The cosmological model with scalar, spin and torsion field (*)

S.-W. KIM
Department of Science Education, Ewha Women's University - Seoul 120-750, Korea

(ricevuto il 30 Luglio 1996)

Summary. — In this paper the Brans-Dicke cosmological model with spin and torsion field is considered. The effects of the spin and the scalar field on the expansion rate and the types of inflation are discussed. Since the torsion field is generated by the scalar field as well as by the spin, the torsion effect by the scalar field still remains even if there are no spins. It is shown that the expansion rate is the power law faster than the extended inflation. The bubble nucleation rates are the same as in the usual Brans-Dicke case.

PACS 04.20 – Classical general relativity.
PACS 98.80.Cq – Particle-theory and field-theory models of the early Universe (including cosmic pancakes, cosmic strings, chaotic phenomena, inflationary universe, etc.).
PACS 01.30.Cc – Conference proceedings.

1. – Introduction

The Einstein-Cartan(EC) theory [1] is a natural extension of Einstein's general theory of relativity. It incorporates the spin properties of matter by including the torsion and describes their influence on the geometric structure of space-time, curvature and torsion. There had been a variety of cosmological models in the context of the EC theory. In the seventies, the torsion was mainly used in order to remove the initial singularity [2, 3]. Later they were used to investigate their effects on the early stage of the Universe [4]. It was shown that in the EC manifold the inflation can arise without false vacuum energy [5]. In that model, the spin plays a dominant role in causing the inflation. Guth's inflationary model [6] was also extended to EC theory [7]. In that model the initial singularity is avoided and the minimal radius of the universe is influenced by the false vacuum energy. Recently, a cosmological model with torsion and spin under rigid rotation has been studied as a generalization of the Gödel universe [8, 9].

(*) Paper presented at the Fourth Italian-Korean Meeting on Relativistic Astrophysics, Rome-Gran Sasso-Pescara, July 9-15, 1995.

Another extension of Einstein's relativity theory is the Brans-Dicke (BD) theory [10]. It is based on Mach's principle and Dirac's conjecture. The scalar field introduced in the BD theory is the reciprocal value of the gravitational constant. There are also several cosmological models using the scalar field [11]. Among the cosmological models, the extended inflation model using the BD-type scalar field is one of the most attentioned models, because of its power law expansion [12]. Even though there remain several problems [13], the BD scalar field has been playing an important role in cosmological models. Recently, perfect-fluid scalar-tensor cosmology is studied for the case of field-dependent parameter [14].

The present rate of the gravitational constant G is about

$$\left(\frac{\dot{G}}{G}\right)_0 = -3H_0 = -t_0^{-1} \sim 10^{-18}\,\mathrm{s}^{-1}$$

for $H_0 \sim 500$ Km/s/Mpc. Since it is proportional to t^{-1}, there is no doubt that the effect is dominant in the early universe. One can expect that spin effects are of equal importance with mass terms whenever the number density $N(=M/K\hbar^2)$ or the critical mass density $\varrho_s(=M^2/K\hbar^2)$ is achieved [7]. For example, $\varrho_s = 10^{47}\,\mathrm{g/cm^3}$ for electrons, and $\varrho_s = 10^{54}\,\mathrm{g/cm^3}$ for neutrons. Thus, the early cosmological standard model has to be reconsidered by taking into account both the scalar and torsion fields.

As a unification of the EC theory and the BD theory, there was the BD theory in space-time with torsion field [15]. The important thing in the unified theory is that the torsion field can be determined by the scalar field as well as by the spin. Thus, even though there are no spins, the torsion by the scalar field is crucial in cosmological models. Therefore, it is most important to investigate the cosmology in the space-time with torsion and BD scalar fields simultaneously. In this paper, we consider the cosmological model in EC manifold with scalar field. The purpose of this paper is to see the effect of two objects when they exist together, and compare their results with others.

