must, therefore, consider a local neighborhood for the LH to operate, the phase-space volume of which satisfies

$$(4) \quad \frac{\hbar}{2} \leq \Delta x \Delta p \ll \frac{m_0 c^3}{2\alpha}.$$

Clearly (4) applies at the semiclassical level where particle trajectories and hence proper accelerations are still defined. The lower bound limits the size of the local phase-space region associated with the LH, as a result of the intrinsic indeterminacy involved according to the orthodox quantum theory, in the simultaneous measurement of position and momentum.

At the semiclassical level, relation (4) provides a proper acceleration scale (which is particle dependent) for the domain of validity of the LH, namely

$$(5) \quad \alpha \leq \alpha_0 = \frac{c^2}{\lambda},$$

where $\lambda = \hbar/m_0 c$ is the Compton wavelength (divided by 2π) of the particle. Thus for $\alpha \ll \alpha_0$ (“small proper accelerations”) the proper acceleration may be neglected and the LH operates. However, for large values of the proper acceleration $\alpha \sim \alpha_0$, the LH fails so that one cannot assume (1). Incidentally, based on different approaches and for varying reasons, a maximal proper acceleration has been proposed for quantum particles in the literature [8-11] which coincides (up to a factor of two) with the characteristic acceleration α_0 defined above.

It is interesting to note the role of the Compton wavelength λ in (5). Classical particles have a vanishing Compton wavelength rendering their characteristic acceleration α_0 practically infinite. For such particles the LH, therefore, becomes a valid assumption. In contrast, in phenomena involving quantum particles, α_0 is finite (but typically very large) and should play a decisive role when $\alpha \sim \alpha_0$, thereby invalidating the LH. This observation, in view of the wave-particle duality, conforms with the generally held view [1, 12, 13] that wave phenomena, and in particular electromagnetic waves, do not generally comply with the LH.

It is instructive to present the foregoing arguments from an intuitive viewpoint. The LH was based on the observation that, however large the proper acceleration of a particle, one can choose sufficiently small segments of its worldline along which the curvature (which is a measure of the particle’s proper acceleration) may be neglected. The straight worldline segments thus obtained pertain to the hypothetical local inertial observers of the LH. Clearly, the larger the curvature of the worldline, the smaller these segments become (as is also evident in (3)). However, for a quantum particle (at the semiclassical level), one cannot shrink segments of its worldline arbitrarily (because of the uncertainty principle), so that one cannot ignore the local curvature when its value is sufficiently large. In other words, the LH breaks down for large proper accelerations which are relevant to semiclassical phenomena. Equation (1) (the ideal-clock hypothesis), therefore, does not hold generally and must be modified.

At the semiclassical level, a natural modification of the LH and hence eq. (1) would, therefore, be to assume that the rate of an accelerated clock is influenced by the “normalized” magnitude of its instantaneous proper acceleration. More precisely, we hypothesize that, locally,

$$(6a) \quad d\sigma = f\left(\frac{\alpha}{\alpha_0}\right) d\tau,$$

where f is a universal function satisfying

$$(6b) \quad \lim_{\alpha/\alpha_0 \rightarrow 0} f\left(\frac{\alpha}{\alpha_0}\right) = 1,$$

so that one restores (1) in the classical limit of small proper acceleration. Needless to say, such a modification changes the structure of the space in which particles live.

An example of this modification can be shown to follow from the work of Caianiello and his co-workers [14, 15]. Based on different arguments, they proposed a model for a geometric realization of quantum mechanics via the line element (in our notation),

$$(7) \quad ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu + \frac{\hbar^2}{m_0^2 c^4} \eta_{\mu\nu} dU^\mu dU^\nu.$$

Here (x^μ, U^μ) are independent coordinates on an eight-dimensional phase-space in which the quantum particle (of mass m_0) is regarded as a four-dimensional hypersurface. To make contact with (6), we now consider the semiclassical level at which particles are point-like and trajectories still defined, with the associated clock rate $d\sigma = ds/c$. Along such trajectories U^μ corresponds to the relativistic 4-velocity of the particle, i.e. $U^\mu = dx^\mu/d\tau$ with $c^2 d\tau^2 = \eta_{\mu\nu} dx^\mu dx^\nu$. This enables us to use the relativistic relation $\eta_{\mu\nu} dU^\mu dU^\nu = -\alpha^2 d\tau^2$ to reduce (7) to the form (6),

$$(8) \quad d\sigma^2 = d\tau^2 \left(1 - \frac{\alpha^2}{\alpha_0^2} \right).$$

Cast in this form, α_0 naturally appears as the maximal proper acceleration of the particle. Note that, because of the constraint $\eta_{\mu\nu} U^\mu U^\nu = c^2$, the particle now lives in a seven-dimensional manifold embedded in the phase-space.

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