Multifractal analysis of 4.1A GeV/c ^{22}Ne **and** 4.5A GeV/c ^{28}Si **collisions with emulsion**

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(ricevuto l'1 Aprile 1997; revisionato il 17 Settembre 1997; approvato il 29 Dicembre 1997)

Summary. — The fractal or intermittent behaviour has been investigated for the non-peripheral collisions of 4.1A GeV/c ^{22}Ne and 4.5A GeV/c ^{28}Si with emulsion using the scaled factorial moments and the multifractals in the pseudorapidity, the azimuthal angle and in the two-dimensional phase spaces. The reduced scaled factorial moments as a function of the bin size in pseudorapidity, azimuthal angle and the pseudorapidity-azimuthal angle spaces have been calculated and have revealed the presence of an intermittent behavior. It has been shown that the magnitude of the anomalous fractal dimension increases with the rank of the moment in all the investigated spaces. In the relation between the intermittency parameter and the rank of the moment, no obvious minimum has been observed. However, in the azimuthal angle space for ^{28}Si data, a flattening is seen around the value 4 for the rank of the moment. The analysis of the present data may indicate the random cascading property of the reactions. The results do not give a clear signal for the coexistence of two phases of the self-similar cascade.

PACS 29.40.Rg – Nuclear emulsions.
PACS 25.70 – Low and intermediate energy heavy-ion reactions.

1. – Introduction

The idea of scaling and self-similarity (intermittency) was applied in the investigation of the turbulent phenomena starting from the late seventies [1-6], although the concept of self-similarity in multiparticle production was known since the sixties, *e.g.*, [7]. Bialas and Peschanski [8, 9] and Hwa [10] were the first to suggest practical tools for studying the properties of the non-statistical fluctuations in the high-energy collisions renewing the experimental [11-48] and the theoretical [49-61] studies of this subject.

Many authors investigated the interpretation of the intermittent behavior as due to the phase transition such as from the quark-gluon plasma (QGP) to the normal hadronic matter, *e.g.*, [50-54]. Some of them studied, specifically, the possibility of a second-order phase transition, in which the correlation length diverges and the system becomes scale invariant [51]. Other authors investigated the self-similar cascade [8, 55] in which the anomalous fractal dimensions are sensitive to the details of the vertex describing the cascade. The connection between the intermittency and the minijets formation has been studied, *e.g.*, [55, 56]. The relationship between the Bose-Einstein correlations and the intermittency has been tested [57-60]. Also, the multifragmentation process has been investigated [18, 61]. Recently, several experiments [11-22, 27, 28, 41, 48, 62] confirmed the power law behavior of the normalized factorial moments predicted by the hypothesis of the intermittency [8-10].

In our former publication [62], the intermittency effect has been studied for the same interactions considered here, for both the pseudorapidity and the azimuthal angle phase space distribution. The goal of the present work is to study the multifractal properties for the two considered interactions in the pseudorapidity and in the azimuthal angle phase spaces using higher statistics and for higher orders. A two-dimensional analysis for the scaled factorial moment has also been performed. The intermittency parameter has been investigated to test the coexistence of two phases in the reactions.

2. – Experimental techniques

Nuclear emulsions of the type Br-2 were exposed to 4.1A GeV/c ^{22}Ne and 4.5A GeV/c ^{28}Si beams at the Dubna Synchrophasotron. The pellicles of emulsion have the dimensions of $20\ \text{cm} \times 10\ \text{cm} \times 600\ \mu\text{m}$ (undeveloped emulsion). The intensity of the beam was $\approx 10^4$ particles/cm^2 and the beam diameter was approximately 1 cm. Along the track, a double scanning has been carried out fast in the forward direction and slow in the backward one.

The scanned beam tracks have been further examined by measuring the delta-electron density [63] on each of them to exclude any track having a charge less than the beam particle charge Z_b. The scanning has been performed giving 4308 and 1322 events for ^{22}Ne and ^{28}Si, respectively.

In the measured interactions, all the charged secondary particles have been classified into the following groups [64]:

1) Shower tracks producing «s-particles», such tracks, having an emission angle $\theta \leqslant 3°$, have been further subjected to multiple scattering measurements for momentum determination [65] in order to separate the produced pions from the singly charged projectile fragments.

2) Grey tracks producing «g-particles».

3) Black tracks producing «b-particles».

4) The «b» and «g» tracks are both called heavily ionizing tracks producing «h-particles».

The determination of the momentum of the s-particles emitted within $\theta \leqslant 3°$ enables the separation of the produced pions from the non-interacting singly charged projectile fragments (protons, deuterons and tritons) [66]. The g-particles emitted within $\theta \leqslant 3°$ and having $L > 2$ cm are considered as projectile fragments having $Z = 2$. The b-particles having $\theta \leqslant 3°$ and $L > 1$ cm are due to heavy projectile fragments $Z \geqslant 3$. The number of delta-electrons has been measured for each of these particles in order to determine the corresponding charge $Z = 3, \ldots Z_b$.