2. – Brans-Dicke theory with torsion

We start from the gravitational Lagrangian

(1) $$I_g = \int \mathrm{d}^4x \sqrt{-g}\left(-\phi R + \omega \frac{\phi^{,\mu}\phi_{,\mu}}{\phi}\right),$$

where

(2) $$R = e_a{}^{\mu} e_b{}^{\nu} R^{ab}{}_{\mu\nu} = e_a{}^{\mu} e_b{}^{\nu}(\omega^{ab}{}_{\mu,\nu} - \omega^{ab}{}_{\nu,\mu} + \omega^{a}{}_{c\nu}\omega^{cb}{}_{\mu} - \omega^{a}{}_{c\mu}\omega^{cb}{}_{\nu})$$

is the scalar curvature. The tetrad field $e^{a\mu}$ and the spin connection $\omega^{ab}{}_{\mu}$ are independent variables. Here, the BD parameter ω is defined as the positive value. The metric is given by $g_{\mu\nu} = \eta_{ab} e^a{}_{\mu} e^b{}_{\nu}$ with $\eta_{ab} = \mathrm{diag}(-1, 1, 1, 1)$ and $g = \det(g_{\mu\nu})$. The torsion tensor is defined as

(3) $$F^a{}_{\mu\nu} = e^a{}_{\mu,\nu} - e^a{}_{\nu,\mu} + \omega^a{}_{c\nu} e^c{}_{\mu} - \omega^a{}_{c\mu} e^c{}_{\nu}.$$

The matter Lagrangian is given as

(4) $$I_M = 16\pi \int d^4x \sqrt{-g}\,\mathscr{L}_M(e_a{}^\mu,\ \omega_{ab}{}^\mu;\ \psi_i)\,.$$

The field equations for each variable can be obtained by varying the total action independently with respect to ϕ, $e_a{}^\mu$, and $\omega_{ab}{}^\mu$. The field equation for ϕ is

(5) $$R - \frac{\omega}{\phi^2}\phi^{,\mu}\phi_{,\mu} + 2\omega\frac{\Delta\phi}{\phi} - 2\frac{\omega}{\phi}F^\mu{}_{\lambda\mu}\phi^{,\lambda} = 0\,,$$

where the generalized d'Alembertian in the EC manifold is defined by

(6) $$\Delta\phi = \phi^{,\mu}{}_{;\mu} = \phi^{,\mu}{}_{,\mu} + \Gamma^\mu{}_{\lambda\mu}\phi^{,\lambda}.$$

Here $\Gamma^\mu{}_{\lambda\mu}$ denotes the linear affine connection which is written as

(7) $$\Gamma^\mu{}_{\lambda\nu} = \begin{Bmatrix}\mu\\ \lambda\nu\end{Bmatrix} - K^\mu{}_{\lambda\nu}\,,$$

where the quantity $\begin{Bmatrix}\mu\\ \lambda\nu\end{Bmatrix}$, the Christoffel symbol computed from the metric tensor $g_{\mu\nu}$, is familiar. The contortion tensor $K^\mu{}_{\lambda\nu}$ is given by

(8) $$K^\mu{}_{\lambda\nu} = \frac{1}{2}(-F^\mu{}_{\lambda\nu} + F_\lambda{}^\mu{}_\nu + F_\nu{}^\mu{}_\lambda)$$

relating $F^\mu{}_{\lambda\nu}$ with

(9) $$F^\mu{}_{\lambda\nu} = \Gamma^\mu{}_{\lambda\nu} - \Gamma^\mu{}_{\nu\lambda} = e_a{}^\mu F^a{}_{\lambda\nu}.$$

The field equation for $e_a{}^\mu$ is

(10) $$G^\lambda{}_\mu = \frac{8\pi}{\phi}T^\lambda{}_\mu - \frac{\omega}{\phi^2}\left(\frac{1}{2}\phi_{,\tau}\phi^{,\tau}\delta^\lambda_\mu - \phi_{,\mu}\phi^{,\lambda}\right),$$

where the Einstein tensor $G^\lambda{}_\mu = R^\lambda{}_\mu - (1/2)\,\delta^\lambda_\mu R$ is asymmetric in general and the canonical energy-momentum tensor is defined by

(11) $$T^\lambda{}_\mu = \frac{1}{\sqrt{-g}}e_a{}^\lambda\frac{\delta}{\delta e_a^\mu}(\sqrt{-g}L_m) = t^\lambda{}_\mu + \tau^\lambda{}_\mu\,.$$

The first term $t_{\mu\nu}$ is the symmetric part and the second term $\tau_{\mu\nu}$ is the antisymmetric part due to intrinsic spin.

