Use of νF_ν spectral energy distributions for multiwavelength astronomy

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(ricevuto il 6 Novembre 1995; approvato il 23 Luglio 1996)

Summary. — Spectral Energy Distribution (SED) plots of $\log(\nu F_\nu)$ *vs.* $\log(\nu)$ have become popular for multiwavelength astronomy because they give the source power per logarithmic frequency interval and thereby directly show the relative energy output in each frequency band. They also allow easy manipulation and integration of black-body spectra. However, usage can be tricky and misuse is common. This paper derives equations for manipulating SEDs.

PACS 95.75.Fg – Spectroscopy and spectrophotometry.

1. – Introduction

Spectral Energy Distributions (SEDs), also called νF_ν plots or Continuum Energy Distributions (CEDs; *e.g.*, [1]), have become the standard for broad-band studies. The usual representation is $\log(\nu F_\nu)$ *vs.* $\log(\nu)$ or $\log(E^2 J_\mathrm{E})$ *vs.* $\log(E)$, where ν is the photon frequency, E is the photon energy, F_ν is the energy flux per unit frequency interval and J_E is the photon number flux per unit energy interval. The value of SEDs is that they give the energy flux per logarithmic interval of frequency (often called the "power per logarithmic bandwidth") and therefore allow easy comparison of source luminosities in different wavelength bands.

The first utilization of νF_ν plots (or the equivalent λF_λ plots, see sect. **2**) was by Gehrz, Ney, Strecker, and Woolf for infrared observations in the early 1970's [2-5]. The original motivation for plotting data in this manner was for ease of manipulation of black-body curves [6]. The shape of a black-body curve is invariant to translation on such log-log plots and the total energy under the Planck function, $B(T)$, is simply related to νF_ν by

$$B(T) = \sigma T^4/\pi = 1.359(\nu F_\nu)_\mathrm{max} \tag{1}$$

(*e.g.*, [7]). Special graph paper has even been printed ("Ney paper") that allows quick νF_ν plotting and scaling.

By the late 1970's νF_ν plots were becoming standard for infrared and multiwavelength studies [8, 9]. In the 1990's their application has now become widespread for broad-band observations of such objects as AGN [1, 10-15], stars [16], young stellar objects [17], compact objects [18], high-energy sources [19], and gamma-ray bursts [20]. The recent proceedings on Infrared Astronomy with ISO [21] contains nine separate papers with νF_ν plots in one form or another. For diffuse studies, νI_ν is commonly used [22, 23] where I_ν is the energy flux per unit frequency interval per steradian.

With the current popularity of SEDs, it is important to re-iterate their exact meaning and understand their correct usage. Definitions and general derivations are given in sect. **2**. Easy-to-use equations for conversion to rest-frame coordinates and luminosities for cosmological objects are given in sect. **3**. See Disney and Sparks [24] for other suggestions concerning flux plots.

2. – Relationships

The quantity $F_\nu(\nu)$ is short-hand for the differential notation $(\mathrm{d}F/\mathrm{d}\nu)(\nu)$ and represents the energy flux from a source at frequency ν per unit interval of photon frequency. Observationally it is approximated by measuring the incident energy flux in small frequency bands and dividing by the width of the frequency band. For photon-counting instruments, particularly in X-ray and gamma-ray astronomy, the measured quantity is the number of photons in a band of photon energy, which is then divided by the width of the band to give the photon number flux per unit interval of photon energy, $J_\mathrm{E}(E) = (\mathrm{d}J/\mathrm{d}E)(E)$.

The quantity νF_ν is related to the flux in logarithmic intervals by

$$\nu F_\nu = \nu \frac{\mathrm{d}F}{\mathrm{d}\nu} = \frac{\mathrm{d}F}{\mathrm{d}(\ln(\nu))} = F_{\ln(\nu)} . \tag{2}$$

Thus, νF_ν is the energy flux per natural logarithmic frequency interval; the common statement that it is the energy flux per decade of photon frequency is incorrect. It is also common to call νF_ν the energy flux per octave, which is close to accurate (factor 2 compared to factor $e = 2.7$) but is still not exact. Since the typical logarithmic plot uses base-10 logarithms, slight care is necessary in interpreting νF_ν plots. For example, integration can be tricky. Two methods for quick νF_ν integration are: 1) divide into bins of size factor 2.7 and sum, or 2) divide into the more convenient factor-10 bins, sum, and multiply by $\ln(10) = 2.3$. Note that, since $|\mathrm{d}(\ln(\lambda))| = |\mathrm{d}(\ln(\nu))|$, the wavelength and frequency fluxes are equal: $\lambda F_\lambda = \nu F_\nu$.

If the objective is to plot the actual energy flux per base-10 logarithmic frequency interval (*i.e.* energy flux per decade), $F_{\log(\nu)}$, the following relation to νF_ν applies:

$$F_{\log(\nu)} = \frac{\mathrm{d}F}{\mathrm{d}(\log(\nu))} = \ln(10)\, \frac{\mathrm{d}F}{\mathrm{d}(\ln(\nu))} = \ln(10)\, \nu F \nu = 2.3\, \nu F \nu . \tag{3}$$

However, to avoid confusion, it is not recommended to use $F_{\log(\nu)}$. There is a long νF_ν tradition and νF_ν is "closer" to the data.