The polar angle θ of each track, *i.e.* the space angle between the direction of the beam and that of the given track has been measured. The azimuthal angle ϕ of each track, *i.e.* the angle between the projection of the given track in the plane normal to the beam (the azimuthal plane) and the perpendicular direction to the beam in this plane (in anticlockwise direction) has also been measured. The uncertainty in θ and/or ϕ equals 0.02°. Only events having $n_h \geqslant 7$ have been selected in order to study the produced charged pions from the non-peripheral collisions with Ag(Br), 1856 and 602 such events have been found for ^{22}Ne and ^{28}Si, respectively.

3. – One-dimensional analysis in the pseudorapidity and in the azimuthal angle spaces

The pseudorapidity (η) given by

$$\eta = -\ln\left(\tan\frac{\theta}{2}\right) \tag{1}$$

has shown to be a good approximation to the rapidity at very high energies, where θ is the angle of emission of the pion in the laboratory system. The pseudorapidity of each shower track is calculated according to eq. (1). The resolution of η equals approximately 0.04. Figure 1 represents the η-distribution for the selected interactions of $^{22}\mathrm{Ne} + \mathrm{Ag(Br)}$ at 4.1*A* GeV/c and $^{28}\mathrm{Si} + \mathrm{Ag(Br)}$ at 4.5*A* GeV/c. From the figure, one may see that the central rapidity region can be restricted to $\eta = 0$–3. The average multiplicities in the given region are 15.52 ± 0.26 and 16.98 ± 0.52, respectively. In analogy, fig. 2 shows the ϕ-distributions for the interactions ^{22}Ne and ^{28}Si with Ag(Br) emulsion nuclei, respectively. In the analysis of the fluctuations in the pseudorapidity distributions, the interval $\Delta\eta$ is divided into M bins of size $\delta\eta$, where $\delta\eta = \Delta\eta/M$. The scaled factorial moment is defined as [8]:

$$F_q = M^{q-1} \sum_{i=1}^{M} n_i(n_i - 1) \dots (n_i - q + 1) / \langle n \rangle^q . \tag{2}$$

The value n_i is the number of shower particles falling within the i-th bin, i running from 1 to M and $\langle n \rangle$ is the mean multiplicity of the shower particles in the $\Delta\eta$ interval.

For a given order q in the pseudorapidity space considered, the F_q moments (given by eq. (2)) calculated for all events are vertically averaged over events to obtain the average scaled factorial moment, $\langle F_q \rangle$.

In order to correct for the non-flatness of the η-distributions in the considered central rapidity region, the averaged scaled factorial moments $\langle F_q \rangle$ [62] are divided by the factor R_q [67], given by

$$R_q = \frac{1}{M} \sum_{i=1}^{M} \frac{M^q \langle n_i \rangle^q}{\langle n \rangle^q} , \tag{3}$$