The field equation for $\omega^{ab}{}_\mu$ is given by

(12) $$F^\mu{}_{\alpha\beta} + \delta^\mu_\beta F^\lambda{}_{\lambda\alpha} - \delta^\mu_\alpha F^\lambda{}_{\lambda\beta} = \frac{8\pi}{\phi}\sigma^\mu{}_{\alpha\beta} + \frac{1}{\phi}(\delta^\mu_\alpha\phi_{,\beta} - \delta^\mu_\beta\phi_{,\alpha})\,.$$

This is not a differential equation but an algebraic relation. The spin angular-momentum tensor is defined by

$$\sigma^{\mu}{}_{\alpha\beta} = \frac{1}{\sqrt{-g}} (e^{a}{}_{\alpha} e^{b}{}_{\beta} - e^{a}{}_{\beta} e^{b}{}_{\alpha}) \frac{\delta}{\delta\omega^{ab}{}_{\mu}} (\sqrt{-g}\mathscr{L}_{m}) \tag{13}$$

and is related to $\tau_{\mu\nu}$

$$\tau_{\mu\nu} = \nabla^{*}_{\alpha} (\sigma^{\alpha}{}_{\mu\nu} - \sigma_{\mu\nu}{}^{\alpha} + \sigma_{\nu\mu}{}^{\alpha}), \tag{14}$$

where the modified divergence is

$$\nabla^{*}_{\mu} A^{\mu} = A^{\mu}{}_{;\,\mu} - F^{\lambda}{}_{\mu\lambda} A^{\mu}. \tag{15}$$

By appropriate rearrangements,

$$F^{\mu}{}_{\alpha\beta} = \frac{8\pi}{\phi} \Sigma^{\mu}{}_{\alpha\beta} + \frac{1}{2\phi} (\delta^{\mu}_{\beta} \phi_{,\,\alpha} - \delta^{\mu}_{\alpha} \phi_{,\,\beta}), \tag{16}$$

where $\Sigma^{\mu}{}_{\alpha\beta}$ is given by

$$\Sigma^{\mu}{}_{\alpha\beta} = \sigma^{\mu}{}_{\alpha\beta} + \frac{1}{2} (\delta^{\mu}_{\alpha} \sigma^{\lambda}{}_{\beta\lambda} - \delta^{\mu}_{\beta} \sigma^{\lambda}{}_{\alpha\lambda}). \tag{17}$$

Note that the fluctuation in the scalar field can also act as a source of the torsion field.

The field equation (5) can be rewritten as

$$\Delta\phi = \frac{4\pi T}{\omega} + F^{\mu}{}_{\lambda\mu} \phi^{,\,\lambda} = \frac{4\pi T}{\omega} + \frac{8\pi}{\phi} \Sigma^{\mu}{}_{\lambda\mu} \phi^{,\,\lambda} + \frac{3}{2\phi} \phi_{,\,\lambda} \phi^{,\,\lambda}. \tag{18}$$

Here the contracted curvature tensor

$$R = -\frac{8\pi T}{\phi} + \frac{\omega}{\phi^{2}} \phi_{,\,\lambda} \phi^{,\,\lambda} \tag{19}$$

is used. The conservation laws for the energy-momentum tensor and the spin angular-momentum tensor are

$$\nabla^{*}_{\lambda} T^{\lambda}{}_{\mu} - F^{\varrho}{}_{\lambda\mu} T^{\lambda}{}_{\varrho} + \frac{1}{2} \sigma^{\varrho}{}_{\alpha\lambda} R^{\alpha\lambda}{}_{\varrho\mu} = 0, \tag{20}$$

$$\nabla^{*}_{\mu} \sigma^{\mu}{}_{\alpha\beta} - (T_{\alpha\beta} - T_{\beta\alpha}) = 0. \tag{21}$$

3. – A cosmological model

Any physical cosmological model needs some basic assumptions about the space-time and the matter. We assume $k = 0$ (flat) Friedmann-Robertson-Walker (FRW) model for the space-time, since our space-time is nearly flat. Therefore, the scalar field can also be assumed to be time-dependent only, according to the space-time structure.

As the matter part, the perfect-fluid model is used and the spins are randomly distributed. Thus, the average value of spins vanishes, while that of the quadratic term of spins does not [3, 5, 9],

$$\langle \sigma \rangle = 0 \,, \qquad \langle \sigma^2 \rangle \neq 0 \,. \tag{22}$$

Later we omit the bracket of the average operation as a matter of convenience.

To estimate the curvature tensor in this model, we should calculate the scalar effect, spin effect, and the spin-scalar effect, from the torsion field to the curvature. However, the spin-scalar terms are vanished by averaging procedures, because all the terms are first order in spin. Since the spin effects are well known in various papers about the EC manifold, it is necessary to calculate only the scalar effect.