For J_E, the first step is to convert from photon flux to energy fux: $F_\mathrm{E} = E J_\mathrm{E}$. Then, in a similar manner to eq. (2),

$$E^2 J_{\mathrm{E}} = EF_{\mathrm{E}} = \frac{\mathrm{d}F}{\mathrm{d}(\ln(E))} . \tag{4}$$

3. – Cosmological sources

We now consider an object at red-shift z in a Friedmann universe. The first subsection concerns the conversion of measured SEDs to rest-frame variables for the object. The second subsection concerns luminosity calculations.

3·1. *Rest-frame conversions.* – For distant sources, it is recommended that SEDs be converted to the rest frame of the emitting object, particularly when the spectra of more than one object are being studied. This facilitates interpretation and comparison of the spectra in terms of physical processes at the source. For flux plots the conversion is done by changing to the rest-frame frequency, ν_0, from the observed frequency, ν: *viz.* $\nu_0 = (1+z)\,\nu$ or $E_0 = (1+z)\,E$. Since $\mathrm{d}(\ln(\nu_0)) = \mathrm{d}(\ln(\nu))$, there is no effect of the frequency shift on νF_ν, and one can simply plot $\nu F_\nu(\nu = \nu_0/(1+z))$ *vs.* ν_0.

It is possible to make a first-order rest-frame correction to the flux itself by taking into account the fact that the photon energy is a factor of $(1+z)$ less in the observer frame than in the rest frame and the unit of time is a factor of $(1+z)$ more. Thus, since flux is energy per unit time, a corrected flux of $(1+z)^2\,\nu F_\nu$ can be used. However, this correction is not recommended since flux is relevant mostly for the observer. In the rest frame, luminosity is a better quantity. Also, as shown below, luminosities are calculated from the observer-frame fluxes and not the rest-frame values. In summary, the recommendation is to convert the frequency scale to the rest-frame but to plot the observed fluxes, $\nu F_\nu(\nu = \nu_0/(1+z))$.

3·2. *Luminosity calculations.* – Luminosities can be easily calculated from νF_ν. For nearby stationary objects emitting isotropically, the luminosity per logarithmic bandwidth is given by $\nu L_\nu = 4\pi D^2 \nu F_\nu$, where D is the distance to the source.

For objects at cosmological red-shift z, the luminosity calculation must include the frequency red-shift and time dilation discussed above. In addition, the emission solid angle at the source for photons that cross a unit area at the observer is changed by the curvature of space as a complicated function of the red-shift, Hubble's constant H_0, and the deceleration parameter q_0 [25]. For small z, the solid angle is increased by a factor $1 + (1+q_0)\,z$.

These three effects are combined into a modified distance variable known as the luminosity distance, D_{L}, which, for a Friedmann universe with curvature constant k equal to ± 1 or 0, is defined as

$$D_{\mathrm{L}} = \frac{c}{H_0 q_0^2}\left[zq_0 + (1-q_0)(1-\sqrt{2q_0 z + 1})\right] \approx \tag{5}$$

$$\approx \frac{cz}{H_0}\left[1 + \frac{1}{2}(1-q_0)\,z\right] \qquad \text{for } z \ll 1 \tag{6}$$

[25, 26], where c is the speed of light. Convenient plots and equations for D_L are given by Zombeck [27]. The luminosity of a source per logarithmic frequency interval is then given by

$$\nu L_\nu(\nu_0) = L_{\ln(\nu)}(\nu_0) = 4\pi D_L^2 \nu F_\nu(\nu) . \tag{7}$$

Note that the luminosity distance contains in it the effects of photon energy red-shift and time dilation. Therefore, the trick is to calculate the luminosity from the measured (observer-frame) flux at the observed frequency. Also, it is important to specify that the ordinate is luminosity per logarithmic frequency interval, νL_ν or $L_{\ln(\nu)}$. A common but confusing practice is to label the ordinate simply "Luminosity", leaving ambiguity as to whether L_ν or νL_ν is being displayed.

4. – Recommendations

This paper develops the proper techniques for interpreting and plotting SEDs. Specific recommendation are:

1) $\nu F_\nu = \mathrm{d}F/\mathrm{d}(\ln(\nu)) = F_{\ln(\nu)}$ (instead of, say, $F_{\log(\nu)}$), but keep in mind that νF_ν is the power per natural logarithmic frequency interval and not the power per decade as often stated.

2) For fluxes of cosmological sources, plot the measured flux, $\nu F_\nu(\nu = \nu_0/(1+z))$, as a function of rest-frame frequency, ν_0.

3) For luminosities of cosmological sources, use the luminosity distance and the *measured* flux to calculate the luminosity: $\nu L_\nu(\nu_0) = 4\pi D_L^2 \nu F_\nu(\nu)$.

4) Be sure to specify whether the luminosity per logarithmic frequency interval, $\nu L_\nu = L_{\ln(\nu)}$, or luminosity per linear frequency interval, L_ν, is being used.

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The author thanks E. Ney for information on the usage and history of νF_ν plots. He is also grateful to J. Mather for interesting discussions on $F_{\ln(\nu)}$ *vs.* $F_{\log(\nu)}$. Discussions with R. Hartman, S. Odenwald, B. Swanenburg and N. Wright and assistance from K. Pollock are appreciated.

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