where

$$\langle n_i \rangle = \frac{1}{N} \sum_{j=1}^{N} n_{i,j} \tag{4}$$

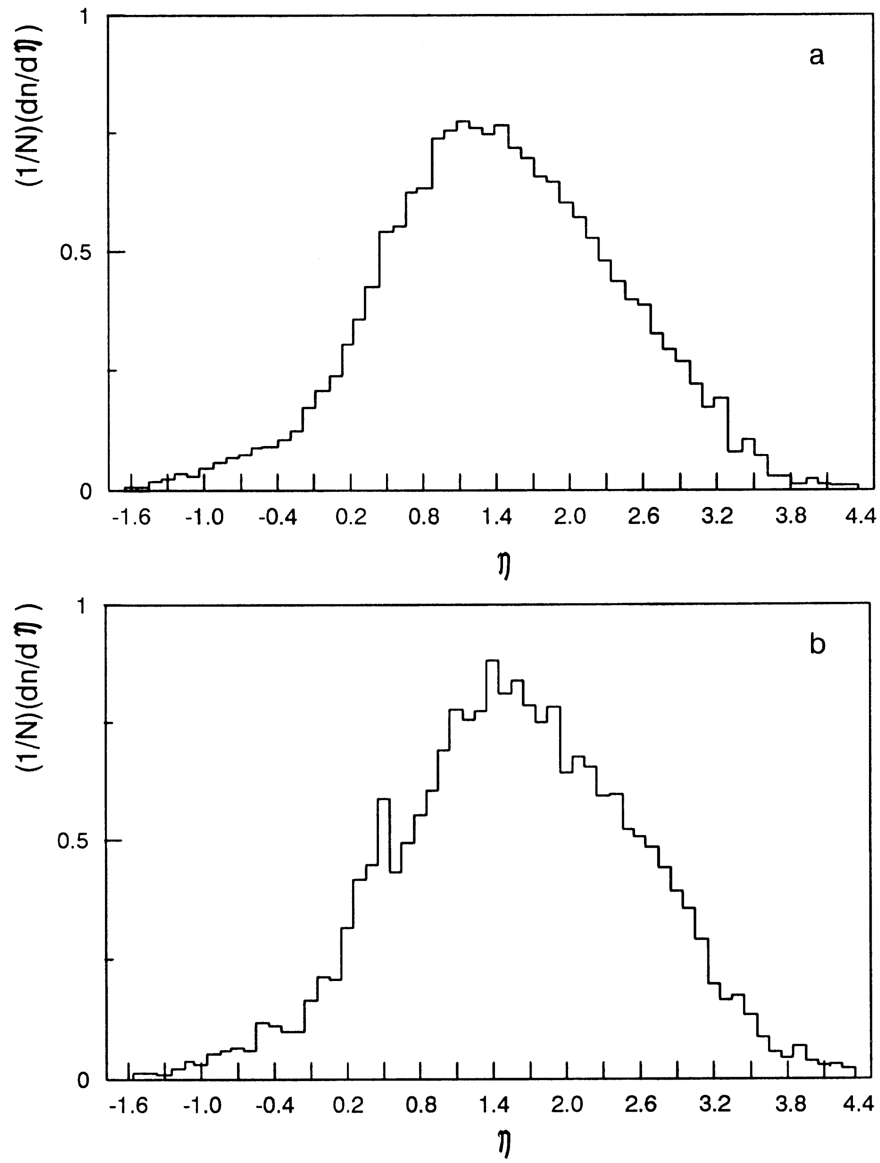

Fig. 1. – Pseudorapidity distribution of the produced charged pions in a) 4.1A GeV/c ^{22}Ne + Ag(Br) and b) 4.5A GeV/c ^{28}Si + Ag(Br) interactions.

and N is the number of events in the considered sample. Thus, the reduced scaled factorial moments are given by

$$\langle F_{qR} \rangle = \frac{\langle F_q \rangle}{R_q} \ . \tag{5}$$

The dependence of the average reduced scaled factorial moment F_{qR} of order q on the

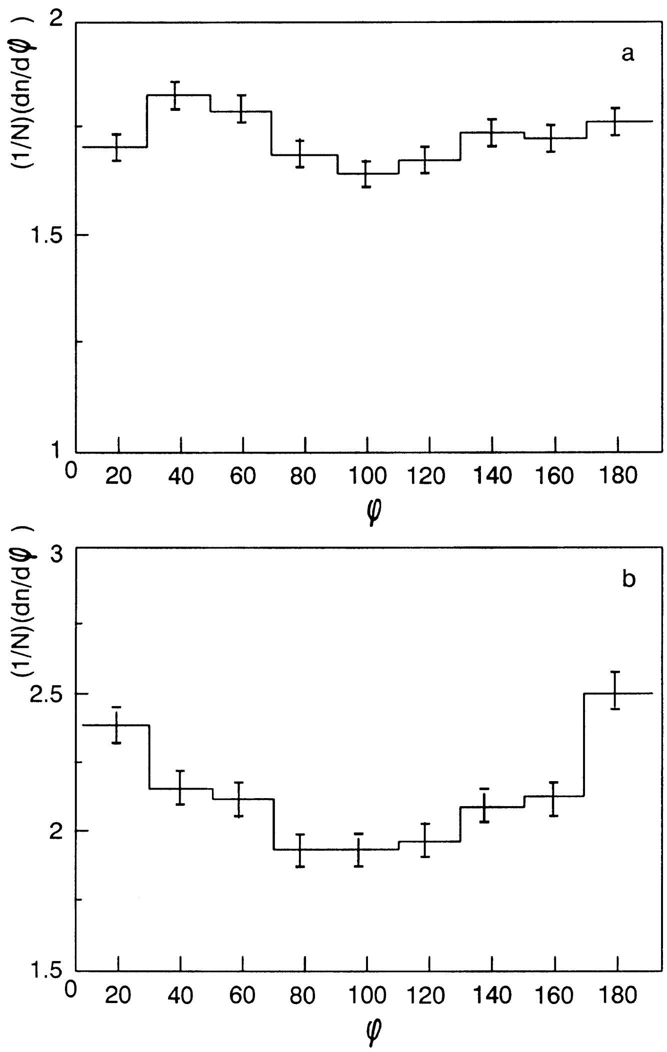

Fig. 2. – Azimuthal angle disribution of the produced charged pions in a) 4.1A GeV/c ^{22}Ne + Ag(Br) and b) 4.5A GeV/c ^{28}Si + Ag(Br) interactions.

bin size δi ($i = \eta$, ϕ and $\eta - \phi$) is given by

$$\ln \langle F_{qR}(\delta i) \rangle = -a_q \ln(\delta i) + b_q\,, \tag{6}$$

where a_q and b_q represent the slope and the intercept of the fitting straight line respectively, are constants for a given q.