The metric in the flat FRW space-time is given as

$$\mathrm{d}s^2 = -\,\mathrm{d}t^2 + R^2(t)(\mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2)\,. \tag{23}$$

The non-vanishing Christoffel symbols are

$$\begin{cases} \begin{Bmatrix} 0 \\ 11 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 22 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 33 \end{Bmatrix} = R\dot{R}\,, \\ \begin{Bmatrix} 1 \\ 01 \end{Bmatrix} = \begin{Bmatrix} 2 \\ 02 \end{Bmatrix} = \begin{Bmatrix} 3 \\ 03 \end{Bmatrix} = \dfrac{\dot{R}}{R} \equiv H\,, \end{cases} \tag{24}$$

where the dot means the differentiation with respect to time t.

When the Einstein tensor calculated by the Christoffel symbol $\begin{Bmatrix} \mu \\ \nu\lambda \end{Bmatrix}$ is noted as $G(\{\})$, the Einstein equation is

$$G_{\mu\nu}(\{\}) = \frac{8\pi}{\phi}(T_{\mu\nu} + \theta_{\mu\nu}) + \Phi_{\mu\nu}, \tag{25}$$

where $\theta_{\mu\nu}$ is the spin effect by torsion as the difference between $G_{\mu\nu}(\{\})$ and $G_{\mu\nu}$ [5]. The additional $\Phi_{\mu\nu}$ is the scalar effect arising during the calculation of curvature tensor in flat FRW model,

$$\Phi_{\mu\nu} = \left(\frac{1}{2}g_{\mu\nu} + \delta^0_\mu\delta^0_\nu\right)\left(\omega + \frac{3}{2}\right)\frac{\dot{\phi}^2}{\phi^2} - (g_{\mu\nu} + \delta^0_\mu\delta^0_\nu)\frac{\ddot{\phi}}{\phi} + \frac{\dot{\phi}}{\phi}(2\delta^i_\mu\delta^i_\nu R^2 H - 3g_{\mu\nu}H) \quad (\text{no summation on } i)\,. \tag{26}$$

When all the source terms except the scalar field are averaged out,

$$\langle t_{\mu\nu} \rangle = (\varrho + P)\, u_\mu u_\nu + P g_{\mu\nu}, \tag{27}$$

$$\langle \tau_{\mu\nu} \rangle = -\frac{8\pi}{\phi}\sigma^2 u_\mu u_\nu, \tag{28}$$

$$\langle \theta_{\mu\nu} \rangle = \frac{4\pi}{\phi}\sigma^2 u_\mu u_\nu - \frac{2\pi}{\phi}\sigma^2 g_{\mu\nu}, \tag{29}$$

where $u_\mu = (-1, 0, 0, 0)$ is the velocity in the comoving coordinates. The energy-momentum tensor is separated into two terms as eq. (11). The spin fluid is Weyssenhoff-fluid type [16] whose spin density and spin current density are defined as

$$\sigma^2 = \frac{1}{2} S_{\mu\nu} S^{\mu\nu}, \qquad \sigma^\alpha{}_{\mu\nu} = u^\alpha S_{\mu\nu}. \tag{30}$$

Using the above assumptions for space-time and matters, the equations of motion are obtained as

$$H^2 = \frac{8\pi}{3\phi}\left(\varrho - \frac{2\pi}{\phi}\sigma^2\right) + \frac{1}{6}\left(\omega + \frac{3}{2}\right)\frac{\dot{\phi}^2}{\phi^2} + H\frac{\dot{\phi}}{\phi}, \tag{31}$$

$$\frac{\ddot{R}}{R} = -\frac{4\pi}{3\phi}\left(\varrho + 3P - \frac{8\pi}{\phi}\sigma^2\right) - \frac{1}{3}\left(\omega + \frac{3}{2}\right)\frac{\dot{\phi}^2}{\phi^2} + \frac{1}{2}\frac{\ddot{\phi}}{\phi}, \tag{32}$$

$$\ddot{\phi} + 3\dot{\phi}H \equiv \frac{4\pi}{\omega}\left(\varrho - 3P - \frac{8\pi}{\phi}\sigma^2\right). \tag{33}$$

There are also conservation laws for the matter and the spin. As usual, we adopt the conservation laws separately [9]:

$$\frac{d}{dt}(\varrho R^3) + P\frac{d}{dt}(R^3) = 0 \quad \rightarrow \quad \dot{\varrho} = -3H(\varrho + P), \tag{34}$$

$$\frac{d}{dt}(\sigma^2 R^3) + \sigma^2\frac{d}{dt}(R^3) = 0 \rightarrow \quad (\dot{\sigma^2}) = -6H\sigma^2. \tag{35}$$

As a constraint, there is the equation of state that relates the energy density to pressure as $P = k\varrho$.

4. – Properties of the cosmological model

To solve the field equations easily, we will try to cancel the spin term because σ^2 has the R^{-6}-nature from the conservation law eq. (35), which arises the complexity of the equation. But it is not easy to get rid of the spin term in the equations. Apparently though it looks as if the effective density and effective pressure $\varrho_{\text{eff}} = \varrho - (2\pi/\phi)\,\sigma^2$, $P_{\text{eff}} = P - (2\pi/\phi)\,\sigma^2$ of ref. [9] can screen the spin term in eq. (31) and eq. (32), the spin term still appears in eq. (33).

If matter can be described as a liquid of unpolarized fermions with spin $\hbar/2$, the spin can be represented in terms of ϱ as

$$\sigma^2 = \frac{\hbar^2}{8}\langle n^2 \rangle = \frac{\hbar^2}{8} A_k^{2/(1+k)} \varrho^{2/(1+k)} \equiv \alpha_k \varrho^{2/(1+k)} \tag{36}$$

from the conservation law. Here n is the particle number density and A_k is a dimensional constant depending on k.

As you can see, it is very hard to get the rigorous solutions analytically. For example, even if $H \to 0$ and ϱ, P, σ^2 are constants, the field equation (33) becomes

$$\phi\ddot{\phi} + A\phi + B = 0 \tag{37}$$

and the solution to this equation is the integral form at best as

$$\sqrt{2}t + C'' = \int \frac{d\phi}{\sqrt{-B \log \phi - A\phi - C'}}\ , \tag{38}$$

where C' and C'' are integration constants. Thus now we can try to analyze the cosmological model approximately at special regions.

When the model approaches the state of $H \cong 0$, eqs. (31) and (33) become

$$0 \cong \frac{8\pi}{\phi}\left(\varrho_0 - \frac{2\pi}{\phi}\sigma_0^2\right) + \frac{1}{2}\left(\omega + \frac{3}{2}\right)\frac{\dot{\phi}^2}{\phi^2}\ , \tag{39}$$

$$\ddot{\phi} \cong \frac{4\pi}{\omega}\left(\varrho_0 - 3P_0 - \frac{8\pi}{\phi}\sigma_0^2\right). \tag{40}$$

Here the matters including spin are assumed to be nearly constant at that time because the scale factor will cease to expand. The approximate solution to eq. (39) near $H \cong 0$ (the bouncing time) is

$$\phi \cong -\left(\frac{4\pi\varrho_0}{\omega + 3/2}\right)t^2 + \frac{2\pi\sigma_0^2}{\varrho_0} \qquad (H \cong 0). \tag{41}$$

This solution also must satisfy eq. (40). By putting the solution eq. (41) into eq. (40), the bouncing time $t_{H=0}$ is obtained by

$$t_{H=0}^2 \cong \frac{\sigma_0^2(\omega + 3/2)}{2\varrho_0} \frac{\dfrac{2\omega}{\omega + 3/2} - 3(1+k)}{\dfrac{2\omega}{\omega + 3/2} + (1 - 3k)}\ . \tag{42}$$

For normal matter of the equation of state $k > 0$, the right-hand side of eq. (42) becomes negative which means the non-existence of the bouncing time. If we consider the special matter that satisfies the equation of state $k < -\frac{\omega + 9/2}{3\omega + 9/2}$, the bouncing time exists. At that time the scalar field is approximately

$$\phi \cong \phi_0 = \frac{8\pi\sigma_0^2}{\varrho_0} \frac{\omega + 3/2}{2\omega + (1 - 3k)(\omega + 3/2)} \tag{43}$$

and the minimum scale factor is

$$R_0 \sim \varrho_0^{-1/3(1+k)}\ . \tag{44}$$

Equation (32) can be rewritten as

$$\frac{\ddot{R}}{R} = -\frac{4\pi}{3\phi}\left(\varrho + 3P - \frac{8\pi}{\phi}\sigma^2\right) - \frac{1}{3}\left(\omega + \frac{3}{2}\right)\frac{\dot{\phi}^2}{\phi^2} + \frac{1}{2}\frac{\ddot{\phi}}{\phi} = \tag{45}$$