All the further analyses for any dependence of the η-distribution on the number of bins are done using the data corrected for the non-flatness of the η-distribution. The dependence of the average reduced scaled factorial moments $\langle F_{qR} \rangle$ on the bin size $\delta\eta$ for the ^{22}Ne and the ^{28}Si projectiles respectively, for $q = 2, 3, \ldots$ and 6, together with the corresponding best fit curves can be seen from fig. 3a), b). The data, in fig. 3, may

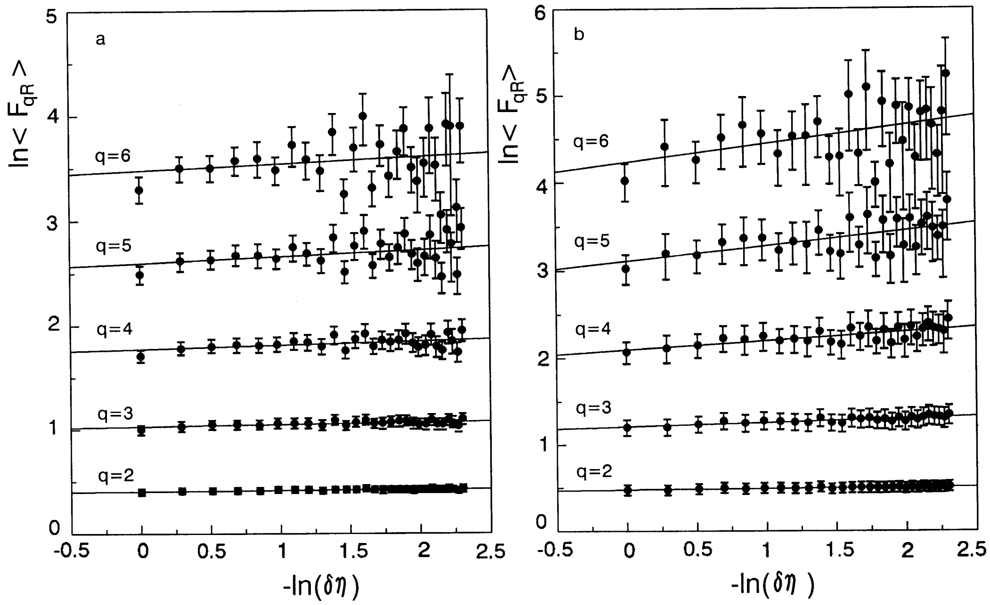

Fig. 3. – Average reduced scaled factorial moments as a function of the bin size $\delta\eta$ for the produced charged pions in a) 4.1A GeV/c ^{22}Ne + Ag(Br) and b) 4.5A GeV/c ^{28}Si + Ag(Br) interactions.

be described by the power law given by eq. (6) and the slopes of the straight lines represent the corrected intermittency exponents which may be used for a quantitative analysis.

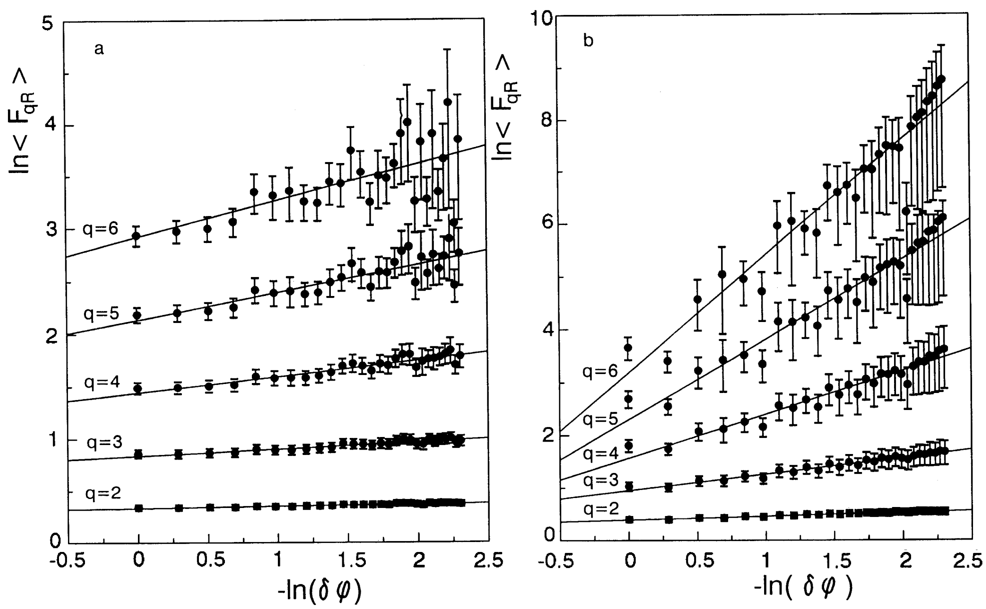

Fig. 4. – Average reduced scaled factorial moments as a function of the bin size $\delta\phi$ in a) 4.1A GeV/c ^{22}Ne + Ag(Br) and b) 4.5A GeV/c ^{28}Si + Ag(Br) interactions.