$$= -\frac{2\pi}{3\phi}\left[\left(\frac{2}{3} - \frac{1}{\omega}\right)\varrho + \left(2 + \frac{3}{\omega}\right)P - \frac{8\pi}{\phi}\left(\frac{2}{3} - \frac{2}{\omega}\right)\sigma^2\right] - \frac{1}{3}\left(\omega + \frac{3}{2}\right)\frac{\dot{\phi}^2}{\phi^2} - \frac{3}{2}H\frac{\dot{\phi}}{\phi}\ .$$

The sign of $\ddot{R}$ is important to see the temporal change of the cosmological model. For any equation of state, the scale factor has the property of $\ddot{R} < 0$, if ω is very large or approaches zero. It is a non-accelerated cosmological model, that is, it is not an inflationary model. If ω has the proper finite value that does not approach infinity or zero and the spin is very large, then $\ddot{R} > 0$, inflation arises. It means that the spin also plays the role of triggering the inflation.

For the inflation type of the model, the sign of $\dot{H}$ should be determined. Equation (45) can be rearranged as

$$\dot{H} = -\frac{4\pi}{\phi}\left(\varrho + P - \frac{4\pi}{\phi}\sigma^2\right) - \frac{1}{2}\left(\omega + \frac{3}{2}\right)\frac{\dot{\phi}^2}{\phi^2} + \frac{1}{2}\frac{\ddot{\phi}}{\phi} + 3H\frac{\dot{\phi}}{\phi} = \tag{46}$$

$$= -\frac{2\pi}{\phi}\left[\left(2 - \frac{1}{\omega}\right)\varrho - \left(2 + \frac{3}{\omega}\right)P - \frac{8\pi}{\phi}\left(1 - \frac{1}{\omega}\right)\sigma^2\right] - \frac{1}{2}\left(\omega + \frac{3}{2}\right)\frac{\dot{\phi}^2}{\phi^2} - \frac{3}{2}H\frac{\dot{\phi}}{\phi}\ .$$

If we look at the equation, we can compare the value of spin with the scalar field. If spin is very large, then $\dot{H} > 0$, that is, the inflation is faster than the exponential inflation. If the scalar field dominates spin and ω is very large, then $\dot{H} < 0$, that is, the power inflation is slower than the exponential inflation. This result reminds us of the extended inflation model [12] in which the power inflation is due to the BD scalar field.

5. – Inflation models

For the case of vacuum equation of state ($\varrho = -P$), eq. (31) and eq. (33) can be combined to remove the matter term including spin as

$$H^2 = \frac{\omega}{6}\frac{\ddot{\phi}}{\phi} + \left(\frac{\omega}{2} + 1\right)H\frac{\dot{\phi}}{\phi} + \frac{1}{6}\left(\omega + \frac{3}{2}\right)\frac{\dot{\phi}^2}{\phi^2}\ . \tag{47}$$

By setting $y = R^{-4/(2+\omega)}\phi$, then eq. (47) becomes

$$\frac{\dot{y}^2}{y^2} = D^2\frac{\dot{\phi}^2}{\phi^2}\left[1 + \frac{8\omega}{3D^2(\omega+2)^2}\frac{\ddot{\phi}}{\dot{\phi}}\frac{\phi}{\dot{\phi}}\right], \tag{48}$$

where $D(\omega) = 1 + (4(2\omega+3))/(3(\omega+2)^2) > 1$. When ω is very large,

$$\frac{\dot{y}}{y} \cong \pm D\frac{\dot{\phi}}{\phi} \pm \frac{4\omega}{3D(\omega+2)^2}\frac{\ddot{\phi}}{\dot{\phi}} \tag{49}$$

to the first order of ω^{-1}. This gives the relation

$$R \sim \phi^{\alpha}\dot{\phi}^{\beta}, \tag{50}$$

where $\alpha = ((\omega+2)/4)(1 \pm D)$ and $\beta = \pm\omega/3D(\omega+2)$. The plus/minus signs are given to α and β simultaneously. Since D is the order of unity, $\alpha \sim O(\omega)$ and $\beta \sim O(1)$.

From the conservation law eq. (34) one has

$$\varrho = \text{const} = \varrho_{\text{v}}, \tag{51}$$

the false vacuum energy density. By combining eqs. (32) and (33) again to cancel the spin term,

$$\frac{8\pi\varrho_{\text{v}}}{\phi} \cong \left(\alpha^2 - \alpha + \omega\alpha + \frac{1}{3}\omega\right)\frac{\dot{\phi}^2}{\phi^2} + \left(2\alpha\beta + \alpha + \frac{\omega}{3} + \omega\beta\right)\frac{\ddot{\phi}}{\phi}. \tag{52}$$

Here the coefficients are given to the order of ω and ω^2 neglecting the constant terms ($O(1)$). The vacuum energy ϱ_{v} is not so small that it cannot be negligible compared with the ω-term. The exact solution to eq. (52) is

$$\phi = \left(\sqrt{\frac{B}{2+4A}}\,t + \sqrt{\phi_0}\right)^2, \tag{53}$$

where

$$A = \frac{\alpha^2 - \alpha + \omega\alpha + \frac{1}{3}\omega}{2\alpha\beta + \alpha + \frac{\omega}{3} + \omega\beta} \quad \text{and} \quad B = \frac{8\pi\varrho_{\text{v}}}{2\alpha\beta + \alpha + \frac{\omega}{3} + \omega\beta}.$$

The initial value of $\phi(t)$ is given as ϕ_0. The t^2-dependence for ϕ is the same as the previous extended inflation.

Thus, the scale factor becomes

$$R \sim t^{2\alpha+\beta}. \tag{54}$$

For large value of ω, the power is

$$2\alpha + \beta \sim \begin{cases} \omega + \dfrac{5}{3}, \\ -1. \end{cases} \tag{55}$$

But the case of -1 can be discarded because this is neither the expansion nor the inflation type. If this result is compared with that of the extended inflation model ($R \sim t^{\omega+1/2}$) [12], it is shown that our modified model inflates faster than the previous extended inflation model. For large ω, however, two kinds of power expansion are similar to each other.

Later at the time of $t \gg \hbar/mc^2 = 10^{-23}$s when the spin effect sufficiently diminishes and the inflation is already finished (inflation era: $10^{-43} \sim 10^{-35}$ s), then there is still the scalar effect by torsion field. By the equation of state $k = 0$, the solutions in the matter-dominated era become

(56) $$\phi \propto t^{f(\omega)},$$

(57) $$R \propto t^{2-f(\omega)/3},$$

(58) $$\frac{4\pi\varrho t^2}{\phi} = f(\omega)\,\omega,$$

where $f(\omega) = 4[3\sqrt{\omega^2+4\omega+2} - 3\omega - 5]/(6\omega - 7)$. As $\omega \to \infty$, $f(\omega) \to 0$ which goes to the standard model [17].

In the spinless case, the equations become

(59) $$H^2 = \frac{8\pi}{3\phi}\varrho + \frac{1}{6}\left(\omega + \frac{3}{2}\right)\frac{\dot\phi^2}{\phi^2} + H\frac{\dot\phi}{\phi},$$

(60) $$\frac{\ddot R}{R} = -\frac{4\pi}{3\phi}(\varrho + 3P) - \frac{1}{3}\left(\omega + \frac{3}{2}\right)\frac{\dot\phi^2}{\phi^2} + \frac{1}{2}\frac{\ddot\phi}{\phi},$$

(61) $$\ddot\phi + 3\dot\phi H = \frac{4\pi}{\omega}(\varrho - 3P).$$

Since the space-time is still EC manifold, they are different from the usual BD cosmological models. Even though there are no spins, the torsion effect by the scalar field still remains.

For the case of vacuum density with equation of state $\varrho = -P$,

(62) $$\begin{cases} \phi = \chi(t+\delta)^2, \\ R = \chi^{n/2}(t+\delta)^n, \end{cases}$$

where $\chi = 8\pi\varrho_{\rm v}/\omega(1+3n)$, $n = 1 + \omega/2 + (\omega^2/4 + 2\omega + 2)^{1/2}$ and $\delta = \sqrt{\phi_0/\chi}$.

The initial value of the ϕ-field is $\phi_0 \sim m_{\rm p}^2$. The power of R is n which is larger than $\omega + 1/2$ of the usual BD theory [12]. For large ω,

(63) $$n \approx \omega + 3,$$

(64) $$\phi \sim t^2,$$

(65) $$R \sim t^{\omega+3}$$

(cf. $R \sim t^{\omega+1/2}$ in BD).