The intermittency analysis has been performed in the azimuthal angle ϕ space around the beam axis as a variable for the produced charged particles in the interval $\phi = 0$–2π ($\Delta\phi = 2\pi$) using eqs. (2)-(6) for ϕ instead of η, for the considered reactions.

Figure 4a), b) shows the relation between the average reduced scaled factorial moments $\langle F_{qR} \rangle$ and the bin size $\delta\phi$ for $q = 2$–6. The average multiplicities, in the considered $\delta\phi$ region, for the studied reactions are 17.31 ± 0.28 and 19.25 ± 0.53, respectively. From figs. 3a), b) and 4a), b) together with the data in the table, it can be seen that the strength of the intermittency, measured by the values of the slope parameter, increases with the order q. The intermittency effect is more pronounced in the ϕ-space than in the η-space. This may be due to the fact that the ϕ-distribution is more flat than the considered η-distribution.

4. – Two-dimensional analysis in the pseudorapidity and azimuthal angle space

Ochs and Wosiek [55, 68] have suggested that the intermittency effects should be more evident if the fluctuations are analyzed simultaneously in the two-dimensional space ($\eta - \phi$). For a quantitative analysis of the normalized multiplicity moments in the two-dimensional space, the ranges of η and ϕ are considered to be $\Delta\eta = 3$ and $\Delta\phi = 2\pi$. The $\Delta\eta\,\Delta\phi$ space has been divided into $M_{\eta-\phi} = M_\eta M_\phi$ intervals and $\delta\eta\,\delta\phi = \Delta\eta\,\Delta\phi / M_\eta M_\phi$ with $M_\eta = M_\phi$.

In the two-dimensional analysis, the reduced scaled factorial moments for the interval $\delta\eta\,\delta\phi$ are calculated similarly as given in eqs. (2)-(6). Figure 5a), b) shows $\ln\langle F_{qR} \rangle$ as a function of $-\ln(\delta\eta \cdot \delta\phi)$ for the considered data for $q = 2$–6. As can be

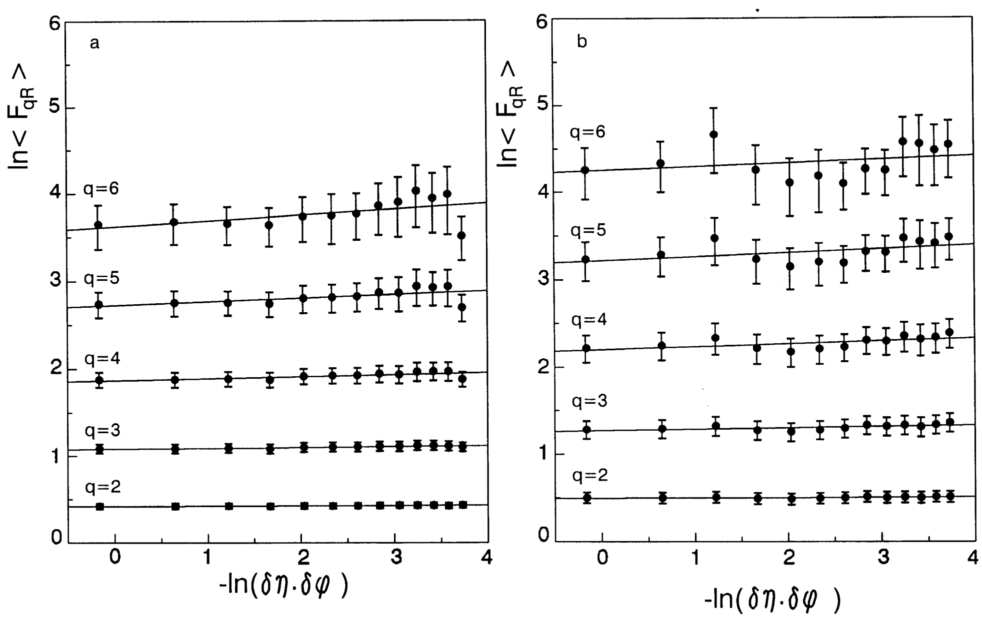

Fig. 5. – Average reduced scaled factorial moments as a function of the bin size $\delta\eta \cdot \delta\phi$ for the produced charged pions in a) 4.1A GeV/c ^{22}Ne + Ag(Br) and b) 4.5A GeV/c ^{28}Si + Ag(Br) interactions.

seen in the figure, the values of the slope parameter, a_q, are compatible with zero, however, the general trend shows that the values of a_q increase with q. This result does not contradict those of refs. [11, 12, 41] which show significant values for a_q. This indicates that the suggestion of refs. [55, 68] is still a matter of debate and more investigations are required.