The probability of a point remaining in the false-vacuum phase during a bubble nucleation process beginning at time $t_{\rm B}$, that is, the bubble nucleation rate converting from false vacuum to the true-vacuum phase is [18]

(66) $$p(t) = \exp\left[-\int_{t_{\rm B}}^{t} {\rm d}t'\,\lambda(t')\,R^3(t')\,\frac{4\pi}{3}\left[\int_{t'}^{t}\frac{{\rm d}t''}{R(t'')}\right]\right],$$

where $\lambda(t)$ is the nucleation rate per unit time per unit volume, approximately constant during the inflationary phase.

In exponential-type inflation, the exponent of the probability is $\sim -(t - t_{\rm B})$, while in power-type inflation, the exponent of $p(t)$ is $\sim -(t^4 - t_{\rm B}^4)$ independent of the value of the power n. It means that as time goes on $p(t) \ll 1$ for power law expansion. Thus the probability in our case is the same as that in the usual BD case. The probability is decreasing much faster than the volume is increasing. The universe is dominated by true vacuum and exits from the false vacuum. It is the well-known solution to the great exit problem in the extended inflation [12].

6. – Conclusion

We discussed about the properties and the solutions of the cosmological model with BD scalar field and torsion in modified EC manifold. We analyzed the qualitative types of expansion, inflation, and after inflation. The spin plays an important role in the inflation model and the scalar field affects the inflation-type model to the extent that it transforms the model from exponential type to power expansion. We also calculated the inflation model with spin and torsion field. It is shown that the cosmological model expands faster than the extended inflationary model. In another paper we will discuss in detail about the inflation-type model and remaining problems to be adjusted.

For the spinless case, there are still torsion effects by ϕ left. The results are never exactly identified with those of the BD theory. However, for $\phi = \text{const}$, the results do naturally approach the results of the EC theory.

* * *

This work was supported in part by the Basic Sciences Research Institute of Ewha Women's University and by the Korea Science and Engineering Foundation by Grant No. 95-0702-04-01-3.

REFERENCES

[1] HEHL F. W., VON DER HEYDE P., KERLICK G. D. and NESTER J., *Rev. Mod. Phys.*, **48** (1976) 393.
[2] TRAUTMANN A., *Nature*, **242** (1973) 7.
[3] KUCHOWICZ B., *Gen. Relativ. Gravit.*, **9** (1978) 511.
[4] NURGALIEV I. S. and PONOMARIEV W. N., *Phys. Lett. B*, **130** (1983) 378.
[5] GASPERINI M., *Phys. Rev. Lett.*, **56** (1986) 2873.
[6] GUTH A. H., *Phys. Rev. D*, **23** (1981) 347.
[7] DEMIANSKI M., DE RITIS R., PLATANIA G., SCUDELLARO P. and STORNAIOLO C., *Phys. Lett. A*, **116** (1986) 13.
[8] BEDRAN M. L., VASCONCELLOS VAIDYA E. P. and SOM M. M., *Nuovo Cimento B*, **87** (1985) 101.
[9] SMALLEY L. L. and KRISCH J. P., *Class. Quantum Grav.*, **11** (1994) 2375; OBUKHOV YU. N. and KOROTKY V. A., *Class. Quantum Grav.*, **4** (1987) 1633.
[10] BRANS C. H. and DICKE R. H., *Phys. Rev.*, **124** (1961) 925.
[11] For example, LINDE A., *Particle Physics and Inflationary Cosmology* (Harwood Academic Publ., New York, N.Y.) 1990.
[12] DAILE LA and STEINHARDT P. J., *Phys. Rev. Lett.*, **62** (1989) 376.
[13] DAILE LA, STEINHARDT P. J. and BERTSCHINGER E., *Phys. Lett. B*, **231** (1989) 213; GOLDWIRTH D. S. and ZAGLAUER H. W., *Phys. Rev. Lett.*, **67** (1991) 3639.
[14] BARROW J. D. and MIMOSO JOSÉ P., *Phys. Rev. D*, **50** (1994) 3746.
[15] KIM S.-W., *Phys. Rev. D*, **34** (1986) 1011.
[16] WEYSSENHOFF J. and RAABE A., *Acta Phys. Polon.*, **9** (1947) 7.
[17] WEINBERG S., *Relativity and Cosmology* (Wiley, New York, N.Y.) 1980.
[18] GUTH A. H. and WEINBERG E. J., *Nucl. Phys. B*, **212** (1983) 321.