5. – Anomalous fractal dimension and intermittency parameter

The generalized dimensions D_q of the fractal and the multifractals are related to the intermittency exponents [69] as

$$D_q = 1 - d_q\,, \tag{7}$$

where

$$d_q = \frac{a_q}{q-1} \tag{8}$$

are known as the anomalous fractal dimensions. The analysis of the present experimental data in fig. 6a), b) shows that the magnitude of the anomalous fractal dimension increases approximately linearly with the increase of the order of the moments for η, ϕ and $(\eta - \phi)$ spaces.

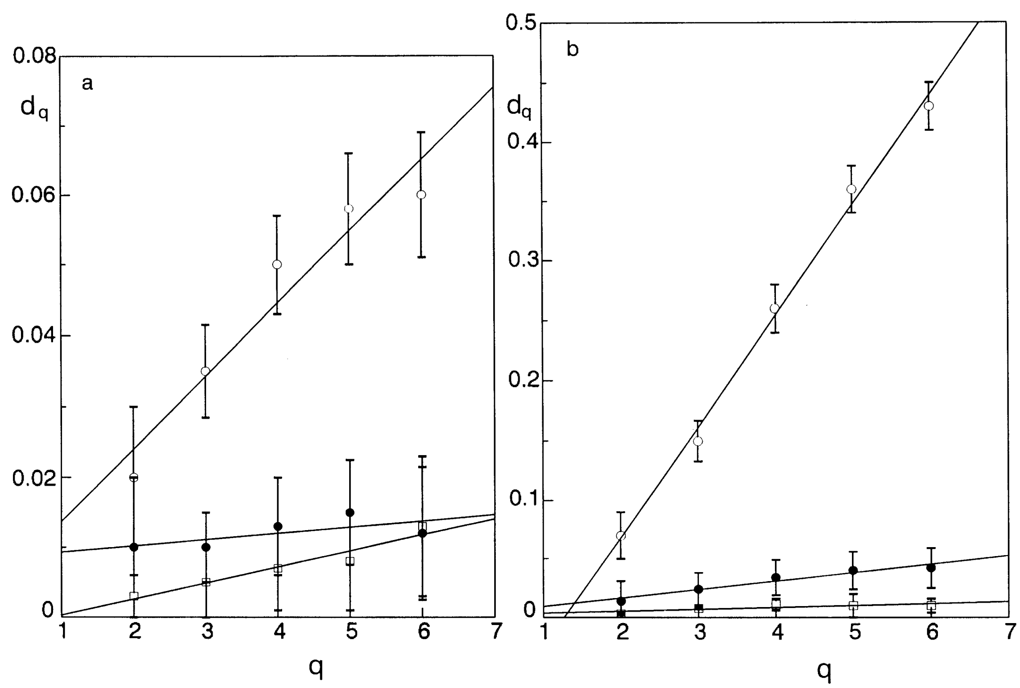

Fig. 6. – Relation between the anomalous fractal dimension d_q and the order of the moment q for the produced charged pions in a) 4.1A GeV/c ^{22}Ne + Ag(Br) and b) 4.5A GeV/c ^{28}Si + Ag(Br) interactions in η (dots), ϕ (open dots) and $\eta - \phi$ (squares) spaces.

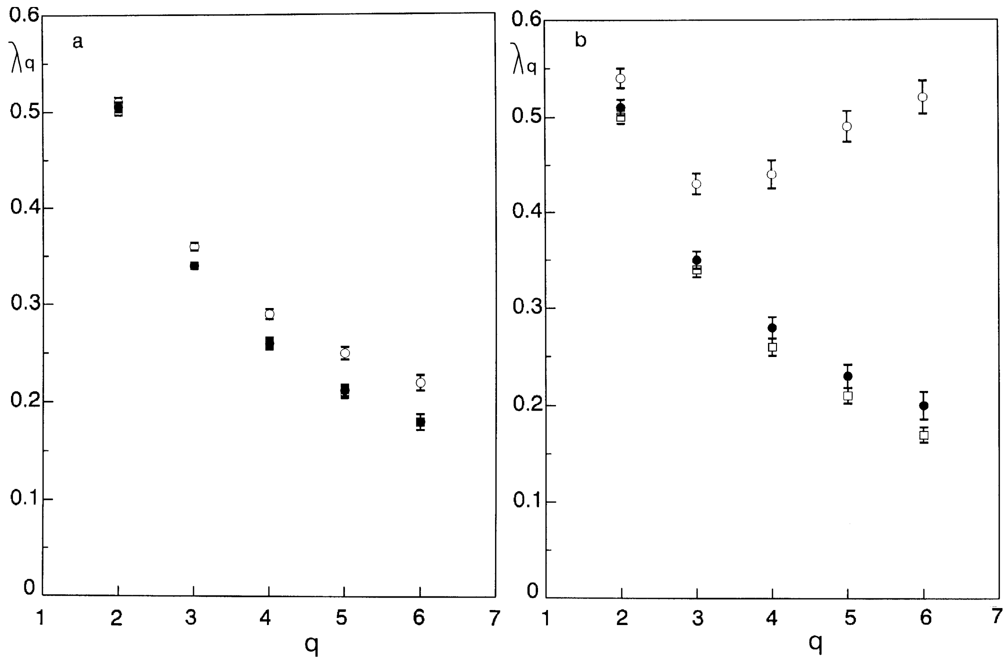

Fig. 7. – Relation between the intermittency parameter λ_q and the order of the moment q for a) the 4.1A GeV/c ^{22}Ne + Ag(Br) and b) the 4.5A GeV/c ^{28}Si interactions in the η (dots), ϕ (open dots) and $\eta - \phi$ (squares) spaces.

It has been suggested that the self-similar cascade can occur in different phases [56], namely the normal phase populated by many relatively small fluctuations and the spin glass phase consisting of few very large fluctuations. The condition for the coexistence of the two phases of the cascade is the presence of a minimum of the intermittency parameter λ_q at a certain value q_c for the order q. The value of λ_q is given by [70, 71]

$$\lambda_q = \frac{a_q + 1}{q} . \tag{9}$$

The regions $q < q_c$ and $q > q_c$ correspond to the normal and to the spin glass phases, respectively. Self-similar multiparticle systems are seen to behave differently in these two regions [70, 71]. Figure 7a), b) shows the relation between the intermittency parameter λ_q and the order q for ^{22}Ne and ^{28}Si, respectively in the η, ϕ and $(\eta - \phi)$ spaces. It can be seen that the value of λ_q decreases with the increase of the order q which agrees with refs. [12, 13, 41]. It may be noticed that the points of η coincide with those of $\eta - \phi$ for the ^{22}Ne reaction, while they are very near to each other for the ^{28}Si interactions. A slight minimum is observed for the ^{28}Si only in the ϕ-space. This may indicate the coexistence of two different phases, *i.e.* the normal and the spin glass phases. However, a strong evidence for this coexistence of the two phases cannot be claimed due to the fact that the minimum has not been observed for the ^{22}Ne and the ^{28}Si reactions in the η and in the $\eta - \phi$ spaces.

6. – Conclusion

A significant statistical study of 4.1*A* GeV/c ^{22}Ne and 4.5*A* GeV/c ^{28}Si interactions with Ag(Br) emulsion nuclei has been carried out. From the present study, it may be concluded that the intermittent behaviour is observed up to the value 6 of the order of the reduced scaled factorial moment in the considered reactions for the pseudorapidity and the azimuthal angle space. For the two-dimensional pseudorapidity-azimuthal angle space it lacks more investigations. For all the considered space, the anomalous fractal dimension increases with the increase of the order of the reduced scaled factorial moment particularly, this is very obvious, in the azimuthal angle space. In the relation between the intermittency parameter and the order of the reduced scaled factorial moment, no statistically significant minimum has been observed, except for the ^{28}Si in the ϕ-space, consequently one cannot claim evidently the coexistence of two different phases.

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TABLE I. – *Values of the intermittency exponent (slope parameter)* a_q *for produced charged pions from interactions in emulsion of diff* *projectiles*: 4.1A GeV/c ^{22}Ne, 4.5A GeV/c ^{28}Si (*present work*), 2.1A GeV/c ^{16}O [11], 800 GeV/c *protons* [12], 200A GeV/c ^{32}S, 200A GeV/c 60A GeV/c ^{16}O *and* 14.5A GeV/c ^{28}Si [41].

Beam, E	Variables	a_2	a_3	a_4	a_5	a_6
^{22}Ne, 4.1	$\Delta\eta = 3$, $N_h > 7$	0.010 ± 0.010	0.020 ± 0.010	0.038 ± 0.022	0.060 ± 0.030	0.057 ± 0
	$\Delta\phi = 2\pi$, $N_h > 7$	0.020 ± 0.010	0.070 ± 0.013	0.150 ± 0.020	0.233 ± 0.032	0.290 ± 0
	$\Delta\eta - \Delta\phi$, $N_h > 7$	0.003 ± 0.007	0.010 ± 0.010	0.020 ± 0.020	0.030 ± 0.030	0.050 ± 0
^{28}Si, 4.5	$\Delta\eta = 3$, $N_h > 7$	0.014 ± 0.017	0.048 ± 0.028	0.102 ± 0.044	0.161 ± 0.063	0.210 ± 0
	$\Delta\phi = 2\pi$, $N_h > 7$	0.070 ± 0.020	0.300 ± 0.034	0.770 ± 0.060	1.430 ± 0.082	2.150 ± 0
	$\Delta\eta - \Delta\phi$, $N_h > 7$	0.003 ± 0.014	0.016 ± 0.023	0.033 ± 0.035	0.040 ± 0.040	0.030 ± 0
^{28}Si, 14.5	$\Delta\eta = 3$, $N_h > 0$	0.006 ± 0.020	0.019 ± 0.080	0.039 ± 0.19	0.072 ± 0.370	
^{32}S, 200	$\Delta\eta = 4$, $N_h > 15$	0.005 ± 0.001	0.014 ± 0.020	0.033 ± 0.005	0.069 ± 0.012	0.131 ± 0
	$\Delta\phi = 2\pi$, $N_h > 15$	0.015 ± 0.001	0.050 ± 0.005	0.171 ± 0.012	0.230 ± 0.029	0.371 ± 0
	$\Delta\eta - \Delta\phi$, $N_h > 15$	0.011 ± 0.001	0.033 ± 0.003	0.065 ± 0.008	0.103 ± 0.020	
^{16}O, 200	$\Delta\eta = 4$, $N_h > 15$	0.007 ± 0.001	0.023 ± 0.002	0.046 ± 0.006	0.086 ± 0.012	0.156 ± 0
	$\Delta\phi = 2\pi$, $N_h > 15$	0.025 ± 0.001	0.082 ± 0.005	0.221 ± 0.017	0.517 ± 0.047	0.917 ± 0
	$\Delta\eta - \Delta\phi$, $N_h > 15$	0.013 ± 0.002	0.071 ± 0.010	0.307 ± 0.046	0.823 ± 0.098	
^{16}O, 60	$\Delta\eta = 4$, $N_h > 15$	0.010 ± 0.001	0.024 ± 0.005	0.049 ± 0.014	0.106 ± 0.032	0.197 ± 0
	$\Delta\phi = 2\pi$, $N_h > 15$	0.020 ± 0.002	0.059 ± 0.005	0.115 ± 0.012	0.195 ± 0.032	0.268 ± 0
	$\Delta\eta - \Delta\phi$, $N_h > 15$	0.008 ± 0.002	0.045 ± 0.008	0.191 ± 0.022	0.522 ± 0.051	
^{16}O, 2.1	$\Delta\eta = 4$, $N_h > 7$	0.091 ± 0.010	0.373 ± 0.041	0.827 ± 0.098		
	$\Delta\phi = 2\pi$, $N_h > 7$	0.184 ± 0.021	0.607 ± 0.058	0.983 ± 0.078		
	$\Delta\eta - \Delta\phi$, $N_h > 7$	0.353 ± 0.035	0.785 ± 0.102	1.088 ± 0.130		
p, 800	$\Delta\eta = 5$, $11 \leqslant N_s \leqslant 20$	0.110 ± 0.004	0.317 ± 0.013	0.619 ± 0.028	0.978 ± 0.057	1.316 ± 0
	$\Delta\eta = 5$, $21 \leqslant N_s \leqslant 30$	0.054 ± 0.003	0.158 ± 0.011	0.341 ± 0.028	0.638 ± 0.067	1.030 ± 0
	$\Delta\eta = 5$, $N_s \geqslant 31$	0.027 ± 0.003	0.066 ± 0.008	0.117 ± 0.019	0.181 ± 0.039	0.260 ± 0
	$\Delta\phi = 2\pi$, $11 \leqslant N_s \leqslant 20$	0.101 ± 0.005	0.283 ± 0.015	0.554 ± 0.037	0.895 ± 0.081	1.227 ± 0
	$\Delta\phi = 2\pi$, $21 \leqslant N_s \leqslant 30$	0.070 ± 0.004	0.189 ± 0.011	0.398 ± 0.027	0.745 ± 0.059	1.208 ± 0
	$\Delta\phi = 2\pi$, $N_s \geqslant 31$	0.055 ± 0.003	0.134 ± 0.008	0.225 ± 0.019	0.320 ± 0.039	0.400 ± 0
	$\Delta\eta - \Delta\phi$, $11 \leqslant N_s \leqslant 20$	0.223 ± 0.005	0.562 ± 0.015	0.946 ± 0.053	1.252 ± 0.156	
	$\Delta\eta - \Delta\phi$, $21 \leqslant N_s \leqslant 30$	0.152 ± 0.004	0.404 ± 0.022	0.721 ± 0.058	1.007 ± 0.106	
	$\Delta\eta - \Delta\phi$, $N_s \geqslant 31$	0.121 ± 0.004	0.298 ± 0.014	0.536 ± 0.058	0.738 ± 0.154	